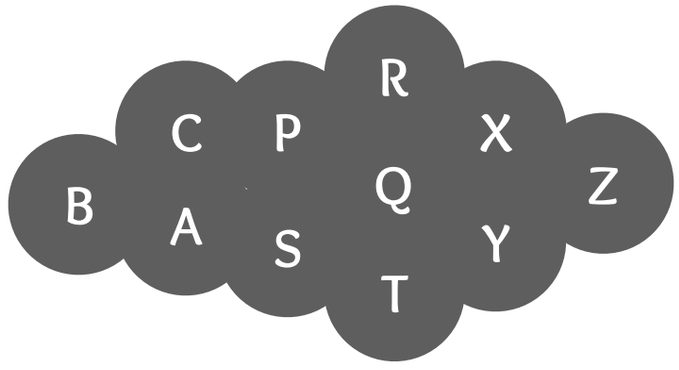
Parameterized Complexity of Party Nominations

Neeldhara Misra, IIT Gandhinagar

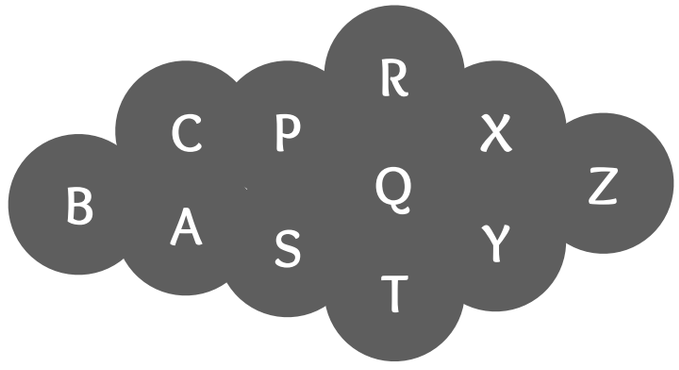
Workshop on Games

Chennai Mathematical Institute

Candidates

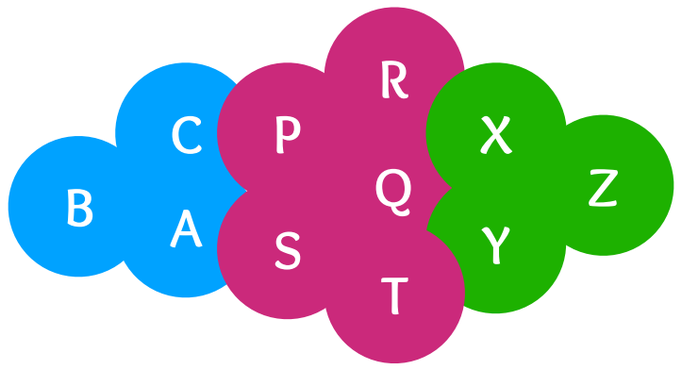


Candidates



Votes

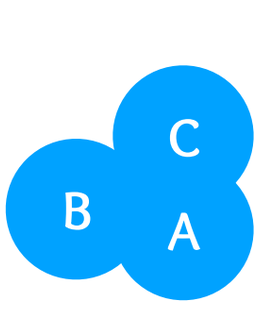
