Critical Relaxed Stable Matchings with Two-Sided Ties

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joint work with Meghana Nasre (IITM) and Prajakta Nimbhorkar(CMI)

CS Theory Seminar

(Chennai Mathematical Institute, Chennai)

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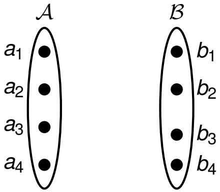
Problem Setup

Stable Marriage Problem with Ties and Critical Agents

- A bipartite graph $G = (\mathcal{A} \cup \mathcal{B}, E)$
  - **Vertex set $\mathcal{A} \cup \mathcal{B}$:** $\mathcal{A} = \{a_1, \ldots, a_{n_1}\}$ $\mathcal{B} = \{b_1, \ldots, b_{n_2}\}$



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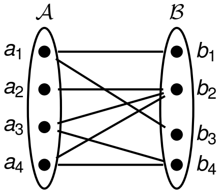

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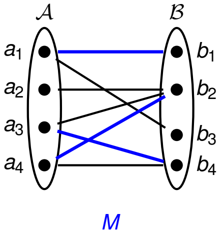

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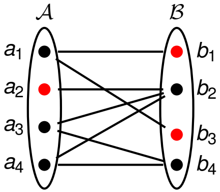

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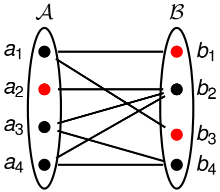

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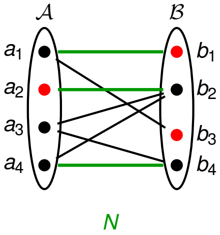

N matches two critical nodes a_2 and b_1

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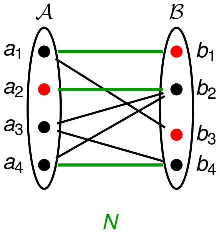

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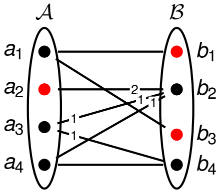

a_3 and a_4 are in a **tie** at **rank 1** for b_2

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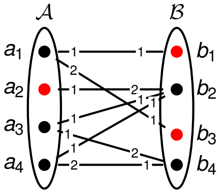

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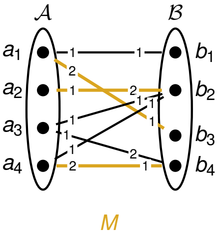

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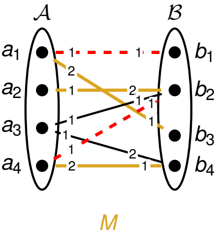

(a_1, b_1) and (a_4, b_2) are blocking pairs

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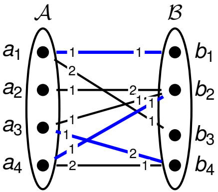

M_s

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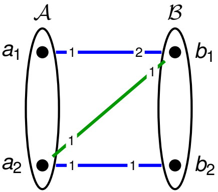

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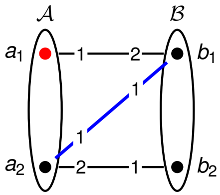

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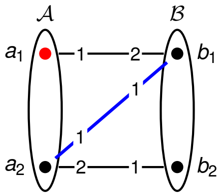

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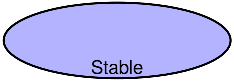

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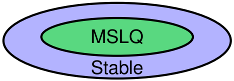

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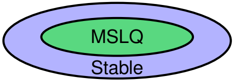

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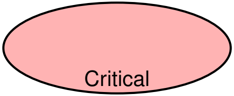

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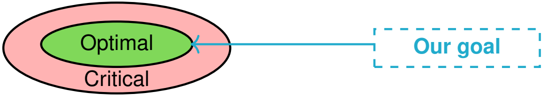

Relaxed Stability [Krishnaa et al., 2020]

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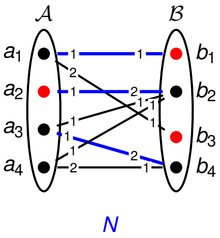

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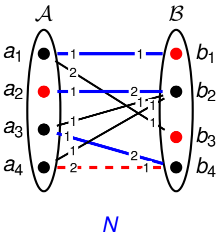

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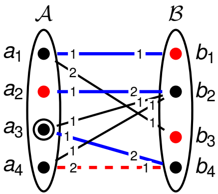

N is not an RSM

(a_4, b_4) is not justified

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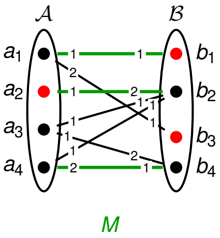

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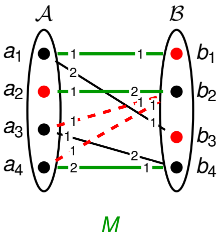
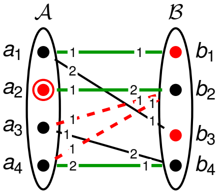


(a_3, b_2) and (a_4, b_2) are blocking pairs

Relaxed Stability

[Krishnaa et al., 2020]

- A matching M is **Relaxed Stable Matching** (RSM) if for every blocking pair (a, b)
  - a is matched to a critical node **or**
  - b is matched to a critical node



M is an RSM

Both blocking pairs are **justified** by b_2

Relaxed Stability

[Krishnaa et al., 2020]

Our goal

- Compute a matching that is **critical** and **relaxed stable**

Relaxed Stability

[Krishnaa et al., 2020]

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- Compute a matching that is **critical** and **relaxed stable**

Observation

- Computing max-size critical Relaxed Stable Matching (RSM) is **NP-hard**

Relaxed Stability

[Krishnaa et al., 2020]

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- Compute a matching that is **critical** and **relaxed stable**

Observation

- Computing max-size critical Relaxed Stable Matching (RSM) is **NP-hard**
  - **Ignore** critical nodes

Our goal

- Compute a matching that is **critical** and **relaxed stable**

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- Computing max-size critical Relaxed Stable Matching (RSM) is **NP-hard**
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  - Problem is **same** as computing max-size stable matching

Relaxed Stability
[Krishnaa et al., 2020]

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- Does a critical relaxed stable matching always exist?

Relaxed Stability

[Krishnaa et al., 2020]

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- Can it be efficiently computed?

Relaxed Stability

[Krishnaa et al., 2020]

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Relaxed Stability

[Krishnaa et al., 2020]

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- Can it be efficiently computed?
- The **best-known** approximation ratio for **max-size stable** matching is $\frac{3}{2}$
  - Can we achieve $\frac{3}{2}$-approximation for max-size critical RSM?

