Playing optimally using memory

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Based on joint work with:
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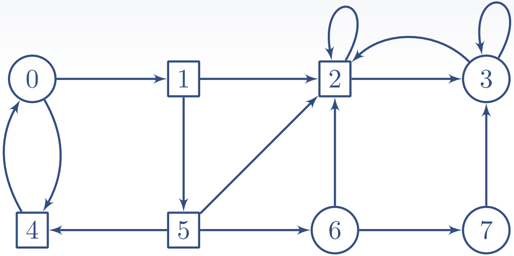
ReLaX Research Lab in Computer Science

Example

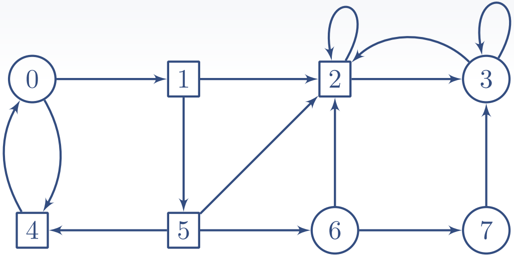


Setting

We consider:

- Finite graphs, a set of colors C, and a mapping from edges to colors.
- Two players, Max (circle) and Min (square).
- A preference relation $\sqsubseteq$ (total preorder) over C^ω for Max.
- Inverse relation $\sqsubseteq^{-1}$ for Min.

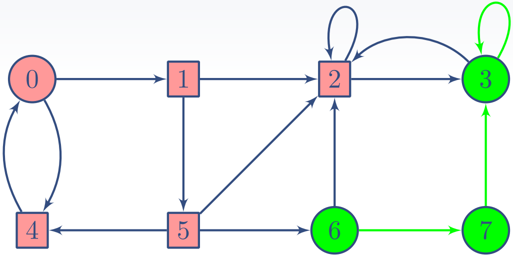
Example



$\sqsubseteq$: visit 3 at least once

Controller synthesis

Example



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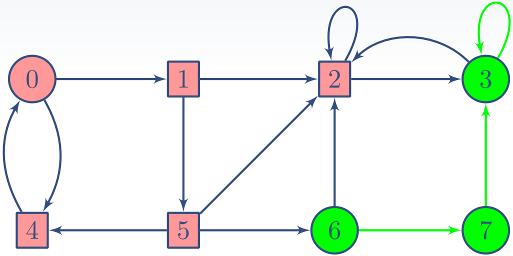

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Design *an optimal* strategy for Max w.r.t. the preference relation $\sqsubseteq$.

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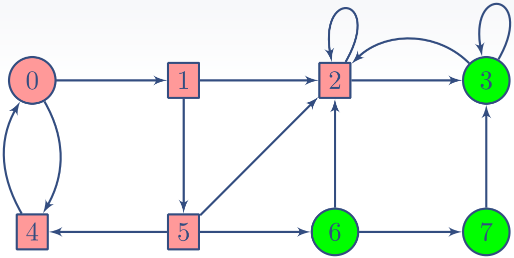

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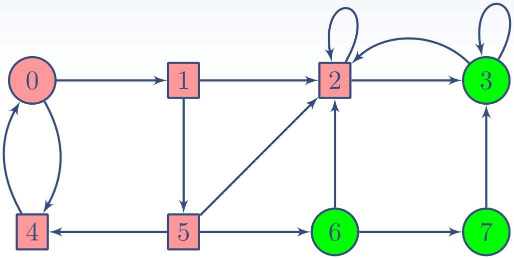

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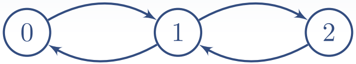
Strategy for Max

$$3 \mapsto 3;\ 7 \mapsto 3;\ 6 \mapsto 7.$$

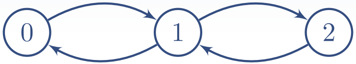
Example



$\sqsubseteq$: infinitely often 2 and infinitely often 0

Complex controller

Example



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Strategy for Max

$$0\ 1 \mapsto 2;\ 2\ 1 \mapsto 0.$$

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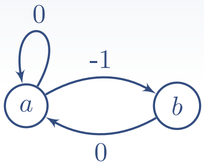

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Very complex controller

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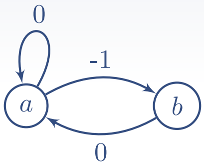

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Strategy for Max

$0 \mapsto 0;$	$0\ 0 \mapsto 1;$	$0\ 0\ 1 \mapsto 0;$	$0\ 0\ 1\ 0 \mapsto 0;$
$0\ 0\ 1\ 0\ 0 \mapsto 0;$	$0\ 0\ 1\ 0\ 0\ 0 \mapsto 0;$	$0\ 0\ 1\ 0\ 0\ 0\ 0 \mapsto 1;$	$0\ 0\ 1\ 0\ 0\ 0\ 0\ 1 \mapsto 0;$
...	...	...	...