Candidates

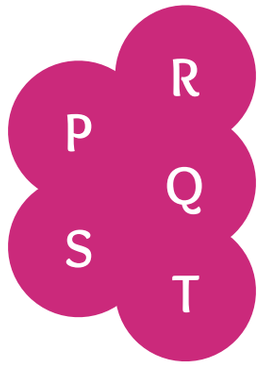


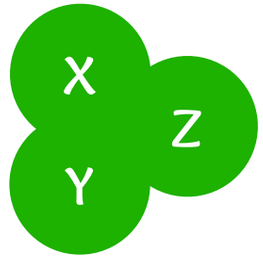
Votes

Z X B C A S P Q Y R T

Candidates

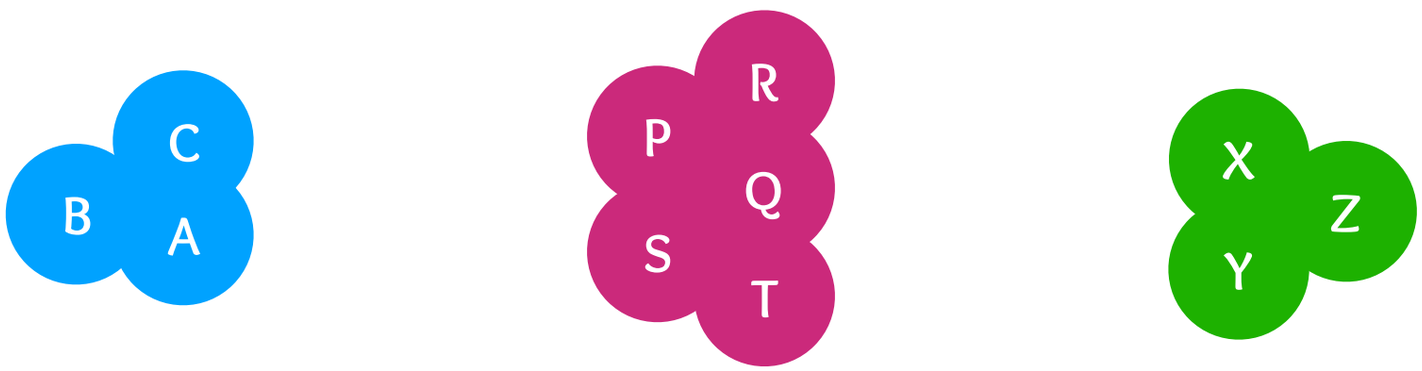






Votes

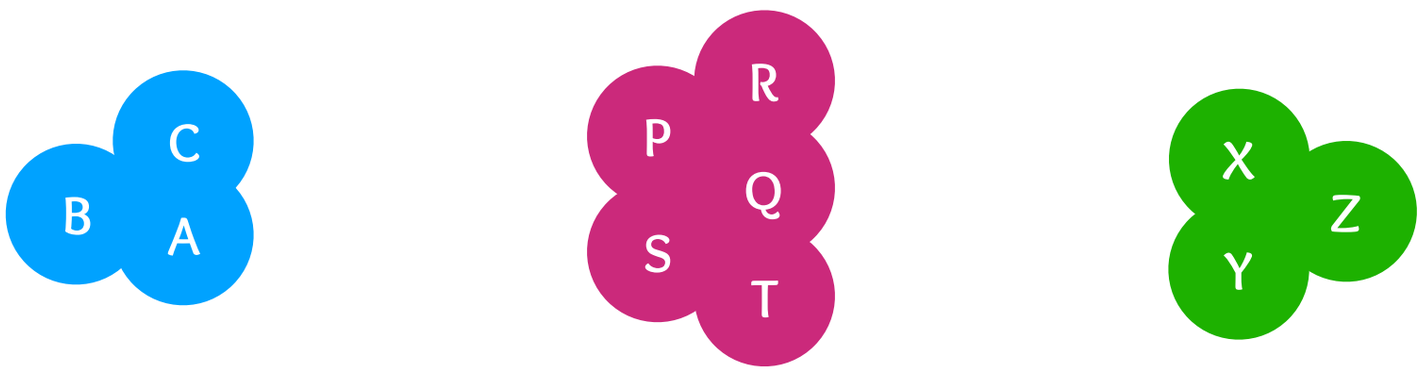
Candidates



The candidates are partitioned into “parties”.