Relaxed Stability

[Krishnaa et al., 2020]

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  - **Ignore** critical nodes
  - Problem is **same** as computing max-size stable matching

- Does a critical relaxed stable matching always exist? ✓
- Can it be efficiently computed? ✓
- The **best-known** approximation ratio for **max-size stable** matching is $\frac{3}{2}$
  - Can we achieve $\frac{3}{2}$-approximation for max-size critical RSM? ✓

Background

Gale-Shapley algorithm [Gale and Shapley, 1962]

- No critical nodes and no ties
- Well-known **linear-time** algorithm for stable matching

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- Vertices in $\mathcal{A}$ propose and vertices in $\mathcal{B}$ accept/reject

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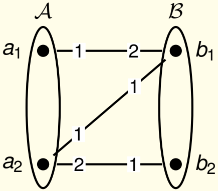

Gale-Shapley algorithm [Gale and Shapley, 1962]

- No critical nodes and no ties
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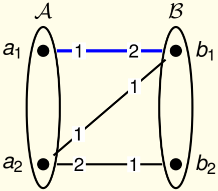


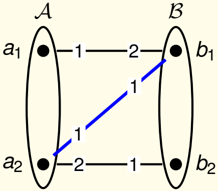
$a_1 \rightarrow b_1$ ✓

b_1 was **unmatched** and hence **accepts** a_2's proposal

Gale-Shapley algorithm

[Gale and Shapley, 1962]

- No critical nodes and no ties
- Well-known **linear-time** algorithm for stable matching
- Vertices in $\mathcal{A}$ propose and vertices in $\mathcal{B}$ accept/reject



$a_1 \rightarrow b_1$ X

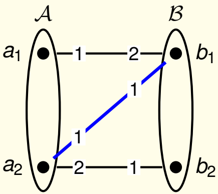
$a_2 \rightarrow b_1$ ✓

b_1 **prefers** a_2 over a_1

Gale-Shapley algorithm

[Gale and Shapley, 1962]

- No critical nodes and no ties
- Well-known **linear-time** algorithm for stable matching
- Vertices in $\mathcal{A}$ propose and vertices in $\mathcal{B}$ accept/reject



$a_1 \rightarrow b_1$ X

$a_2 \rightarrow b_1$ ✓

All $a \in \mathcal{A}$ are either **matched** or **exhausted** their preference list

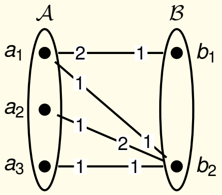
Gale-Shapley algorithm

[Gale and Shapley, 1962]

- No critical nodes and no ties
- Well-known **linear-time** algorithm for stable matching
- Vertices in $\mathcal{A}$ propose and vertices in $\mathcal{B}$ accept/reject
- Algorithm outputs a stable matching M

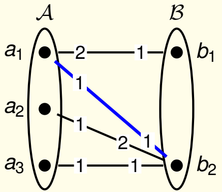
Ties in Preference Lists

- Tied lists on the receiving ($\mathcal{B}$) side and strict list on the proposing ($\mathcal{A}$) side



Király's algorithm: Version I [Király, 2011]

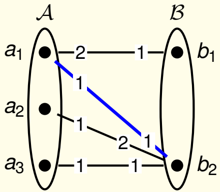
- Tied lists on the receiving ($\mathcal{B}$) side and strict list on the proposing ($\mathcal{A}$) side
- Execute Gale-Shapley algorithm



$a_1 \rightarrow b_2$ ✓

b_2 was **unmatched** and hence **accepts** a_1's proposal

- Tied lists on the receiving ($\mathcal{B}$) side and strict list on the proposing ($\mathcal{A}$) side
- Execute Gale-Shapley algorithm



$a_1 \rightarrow b_2$ ✓

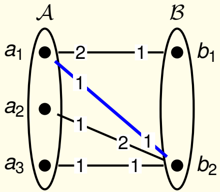
$a_2 \rightarrow b_2$ X

b_2 **rejects** a_2's proposal as a_2 is **not better** than a_1 for b_2

Király's algorithm: Version I

[Király, 2011]

- Tied lists on the receiving ($\mathcal{B}$) side and strict list on the proposing ($\mathcal{A}$) side
- Execute Gale-Shapley algorithm



$a_1 \rightarrow b_2$ ✓

$a_2 \rightarrow b_2$ X

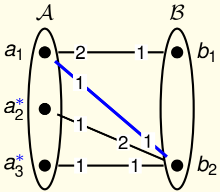
$a_3 \rightarrow b_2$ X

b_2 **rejects** a_3's proposal as a_3 is **not better** than a_1 for b_2

Király's algorithm: Version I

[Király, 2011]

- Tied lists on the receiving ($\mathcal{B}$) side and strict list on the proposing ($\mathcal{A}$) side
- Execute Gale-Shapley algorithm
- Unmatched vertices on $\mathcal{A}$-side gets one more chance to propose with a '$*$' status



$a_1 \rightarrow b_2$ ✓

$a_2 \rightarrow b_2$ X

$a_3 \rightarrow b_2$ X

a_2 and a_3 get '$*$' **status** as they exhausted their preference lists

Király's algorithm: Version I

[Király, 2011]

- Tied lists on the receiving ($\mathcal{B}$) side and strict list on the proposing ($\mathcal{A}$) side
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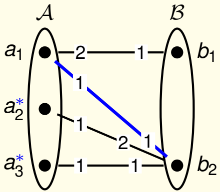
- Unmatched vertices on $\mathcal{A}$-side gets one more chance to propose with a '$*$' status
- $*$ status of a improves its rank by ϵ in all its neighbours' preference lists ($1 > \epsilon > 0$)



$a_1 \rightarrow b_2$ ✓

$a_2 \rightarrow b_2$ X

$a_3 \rightarrow b_2$ X

$a_2^* \rightarrow b_2$ X

b_2 **rejects** a_2^*'s proposal as a_2^* is **not better** than a_1 for b_2

Király's algorithm: Version I

[Király, 2011]

- Tied lists on the receiving ($\mathcal{B}$) side and strict list on the proposing ($\mathcal{A}$) side
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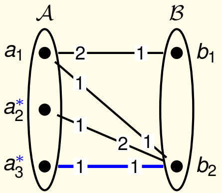


$a_1 \rightarrow b_2$ X

$a_2 \rightarrow b_2$ X

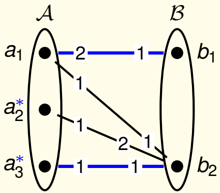
$a_3 \rightarrow b_2$ X

$a_2^* \rightarrow b_2$ X

$a_3^* \rightarrow b_2$ ✓

b_2 **accepts** a_3^*'s proposal and rejects a_1 as a_3^* is better than a_1 for b_2

- Tied lists on the receiving ($\mathcal{B}$) side and strict list on the proposing ($\mathcal{A}$) side
- Execute Gale-Shapley algorithm
- Unmatched vertices on $\mathcal{A}$-side gets one more chance to propose with a '$*$' status
- $*$ status of a improves its rank by ϵ in all its neighbours' preference lists ($1 > \epsilon > 0$)



