MINIMAL RESOLUTIONS OF IDEALS DEFINING MONOMIAL CURVES

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Abstract

Let $\mathbf{a} = (a_0, a_1, \ldots, a_n)$ be a sequence of positive integers and $\phi : R = k[x_0, \ldots, x_n] \to k[t]$, $\phi(x_i) = t^{a_i}$ be the parametrization of the monomial curve $C(\mathbf{a})$. Then $I(\mathbf{a}) = \ker \phi$ is the ideal defining the monomial curve and $R/I(\mathbf{a})$ is isomorphic to the semigroup ring $k[t^{a_0}, \ldots, t^{a_n}]$. We will explicitly construct the minimal resolutions of $R/I(\mathbf{a})$ when $\mathbf{a}$ is an arithmetic sequence. The construction is by iteration of minimized mapping cones each of which is the minimal resolution of a Cohen-Macaualy algebras R/I_i. In fact, we will show that they are all minimal algebra resolutions. One consequence of this resolution is the proof of periodicity conjecture for monomial curves defined by arithmetic sequences. Given $\mathbf{a}$ a sequence of positive integers, we have a series of monomial curves $\mathbf{a} + \mathbf{j} = (a_0 + j, a_1 + j, \ldots, a_n + j)$. A conjecture of Herzog and Srinivasan states that the betti numbers of $C(\mathbf{a}+\mathbf{j})$ are eventually periodic in j. We will prove this for the arithmetic sequences as well as for complete intersections in the sense that for large j, $C(\mathbf{a} + \mathbf{j})$ is a complete intersection if and only if $C(\mathbf{a} + (\mathbf{j} + \mathbf{a_n} - \mathbf{a_0}))$ is a complete intersection. Similar results hold for Gorenstein curves when $n \leq 3$. These results are from my joint work with Gimenez, Sengupta and Jayanthan.