Votes

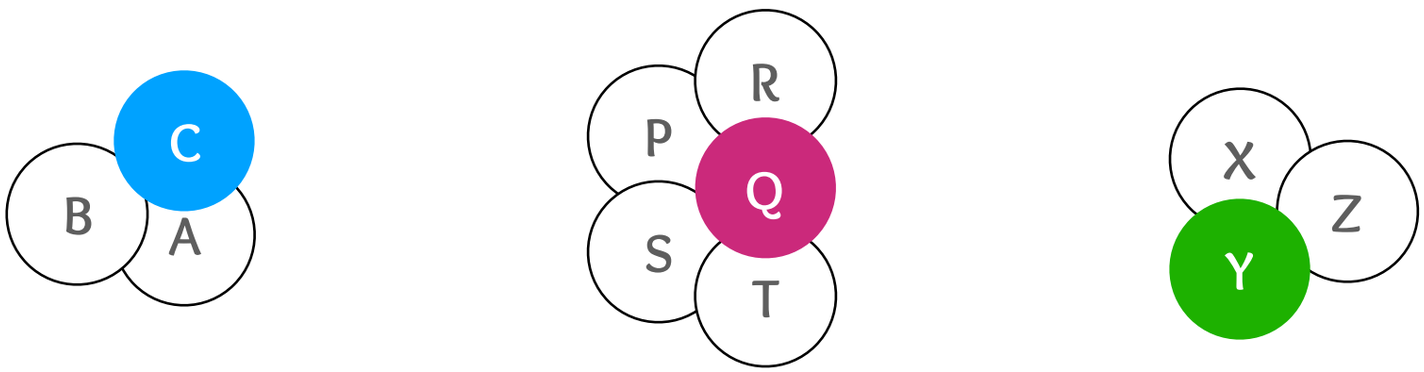
Candidates



Every party nominates a candidate.

Votes

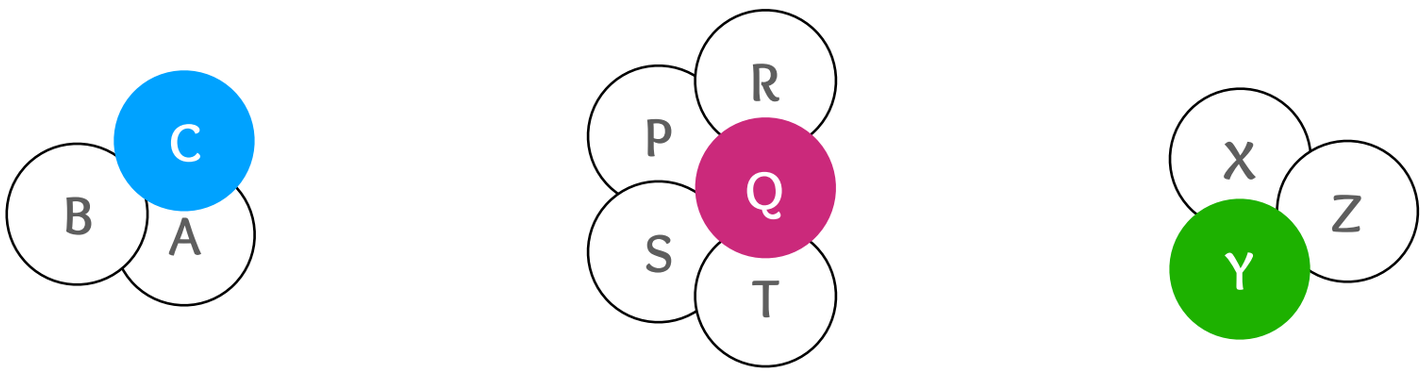
Candidates



Every party nominates a candidate.

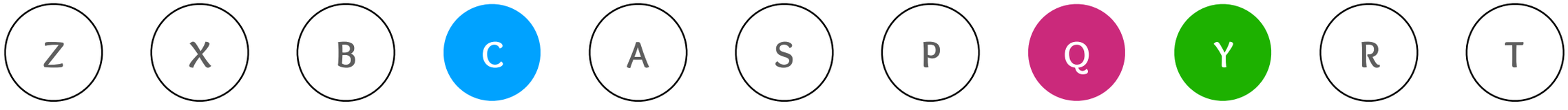
Votes

Candidates



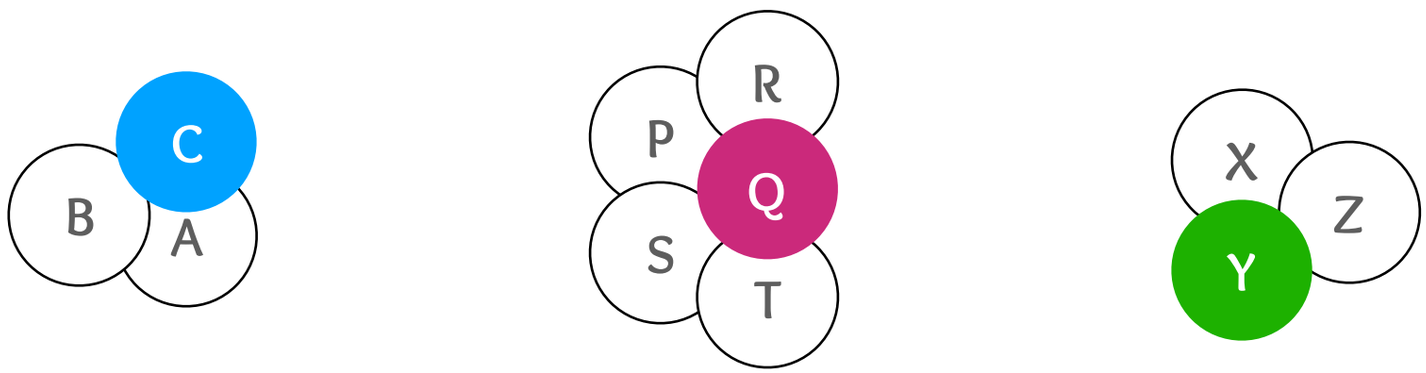
Every party nominates a candidate.