$a_1 \rightarrow b_2$ X
$a_2 \rightarrow b_2$ X
$a_3 \rightarrow b_2$ X
$a_2^* \rightarrow b_2$ X
$a_3^* \rightarrow b_2$ ✓
$a_1 \rightarrow b_1$ ✓

b_1 was **unmatched** and hence **accepts** a_1's proposal

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[Király, 2011]

- Tied lists on the receiving ($\mathcal{B}$) side and strict list on the proposing ($\mathcal{A}$) side
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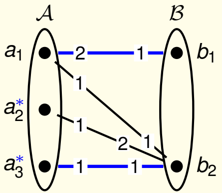
- Unmatched vertices on $\mathcal{A}$-side gets one more chance to propose with a '$*$' status
- $*$ status of a improves its rank by ϵ in all its neighbours' preference lists ($1 > \epsilon > 0$)
- $\frac{3}{2}$-approximation algorithm of the max-size stable matching



$a_1 \rightarrow b_2$ X

$a_2 \rightarrow b_2$ X

$a_3 \rightarrow b_2$ X

$a_2^* \rightarrow b_2$ X

$a_3^* \rightarrow b_2$ ✓

$a_1 \rightarrow b_1$ ✓

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Király's algorithm: Version II

- Tied lists on the proposing ($\mathcal{A}$) side and strict lists on the receiving ($\mathcal{B}$) side

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- Tied lists on the proposing ($\mathcal{A}$) side and strict lists on the receiving ($\mathcal{B}$) side
  - **Uncertain proposal** (a, b)**:** b is k^{th}-ranked nbr of a, $\exists b' \neq b$ at rank k, and b' is unmatched

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    - (ii) if all vertices at rank k are matched, then b is **unproposed** vertex by a or

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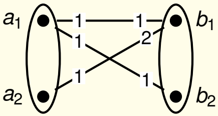

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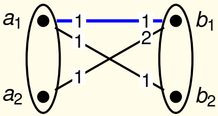

$a_1 \rightarrow b_1$ ✓

b_1 was **unmatched** and hence **accepts** a_1's proposal

Király's algorithm: Version II

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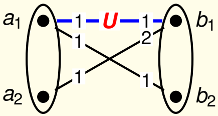

$a_1 \rightarrow b_1$ ✓

(a_1, b_1) is **uncertain**

Király's algorithm: Version II

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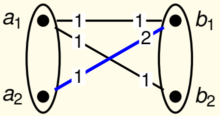
a_1 marks b_1



$a_1 \rightarrow b_1$ X

$a_2 \rightarrow b_1$ ✓

b_1 **rejects** a_1 and accepts a_2's proposal as (a_1, b_1) was **uncertain**

Király's algorithm: Version II

- Tied lists on the proposing ($\mathcal{A}$) side and strict lists on the receiving ($\mathcal{B}$) side
  - **Uncertain proposal (a, b):** b is k^{th}-ranked nbr of a, $\exists b' \neq b$ at rank k, and b' is unmatched
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  - **Favourite nbr b of a:** $k =$ min rank for a at which marked or unmatched nbrs exist
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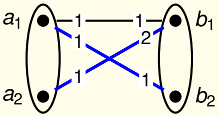
a_1 marks b_1



$a_1 \rightarrow b_1$ X

$a_2 \rightarrow b_1$ ✓

$a_1 \rightarrow b_2$ ✓

b_2 was **unmatched** and hence **accepts** a_1's proposal

Király's algorithm [Király, 2013]

- Tied lists on both sides

Király's algorithm [Király, 2013]

- Tied lists on both sides
  - **Combine** the two ideas: **Version I** and **Version II**

Király's algorithm [Király, 2013]

- Tied lists on both sides
  - **Combine** the two ideas: **Version I** and **Version II**
  - Run the algorithm for tied lists on the proposing ($\mathcal{A}$) side

Király's algorithm

[Király, 2013]

- Tied lists on both sides
  - **Combine** the two ideas: **Version I** and **Version II**
  - Run the algorithm for tied lists on the proposing ($\mathcal{A}$) side
  - If any $a \in \mathcal{A}$ remained unmatched then it gets a $*$ status

Király's algorithm

[Király, 2013]

- Tied lists on both sides
  - **Combine** the two ideas: **Version I** and **Version II**
  - Run the algorithm for tied lists on the proposing ($\mathcal{A}$) side
  - If any $a \in \mathcal{A}$ remained unmatched then it gets a $*$ status
  - Stop when each $a \in \mathcal{A}$ is either matched or exhausted its pref list as a $*$ status vertex

Király's algorithm

[Király, 2013]

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- b must be matched

Király's algorithm [Király, 2013]

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- Suppose $\exists$ a blocking pair (a, b) w.r.t. the output matching M
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- a must be matched: If not, a must have proposed to b at least twice
  - $b \in \mathcal{B}$ cannot be in an uncertain proposal after receiving its second proposal
  - $M(b)$ is not worse than a – contradiction

Király's algorithm

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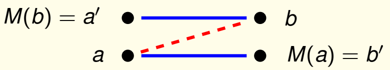

Király's algorithm

[Király, 2013]

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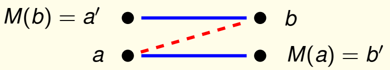
- a must have proposed to b and b rejected it
  - (a, b) was not uncertain: $a' = M(b)$ is not worse than a for b



Király's algorithm

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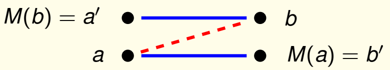
  - (a, b) was not uncertain: $a' = M(b)$ is not worse than a for b
  - (a, b) was uncertain: a proposed b again before proposing to $M(a)$



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  - Output matching M is a **stable** matching
  - $\frac{3}{2}$**-approximation** of maximum size stable matching

Király's algorithm

[Király, 2013]

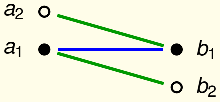
$\frac{3}{2}$-approximation of maximum size stable matching M^*

Proof idea: No 1 or 3-length aug-path w.r.t. M in $(M \oplus M^*)$

$\frac{3}{2}$-approximation of maximum size stable matching M^*

Proof idea: No 1 or 3-length aug-path w.r.t. M in $(M \oplus M^*)$

- Suppose such 3-length augmenting path exists



Király's algorithm [Király, 2013]

$\frac{3}{2}$-approximation of maximum size stable matching M^*

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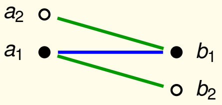
b_2 is unmatched in M



Király's algorithm

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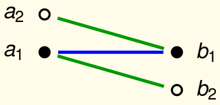


b_2 is unmatched in M

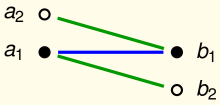
No proposal sent to b_2

a_1 without $*$ status

$\frac{3}{2}$-approximation of maximum size stable matching M^*

Proof idea: No 1 or 3-length aug-path w.r.t. M in $(M \oplus M^*)$

- Suppose such 3-length augmenting path exists
- **Claim 1:** a_1 prefers b_1 over b_2



