PROBLEM SET 5

TOPICS IN MANIFOLDS, SPRING 2016

Problem 1. Consider the horocycle $C = \{x + iy \in \mathbb{H}^2 : x^2 + (y - \frac{1}{2})^2 = (\frac{1}{2})^2\}$, where $\mathbb{H}^2$ denotes the upper half plane with the hyperbolic metric ds. Let $\mathbb{E}^1$ denote the real line $\mathbb{R}^1$ with the standard Euclidean metric dt. Produce an explicit isometry from $\mathbb{E}^1 \to C$; i.e. produce a continuous map $\gamma : \mathbb{E}^1 \to C$ such that $\gamma^*(ds) = dt$.

Problem 2. This problem refers to the hyperbolic polygon R in Figure 5.12 from Stillwell.

(a) Show that the result of identifying edges of R as shown in the figure is topologically the torus with one puncture (see Stillwell 5.4.1).
(b) Find explicit hyperbolic translations g, h which realize the identifications shown in the figure, and show that they generate a free group (see Stillwell 5.4.2).
(c) Show that R is the Dirichlet region for the group $\langle g, h \rangle$ (see Stillwell 5.8.1).

Problem 3. By a suitable choice of hyperbolic polygon, show that the sphere with $n \geq 4$ punctures is a hyperbolic surface (see Stillwell 5.5.1).