Votes



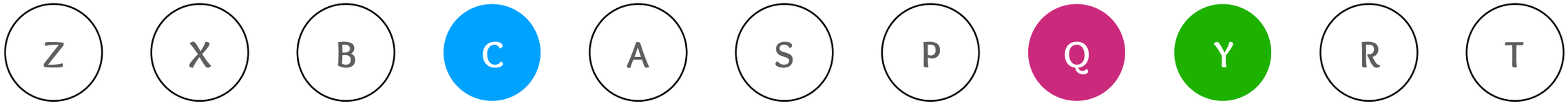
The votes are “projected” on the nominees.

Candidates



Every party nominates a candidate.

Votes



The winner is declared based on the plurality voting rule.

The Problem

Assuming **complete knowledge** about the votes,
how do parties **select their nominees**?

An Example

Candidates A B X

6	A	X	B
4	B	X	A
1	X	B	A

Votes

An Example

Candidates

A B X

6	A	X	B
4	B	X	A
1	X	B	A

Votes

An Example

Candidates

6	A	X	B
4	B	X	A
1	X	B	A

Votes

The Problem

Assuming **complete knowledge** about the votes,
how do parties **select their nominees**?

The Problem

Assuming **complete knowledge** about the votes,
how do parties **select their nominees**?

What do the parties know about
other nominees?

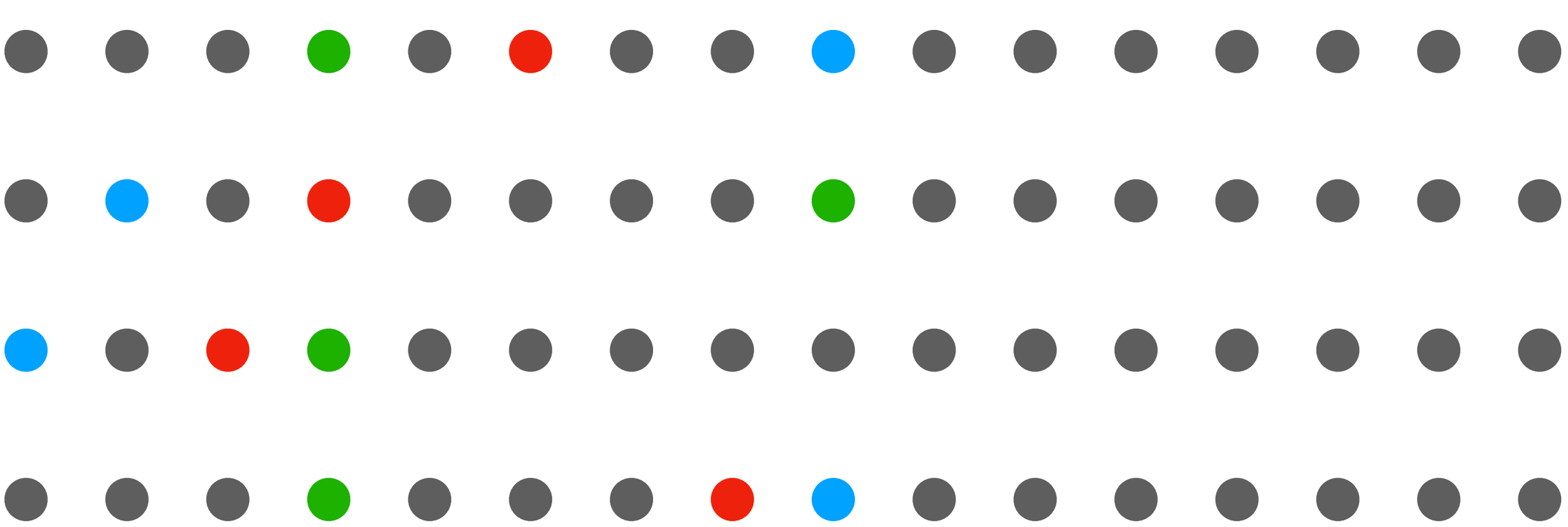
Plurality

Plurality

If we know who the other parties are nominating,
it is easy to “evaluate” a candidate in our party.