$\boxed{\text{No proposal sent to } b_2}$

$\boxed{a_1 \text{ without } * \text{ status}}$

$\frac{3}{2}$-approximation of maximum size stable matching M^*

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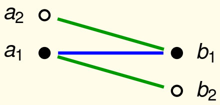
- Suppose such 3-length augmenting path exists
- **Claim 1:** a_1 prefers b_1 over b_2
  - Suppose not then b_1 and b_2 are tied



No proposal sent to b_2

a_1 without $*$ status

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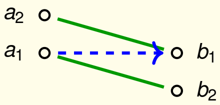

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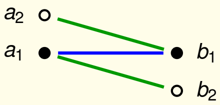

No proposal sent to b_2

a_1 without $*$ status

(a_1, b_1) is uncertain

$\frac{3}{2}$-approximation of maximum size stable matching M^*

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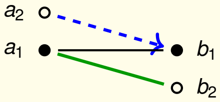
- Suppose such 3-length augmenting path exists
- **Claim 1:** a_1 prefers b_1 over b_2
  - Suppose not then b_1 and b_2 are tied
  - b_1 received other proposals



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(a_1, b_1) is uncertain

Király's algorithm

[Király, 2013]

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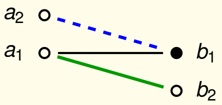

No proposal sent to b_2

a_1 without $*$ status

(a_1, b_1) is uncertain

b_1 rejected a_1

$\frac{3}{2}$-approximation of maximum size stable matching M^*

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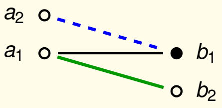
- Suppose such 3-length augmenting path exists
- **Claim 1:** a_1 prefers b_1 over b_2
  - Suppose not then b_1 and b_2 are tied
  - b_1 received other proposals
  - a_1 must propose b_2 before proposing to b_1



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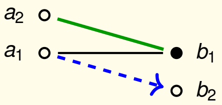


No proposal sent to b_2

a_1 without $*$ status

(a_1, b_1) is uncertain

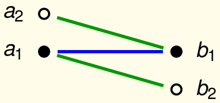
b_1 rejected a_1

a_1 proposed b_2

$\frac{3}{2}$-approximation of maximum size stable matching M^*

Proof idea: No 1 or 3-length aug-path w.r.t. M in $(M \oplus M^*)$

- Suppose such 3-length augmenting path exists
- **Claim 1:** a_1 prefers b_1 over b_2
- **Claim 2**: b_1 prefers a_1 over a_2



Király's algorithm

[Király, 2013]

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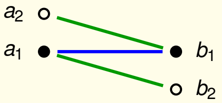
- Suppose such 3-length augmenting path exists
- **Claim 1:** a_1 prefers b_1 over b_2
- **Claim 2**: b_1 prefers a_1 over a_2
  - **Observation:** $b \in \mathcal{B}$ is not part of an uncertain proposal after receiving its second proposal



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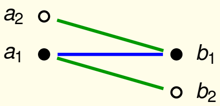
$\boxed{b_1 \text{ received} \geq 3 \text{ proposals}}$



$\frac{3}{2}$-approximation of maximum size stable matching M^*

Proof idea: No 1 or 3-length aug-path w.r.t. M in $(M \oplus M^*)$

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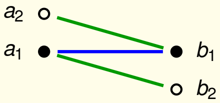

b_1 received ≥ 3 proposals

a_1 did not get $*$ status

$\frac{3}{2}$-approximation of maximum size stable matching M^*

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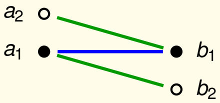


$\boxed{b_1 \text{ received} \geq 3 \text{ proposals}}$

$\boxed{a_1 \text{ did not get } * \text{ status}}$

$\boxed{b_1 \text{ rejected } a_2^* \text{ but not } a_1}$

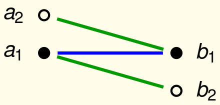
Király's algorithm

[Király, 2013]

$\frac{3}{2}$-approximation of maximum size stable matching M^*

Proof idea: No 1 or 3-length aug-path w.r.t. M in $(M \oplus M^*)$

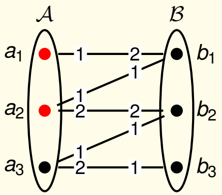
- Suppose such 3-length augmenting path exists
- **Claim 1:** a_1 prefers b_1 over b_2
- **Claim 2**: b_1 prefers a_1 over a_2
- M^* is not stable – a contradiction



Critical Nodes

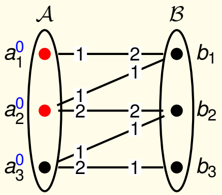
Feasible RSM: Multi-level Gale-Shapley algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$



Feasible RSM: Multi-level Gale-Shapley algorithm

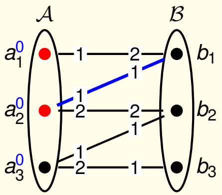
- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$
- All $a \in \mathcal{A}$ are at level 0 to begin with



All $a \in \mathcal{A}$ are **initially** at level **0**

Feasible RSM: Multi-level Gale-Shapley algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$
- All $a \in \mathcal{A}$ are at level 0 to begin with
- Execute Gale-Shapley algorithm



$a_2^0 \rightarrow b_1$ ✓

b_1 accepts a_2^0's proposal as it was unmatched

Feasible RSM: Multi-level Gale-Shapley algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$
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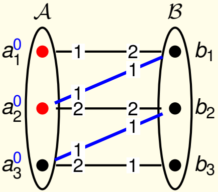
- Execute Gale-Shapley algorithm



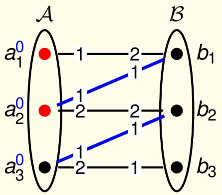
$a_2^0 \rightarrow b_1$ ✓

$a_3^0 \rightarrow b_2$ ✓

b_2 accepts a_3^0's proposal as it was unmatched

Feasible RSM: Multi-level Gale-Shapley algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$
- All $a \in \mathcal{A}$ are at level 0 to begin with
- Execute Gale-Shapley algorithm



$a_2^0 \rightarrow b_1$ ✓

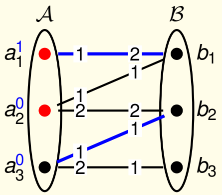
$a_3^0 \rightarrow b_2$ ✓

$a_1^0 \rightarrow b_1$ X

b_1 rejects a_1^0's proposal as $M(b_1) = a_2^0$ is better preferred

Feasible RSM: Multi-level Gale-Shapley algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$
- All $a \in \mathcal{A}$ are at level 0 to begin with
- Execute Gale-Shapley algorithm
- Unmatched critical vertices raise their level and propose again



