PROBLEM SET 4

TOPICS IN MANIFOLDS, SPRING 2016

Problem 1. Give a euclidean geometric construction showing the existence and uniqueness of an $\mathbb{H}^2$-line through any $z_1, z_2 \in \mathbb{H}^2$ (Stillwell 4.2.3).

Problem 2. Define an $\mathbb{H}^2$-circle to be the set of points $\mathbb{H}^2$ equidistant to a given point. Show that all $\mathbb{H}^2$-circles are in fact eucliden circles in $\mathbb{H}^2$ (for hints see Stillwell 4.2.4).

Problem 3. Show that the $\mathbb{D}^2$-circumference of a $\mathbb{D}^2$-circle of $\mathbb{D}^2$-radius ρ is $2\pi \sinh \rho$ (Stillwell 4.2.5).

Problem 4. Show that the $\mathbb{S}^2$-circumference of a $\mathbb{S}^2$-circle of $\mathbb{S}^2$-radius ρ is $2\pi \sin \rho$ (Stillwell 4.2.6).

Problem 5. Show that the $\mathbb{D}^2$-distance between $w_1, w_2 \in \mathbb{D}^2$ is

$$2 \tanh^{-1} \left| \frac{w_2 - w_1}{1 - \bar{w_1} w_2} \right|$$

(see Stillwell 4.4.1).

Problem 6. Show that any $\mathbb{D}^2$-isometry is of the form

(rotation about 0)(limit rotation about 1)(rotation about 0)

(see Stillwell 4.4.4 and 4.4.5 for hints).