Recap

Controller synthesis

Design *an optimal* strategy for Max w.r.t. the preference relation $\sqsubseteq$.

Simple Controller

Decision making depends on the current state; *memoryless* strategies.

Complex Controller

Decision making depends on a bounded history; *finite memory* strategies.

Very complex Controller

Decision making depends on the full history; *infinite memory* strategies.

Memoryless determinacy

Definition

Both players can play *optimally* using *memoryless* strategies w.r.t $\sqsubseteq$ and $\sqsubseteq^{-1}$.

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GZ-Criterion

In 2005, Gimbert & Zielonka characterize the preference relations for which memoryless *optimal strategies* exist for both players.

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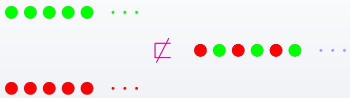
Monotony

Selectivity



Furthermore

Lifting corollary (Gimbert, Zielonka'05)

Let $\sqsubseteq$ be a preference relation, assume that:

i. In *all* Max-*arenas* memoryless optimal strategies exist.

ii. In *all* Min-*arenas* memoryless optimal strategies exist (w.r.t. $\sqsubseteq^{-1}$).

Then, both players have memoryless optimal strategies in *all two-player arenas*.

Remark

Establishing *i.* and *ii.* is usually "easy".

Our hope

Extends all of the above to *finite memory determinacy*.

Finite memory determinacy

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$\sqsubseteq$ is *finite memory determined* if finite memory optimal strategies suffice for both players.

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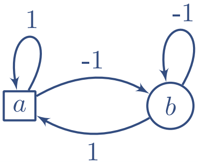

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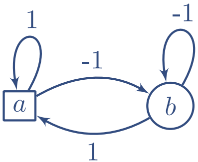

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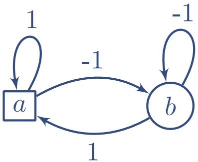

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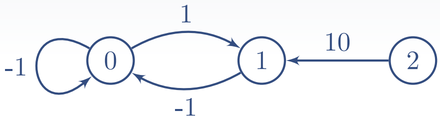
- Max needs *infinite memory* to play optimally.
- In both the one-player versions *finite memory* optimal strategies exist.

Arena dependent V.S Arena independent finite memory

ADFM



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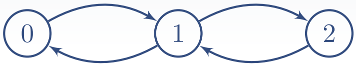
AIFM



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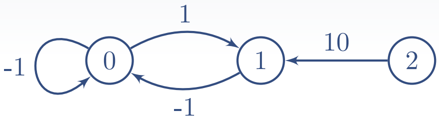

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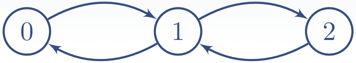

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Our contribution

A characterization of the *arena independent* finite memory determined preference relations.

Arena independent finite memory

Memory structure

An automaton-like formalism that given a color and a memory state, updates to the new memory state.

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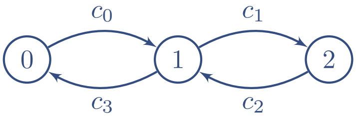
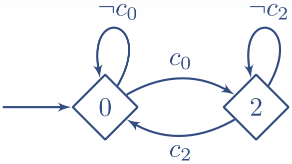
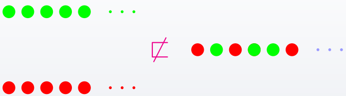




$\mathcal{M}$-Monotony

w and w' lead to the same memory state

$\mathcal{M}$-Selectivity



Interleaving modulo memory states

Results

Theorem (Bouyer, Le Roux, O., Randour, Vandenhove'20)

Let $\sqsubseteq$ be a preference relation and let $\mathcal{M}$ be a memory structure, then both players have optimal arena independant finite memory strategies based on a memory structure $\mathcal{M}$ in all games if and only if $\sqsubseteq$ and $\sqsubseteq^{-1}$ are $\mathcal{M}$-monotone and $\mathcal{M}$-selective.

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Remark

The memory structure in the two-player case is the product of the memory structure in the one-player case.

Comments

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Follows the steps of Gimbert & Zielonka's proof.

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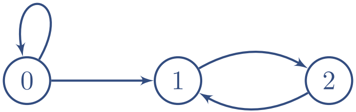

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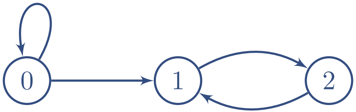

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Crucial steps

i. If $\sqsubseteq$ is $\mathcal{M}$-monotone and $\mathcal{M}$-selective, then for any arena $\mathcal{A}$, $\mathcal{A} \times \mathcal{M}$ is a *covered* arena.

ii. In *covered* arenas it is possible to play optimally with memoryless strategies.

Conclusion

Results

- Characterization of *AIFM*-determinacy.
- A lifting corollary in the context of *AIFM*-optimal strategies.

Future directions

- Characterization of *ADFM*-determinacy.
- More general arenas e.g., *stochastic games*.