$a_2^0 \rightarrow b_1$ X

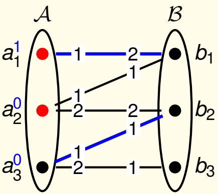
$a_3^0 \rightarrow b_2$ ✓

$a_1^1 \rightarrow b_1$ ✓

b_1 prefers a_1^1 as it is at a higher level

Feasible RSM: Multi-level Gale-Shapley algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$
- All $a \in \mathcal{A}$ are at level 0 to begin with
- Execute Gale-Shapley algorithm
- Unmatched critical vertices raise their level and propose again
- Vertices at higher level are preferred more by $b \in \mathcal{B}$ than those at lower level



$a_2^0 \rightarrow b_1$ X

$a_3^0 \rightarrow b_2$ ✓

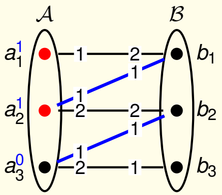
$a_1^1 \rightarrow b_1$ ✓

$a_2^0 \rightarrow b_2$ X

b_2 rejects a_2^0's proposal as $M(b_2) = a_3^0$ is better preferred

Feasible RSM: Multi-level Gale-Shapley algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$
- All $a \in \mathcal{A}$ are at level 0 to begin with
- Execute Gale-Shapley algorithm
- Unmatched critical vertices raise their level and propose again
- Vertices at higher level are preferred more by $b \in \mathcal{B}$ than those at lower level



$a_2^0 \rightarrow b_1$ X

$a_3^0 \rightarrow b_2$ ✓

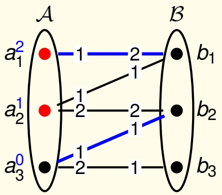
$a_1^1 \rightarrow b_1$ X

$a_2^1 \rightarrow b_1$ ✓

b_1 prefers a_2^1 better than a_1^1

Feasible RSM: Multi-level Gale-Shapley algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$
- All $a \in \mathcal{A}$ are at level 0 to begin with
- Execute Gale-Shapley algorithm
- Unmatched critical vertices raise their level and propose again
- Vertices at higher level are preferred more by $b \in \mathcal{B}$ than those at lower level



$a_2^0 \rightarrow b_1$ X

$a_3^0 \rightarrow b_2$ ✓

$a_1^1 \rightarrow b_1$ X

$a_2^1 \rightarrow b_1$ X

$a_1^2 \rightarrow b_1$ ✓

b_1 prefers a_1^2 as it is at a higher level

Feasible RSM: Multi-level Gale-Shapley algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$
- All $a \in \mathcal{A}$ are at level 0 to begin with
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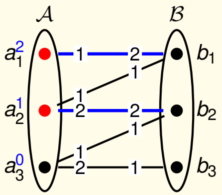

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$a_1^2 \rightarrow b_1$ ✓

$a_2^1 \rightarrow b_2$ ✓

b_2 prefers a higher level neighbour a_2^1

Feasible RSM: Multi-level Gale-Shapley algorithm

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- All $a \in \mathcal{A}$ are at level 0 to begin with
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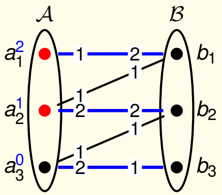

$a_2^0 \rightarrow b_1$ X

$a_3^0 \rightarrow b_2$ X

$a_1^1 \rightarrow b_1$ X

$a_2^1 \rightarrow b_1$ X

$a_1^2 \rightarrow b_1$ ✓

$a_2^1 \rightarrow b_2$ ✓

$a_3^0 \rightarrow b_3$ ✓

b_3 accepts a_3^0's proposal as it was unmatched

Feasible RSM: Multi-level Gale-Shapley algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$
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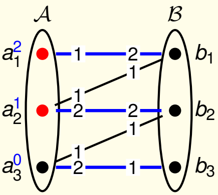

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$a_2^1 \rightarrow b_1$ X

$a_1^2 \rightarrow b_1$ ✓

$a_2^1 \rightarrow b_2$ ✓

$a_3^0 \rightarrow b_3$ ✓

The highest level achieved $= 2 = |C|$

Feasible RSM: Multi-level Gale-Shapley algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$
- All $a \in \mathcal{A}$ are at level 0 to begin with
- Execute Gale-Shapley algorithm
- Unmatched critical vertices raise their **level up to** $|C|$ and propose again
- Vertices at higher level are preferred more by $b \in \mathcal{B}$ than those at lower level

Correctness

Assuming G admits a feasible matching:
Claim 1: Output matching M is feasible.
Claim 2: Output matching M is relaxed stable.

Properties of the output matching

- Each $a \in \mathcal{A}$ is assigned a level at the end of the algorithm

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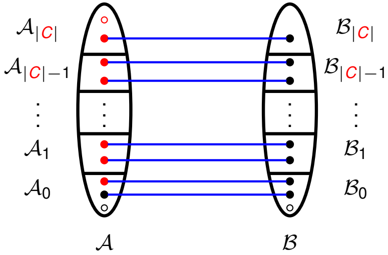

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- Each $a \in \mathcal{A}$ is assigned a level at the end of the algorithm
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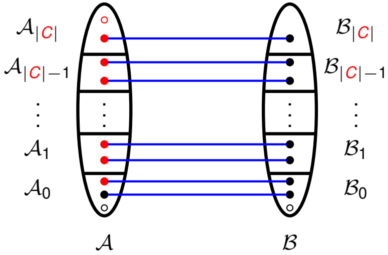

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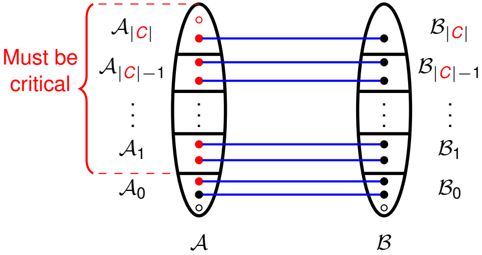

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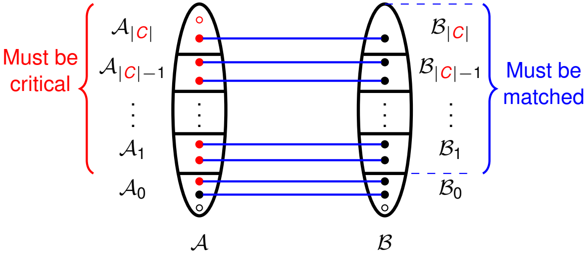

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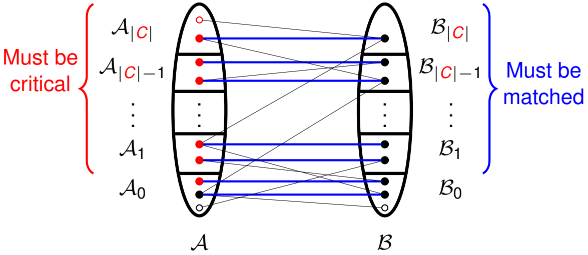

