# PROBLEM SET 6

TOPICS IN MANIFOLDS, SPRING 2016

**Problem 1.** Consider the following five tessellations: the square and triangle tessellations of $\mathbb{E}^2$; the icosahedral, octahedral, and tetrahedral tessellations of $\mathbb{S}^2$. For each of these tessellations, let $\Gamma^+$ be its group of orientation preserving symmetries:

(a) Draw a fundamental region for $\Gamma^+$.
(b) Draw the Caley graph of $\Gamma^+$.
(c) Give a presentation for the group $\Gamma^+$.

(See problems $7.1.1 - 7.1.4$ in Stillwell).

**Problem 2.** Find a presentation of the orientation preserving subgroup of the group generated by reflections in the sides of a $(p, q, r)$ triangle. (See problem 7.3.1 in Stillwell).

**Problem 3.** (a) Show that reflections in the sides of the $(2, 3, \infty)$ triangle with vertices $i, \omega := \frac{1}{2} + \frac{\sqrt{3}}{2}i, \infty$ induce the side-pairing transformations $z \mapsto 1 + z$ and $z \mapsto -\frac{1}{z}$ in its double.

(b) Verify that the group $\Gamma$ generated by these transformations has the free group $F_2$ as a subgroup. What is the index of $F_2$ in $\Gamma$?

(c) Show that the group $\Gamma$ has a presentation

$$\langle g, h | g^2 = h^3 = 1 \rangle$$

(See problem 7.3.6 and 7.3.7 in Stillwell).

**Problem 4.** Show that 336 is the correct number of $(2, 3, 7)$ triangles to tessellate a hyperbolic surface of genus 3. (See problem 7.3.4 in Stillwell).

**Problem 5.** Show that a hyperbolic surface of genus 2 can be tessellated symmetrically by 96 $(2, 3, 8)$ triangles. (See problem 7.3.5 in Stillwell).