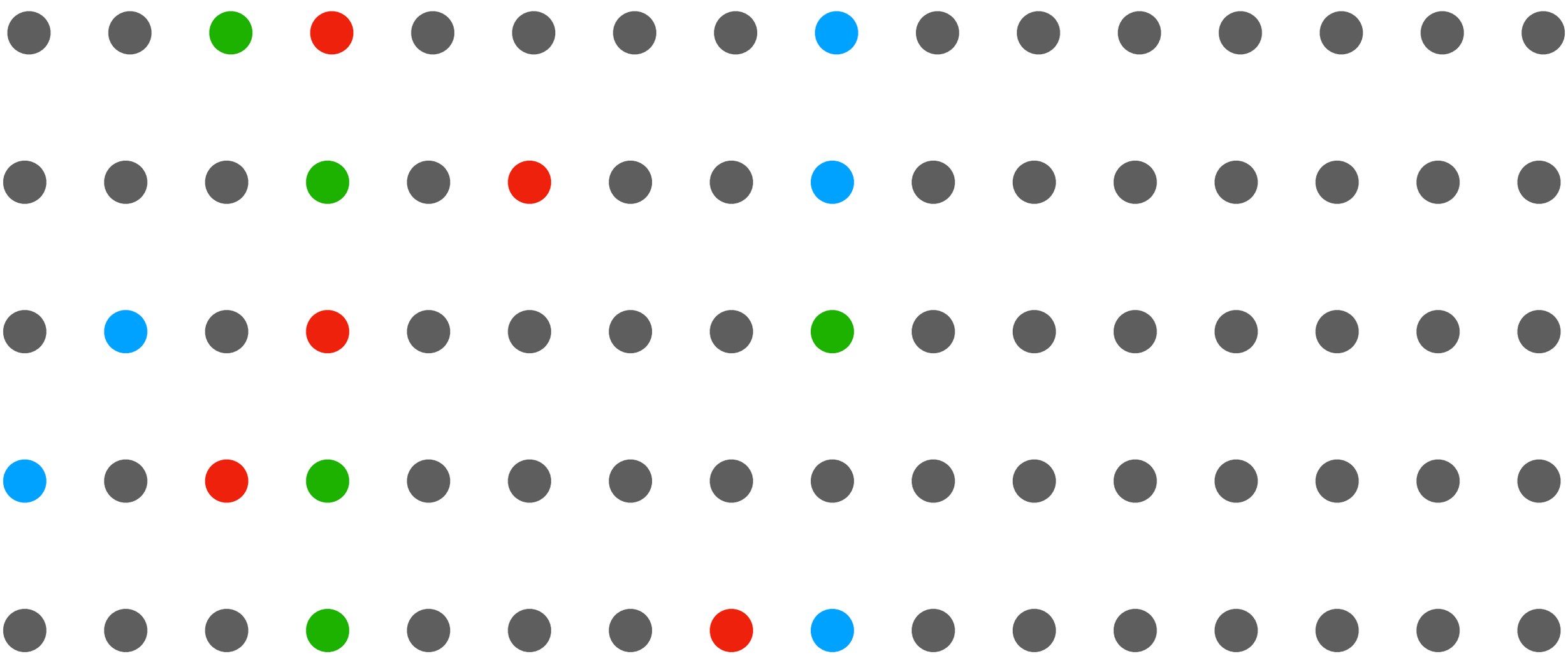
Plurality

If we know who the other parties are nominating,
it is easy to "evaluate" a candidate in our party.