Properties of the output matching

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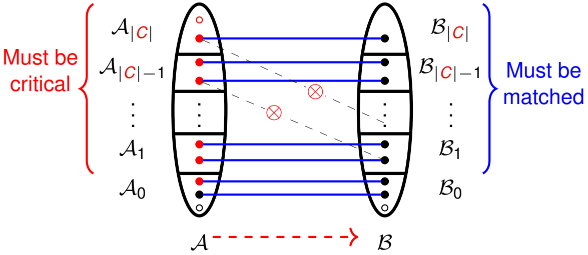

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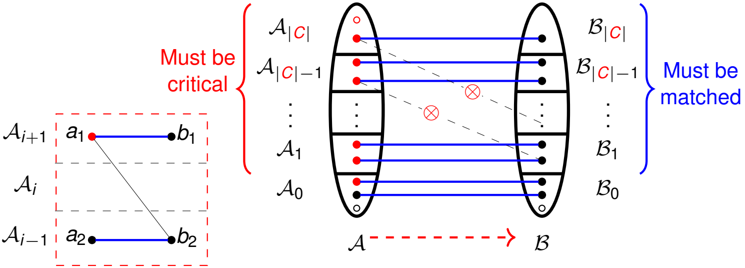

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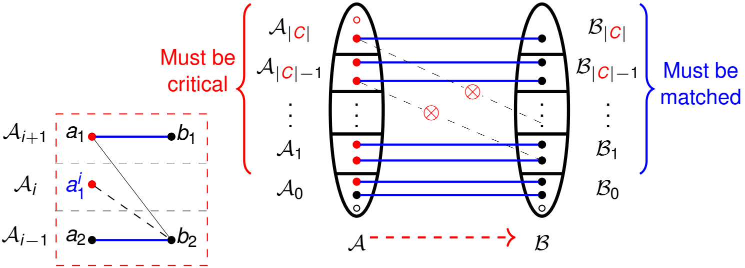

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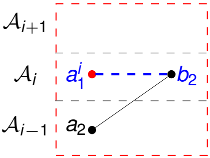

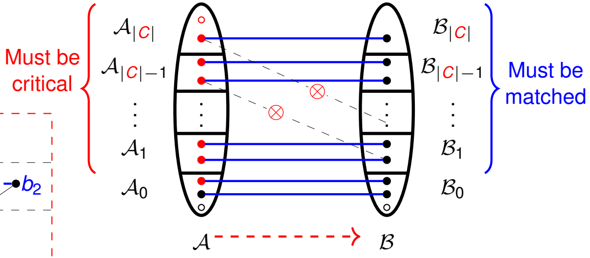

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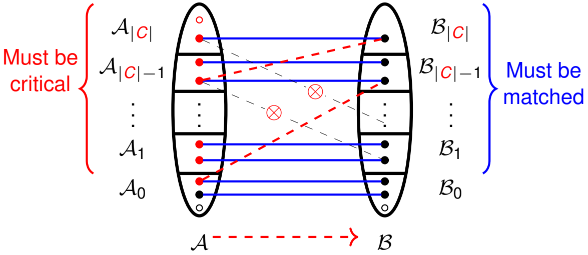

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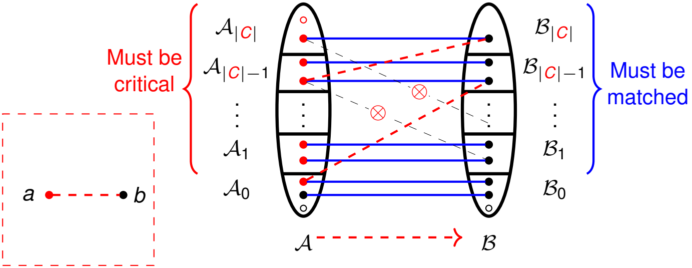

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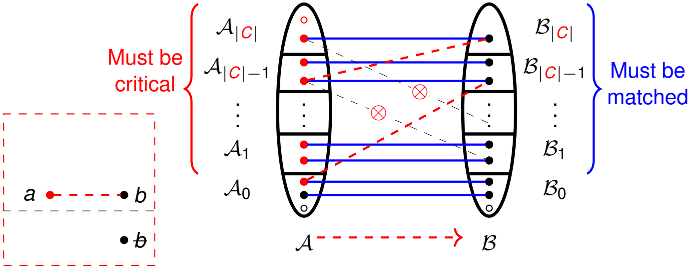

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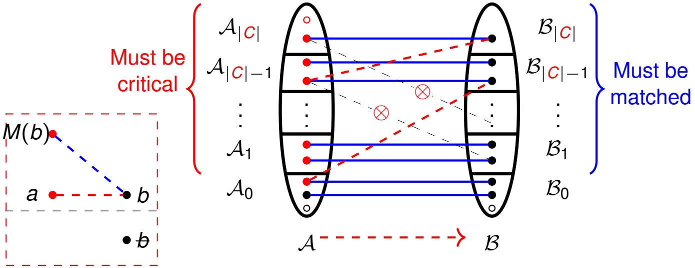

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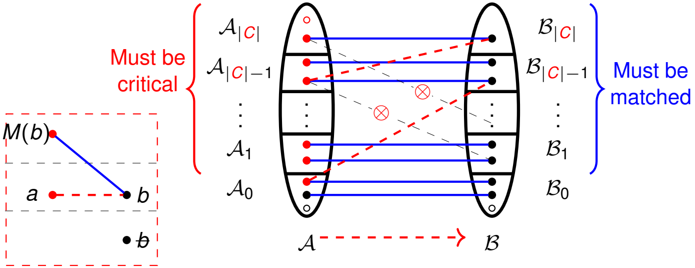

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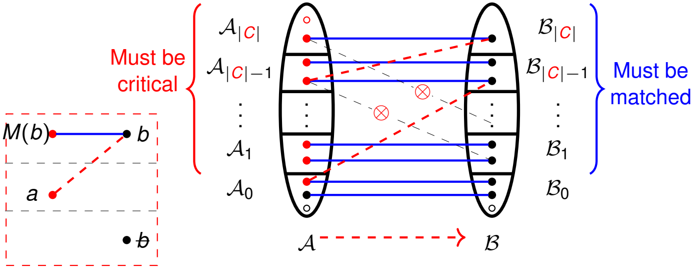

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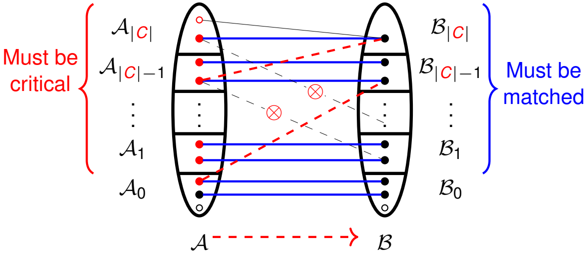
- All neighbours of unmatched critical a are in $\mathcal{B}_{|C|}$



