# PROBLEM SET 11

INTRODUCTION TO MANIFOLDS

**Problem 1.** Compute the de Rham cohomology ring of the $n$-sphere $S^n$.

**Problem 2.** Compute the de Rham cohomology ring of the multiply-punctured Euclidean space $\mathbb{R}^n \setminus \{p_1, \ldots, p_m\}$, where $m \geq 1$.

**Problem 3.** Compute the de Rham cohomology ring of the $n$-dimensional torus $T^n$.

**Problem 4.** Compute the de Rham cohomology vector spaces of the real projective space $\mathbb{RP}^n$.

**Problem 5.** Compute the de Rham cohomology vector spaces $H^k(\Sigma_g)$ for $k \geq 0$, where $\Sigma_g$ is the compact orientable surface of genus $g$.

**Problem 6.** An open cover of a manifold $M$ is a *good cover* if every finite intersection of open sets is contractible. Prove that if a manifold $M$ has a finite good cover, then its de Rham cohomology vector spaces $H^k(M)$ are finite dimensional for all $k \geq 0$.

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