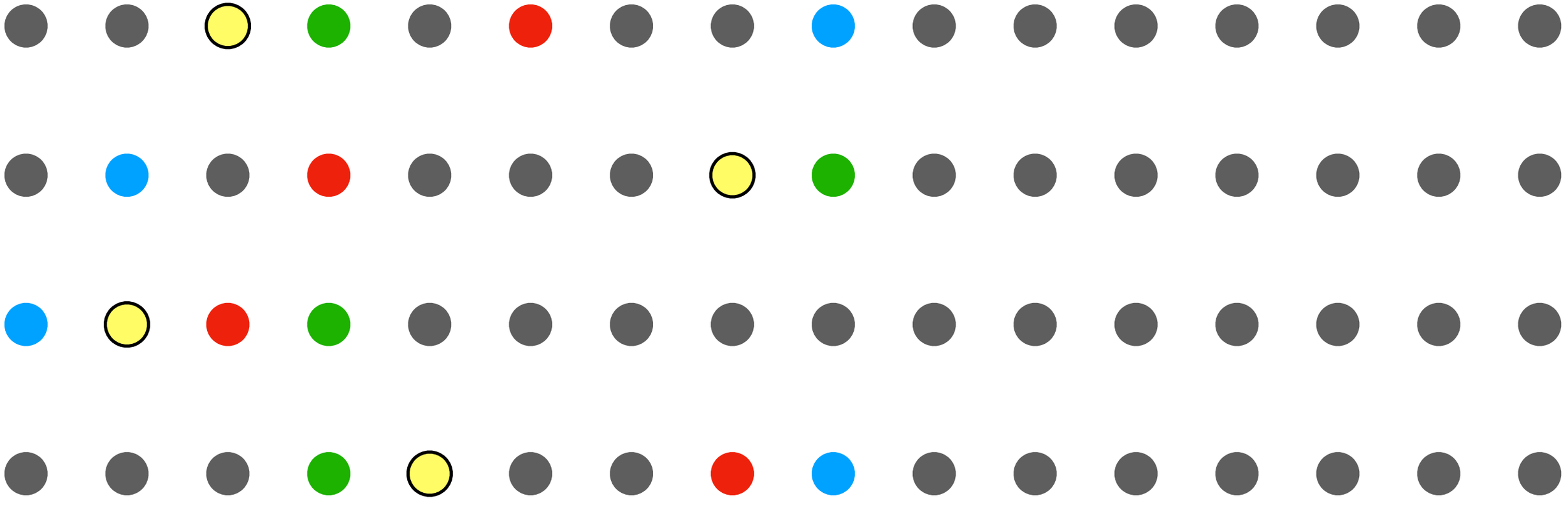
Plurality

If we know who the other parties are nominating,
it is easy to "evaluate" a candidate in our party.



Plurality

If we know who the other parties are nominating,
it is easy to "evaluate" a candidate in our party.



A more natural scenario

We have ***no idea*** who the other nominees are.

A more natural scenario

We have ***no idea*** who the other nominees are.

The Optimist's Question

Do we have a superstar candidate who ensures a party win,
irrespective of who is nominated from the other parties?

A more natural scenario

We have ***no idea*** who the other nominees are.

The Optimist's Question

Do we have a superstar candidate who ensures a party win,
irrespective of who is nominated from the other parties?

The Pessimist's Question

Do we have a promising candidate who makes the party win
in **at least one** of the many possible parallel universes?

A more natural scenario

We have ***no idea*** who the other nominees are.

Necessary President

Do we have a superstar candidate who ensures a party win,
irrespective of who is nominated from the other parties?

The Pessimist's Question

Do we have a promising candidate who makes the party win
in **at least one** of the many possible parallel universes?

A more natural scenario

We have ***no idea*** who the other nominees are.

Necessary President

Do we have a superstar candidate who ensures a party win,
irrespective of who is nominated from the other parties?

Possible President

Do we have a promising candidate who makes the party win
in **at least one** of the many possible parallel universes?

Known Results

Necessary President

Do we have a superstar candidate who ensures a party win, **irrespective of who is nominated** from the other parties?

Piotr Faliszewski, Laurent Gourvès, Jérôme Lang, Julien Lesca, Jèrôme Monnot.
How hard is it for a party to nominate an election winner? AAAI 2016

Known Results

Necessary President

Do we have a superstar candidate who ensures a party win, **irrespective of who is nominated** from the other parties?

co-NP complete even when the size of the largest party is two.