Correctness

Output matching is feasible

Proof Sketch:

- By contradiction

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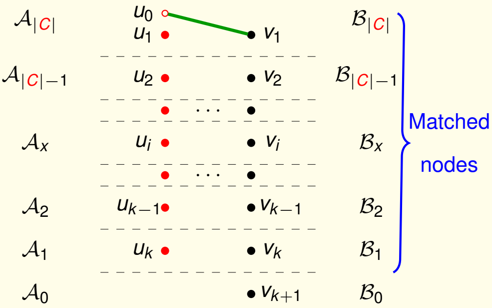

All neighbours of u_0 are in $\mathcal{B}_{|C|}$

Correctness

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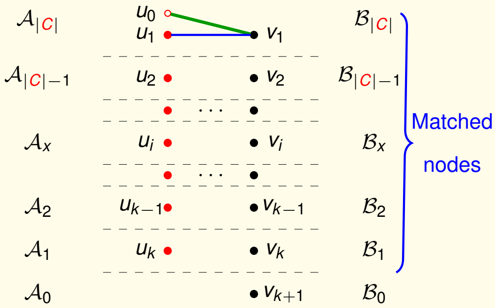

All neighbours of u_0 are in $\mathcal{B}_{|C|}$

All matched edges in M are horizontal

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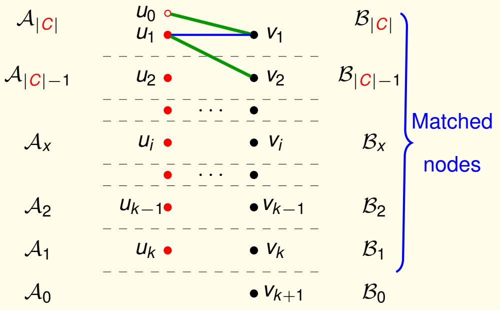

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No steep downward edge

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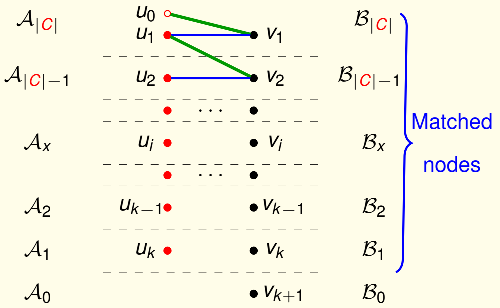

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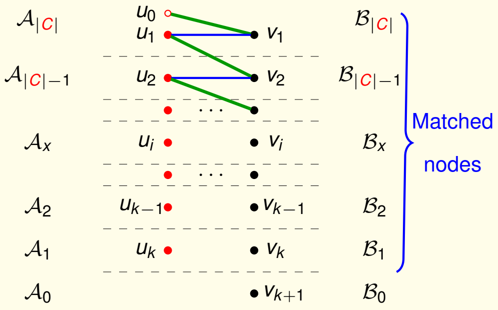

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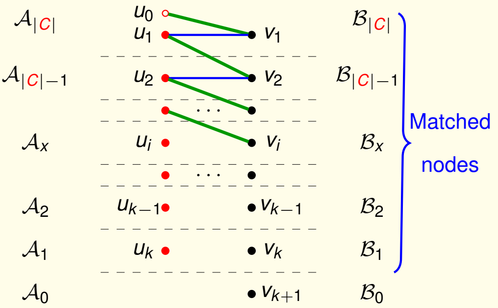

All neighbours of u_0 are in $\mathcal{B}_{|C|}$

All matched edges in M are horizontal

No steep downward edge

Path must end in $\mathcal{A}_0 \cup \mathcal{B}_0$

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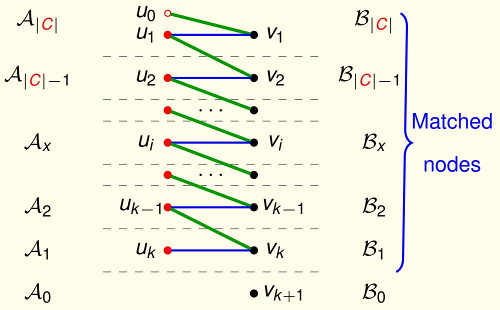

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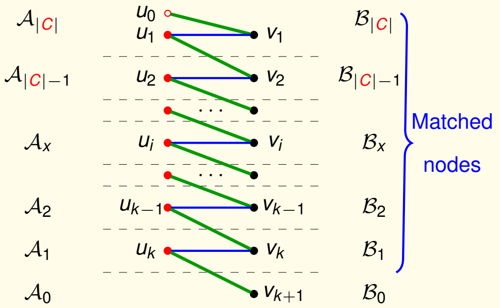

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$\#$critical vertices $> |C|$

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Output matching M of our algorithm is Relaxed Stable Matching

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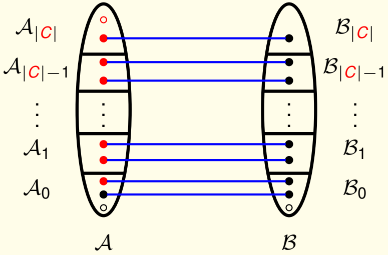
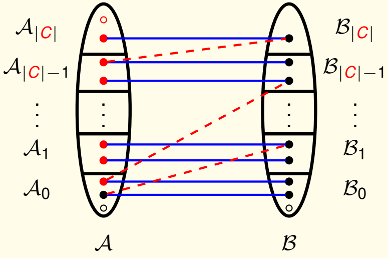


Correctness

Output matching M of our algorithm is Relaxed Stable Matching

All blocking edges w.r.t. M are upwards



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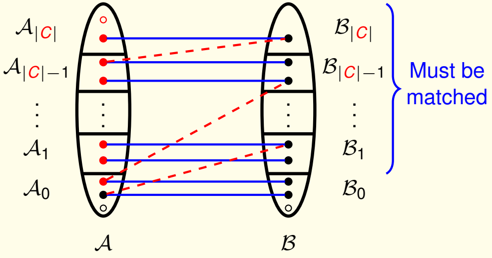

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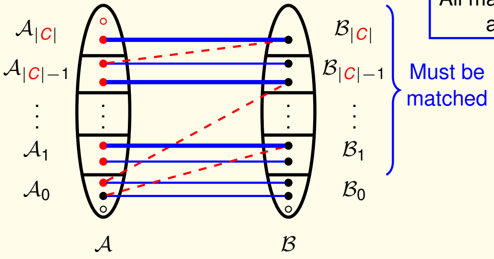

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Correctness

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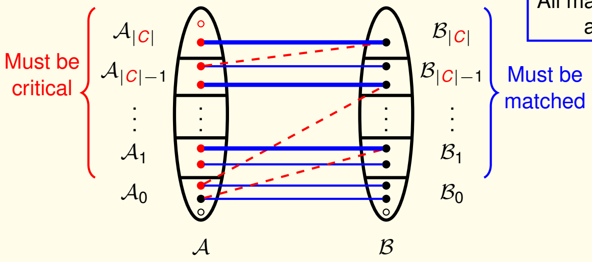

All blocking edges w.r.t. M are upwards

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Correctness

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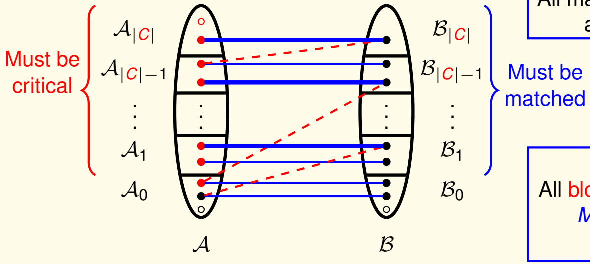


All blocking edges w.r.t. M are upwards

All matched edges in M are horizontal

All blocking edges w.r.t. M are **justified**

Algorithm's Outline

An Evolving Perspective

Summary of our algorithm

- **Assumptions:** (i) Strict lists and (ii) No critical nodes

Gale-Shapley Level

No critical nodes

All vertices in $\mathcal{A}$ propose to **all** neighbours in $\mathcal{B}$

Summary of our algorithm

- **Assumptions:** (i) Strict lists and (ii) $C \subseteq \mathcal{A}$

Higher level (Level $1, 2, \ldots, |C|$)

Critical nodes on $\mathcal{A}$-side

Critical vertices in $\mathcal{A}$ propose to **all** neighbours in $\mathcal{B}$

Gale-Shapley level (Level 0)

No critical nodes

All vertices in $\mathcal{A}$ propose to **all** neighbours in $\mathcal{B}$

Summary of our algorithm

- **Assumptions:** (i) **Tied lists** and (ii) $C \subseteq \mathcal{A}$

Higher level (Level $1, 2, \ldots, |C|$)

Critical nodes on $\mathcal{A}$-side

Critical vertices in $\mathcal{A}$ propose to **all** neighbours in $\mathcal{B}$

Király's algorithm (Level 0 and 0^*)

All vertices in $\mathcal{A}$ propose to all neighbours in $\mathcal{B}$

Summary of our algorithm

- **Assumptions:** (i) Tied lists, (ii) $C \subseteq \mathcal{A} \cup \mathcal{B}$ and (iii) $|\mathcal{A} \cap C| = s$ and $|\mathcal{B} \cap C| = t$

Higher level (Level $t, \ldots, s + t$)

Critical nodes on $\mathcal{A}$-side

Critical vertices in $\mathcal{A}$ propose to **all** neighbours in $\mathcal{B}$

Király's algorithm (Level t and t^*)

All vertices in $\mathcal{A}$ propose to all neighbours in $\mathcal{B}$

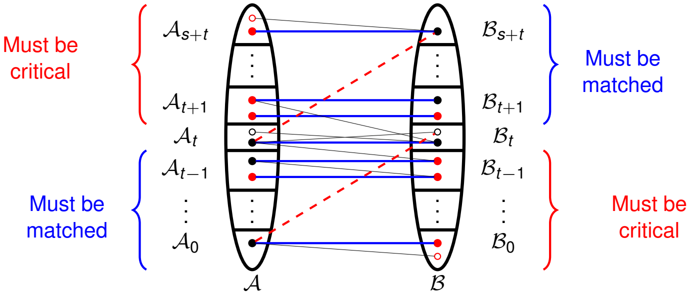
Lower level (Level $0, 1, \ldots, t - 1$)

Critical nodes on $\mathcal{B}$-side

All vertices in $\mathcal{A}$ propose to **critical** neighbours in $\mathcal{B}$

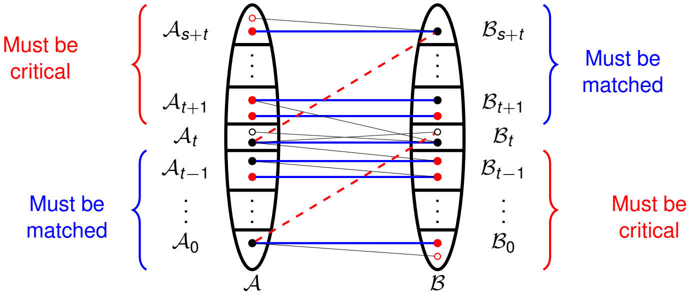
Summary of our algorithm

- M is critical



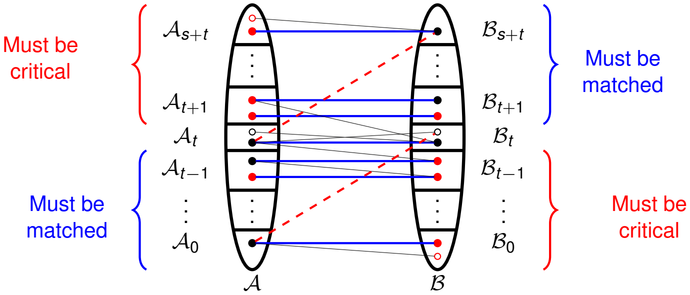
Correctness

- M is critical
- M is Relaxed Stable Matching (RSM)



Correctness

- M is critical
- M is Relaxed Stable Matching (RSM)
- $|M| \geq \frac{3}{2} \cdot |M^*|$ for any Max-size Critical Relaxed Stable Matching M^*



Conclusion

- For an instance $G = (\mathcal{A} \cup \mathcal{B}, E, C)$ with ties on both sides and $C \subseteq \mathcal{A} \cup \mathcal{B}$

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- Computing maximum size critical RSM is **NP-Hard**

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- $\frac{3}{2}$**-approximation** of the max-size critical RSM

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- For an instance $G = (\mathcal{A} \cup \mathcal{B}, E, C)$ with ties on both sides and $C \subseteq \mathcal{A} \cup \mathcal{B}$
- Critical Relaxed Stable Matching (RSM) **always** exists
- Computing maximum size critical RSM is **NP-Hard**
- $\frac{3}{2}$**-approximation** of the max-size critical RSM
- Natural extension is to the many-to-many setting

Conclusion

- For an instance $G = (\mathcal{A} \cup \mathcal{B}, E, C)$ with ties on both sides and $C \subseteq \mathcal{A} \cup \mathcal{B}$
- Critical Relaxed Stable Matching (RSM) **always** exists
- Computing maximum size critical RSM is **NP-Hard**
- $\frac{3}{2}$**-approximation** of the max-size critical RSM
- Natural extension is to the many-to-many setting

Thank You!

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