Piotr Faliszewski, Laurent Gourvès, Jérôme Lang, Julien Lesca, Jèrôme Monnot.
How hard is it for a party to nominate an election winner? AAAI 2016

Known Results

Necessary President

Do we have a superstar candidate who ensures a party win, **irrespective of who is nominated** from the other parties?

co-NP complete even when the size of the largest party is two.

polynomial-time when the profiles are single-peaked.

Piotr Faliszewski, Laurent Gourvès, Jérôme Lang, Julien Lesca, Jèrôme Monnot.
How hard is it for a party to nominate an election winner? AAAI 2016

Known Results

Possible President

Do we have a promising candidate who makes the party win in **at least one** of the many possible parallel universes?

Piotr Faliszewski, Laurent Gourvès, Jérôme Lang, Julien Lesca, Jèrôme Monnot.
How hard is it for a party to nominate an election winner? AAAI 2016

Known Results

Possible President

Do we have a promising candidate who makes the party win in **at least one** of the many possible parallel universes?

NP-complete even when the size of the largest party is two.

Piotr Faliszewski, Laurent Gourvès, Jérôme Lang, Julien Lesca, Jèrôme Monnot.
How hard is it for a party to nominate an election winner? AAAI 2016

Known Results

Possible President

Do we have a promising candidate who makes the party win in **at least one** of the many possible parallel universes?

NP-complete even when the size of the largest party is two.

NP-complete *also* when the profiles are 1D-Euclidean.

Piotr Faliszewski, Laurent Gourvès, Jérôme Lang, Julien Lesca, Jèrôme Monnot.
How hard is it for a party to nominate an election winner? AAAI 2016

Known Results

Possible President

Do we have a promising candidate who makes the party win in **at least one** of the many possible parallel universes?

NP-complete even when the size of the largest party is two.

NP-complete *also* when the profiles are **1D-Euclidean**.

(a subclass of single-peaked & single-crossing profiles)

Piotr Faliszewski, Laurent Gourvès, Jérôme Lang, Julien Lesca, Jèrôme Monnot.
How hard is it for a party to nominate an election winner? AAAI 2016

This Talk

This Talk

Possible President

This Talk

Possible President

NP-complete even when the size of the largest party is two.

NP-complete *also* when the profiles are 1D-Euclidean.

Piotr Faliszewski, Laurent Gourvès, Jérôme Lang, Julien Lesca, Jèrôme Monnot.
How hard is it for a party to nominate an election winner? AAAI 2016

This Talk

Possible President

NP-complete even when the size of the largest party is two,

and the profiles are 1D-Euclidean.

This Talk

Possible President

NP-complete even when the size of the largest party is two,

and the profiles are 1D-Euclidean.

(A stronger hardness result.)

This Talk

Possible President

NP-complete even when the size of the largest party is two,

and the profiles are 1D-Euclidean.

(A stronger hardness result.)

XP and **W[2]-hard** parameterized by the **number of parties.**

This Talk

Possible President

NP-complete even when the size of the largest party is two,

and the profiles are 1D-Euclidean.

(A stronger hardness result.)

XP and **W[2]-hard** parameterized by the **number of parties**.

FPT parameterized by **number of parties** on **1D-Euclidean profiles**.

This Talk

Possible President

NP-complete even when the size of the largest party is two,

and the profiles are 1D-Euclidean.

(A stronger hardness result.)

XP and **W[2]-hard** parameterized by the **number of parties.**

FPT parameterized by **number of parties** on **1D-Euclidean profiles.**

(Parameterized Results)

This Talk

Necessary President

This Talk

Necessary President

co-NP complete even when the size of the largest party is two.

polynomial-time when the profiles are single-peaked.

Piotr Faliszewski, Laurent Gourvès, Jérôme Lang, Julien Lesca, Jèrôme Monnot.
How hard is it for a party to nominate an election winner? AAAI 2016

This Talk

Necessary President

co-NP complete even when the size of the largest party is two.

polynomial-time when the profiles are single-peaked.

polynomial-time when the profiles are single-crossing.

Piotr Faliszewski, Laurent Gourvès, Jérôme Lang, Julien Lesca, Jèrôme Monnot.
How hard is it for a party to nominate an election winner? AAAI 2016

Talk Outline

Introduction

Preliminaries

High-level methodology

W[2]-hardness parameterized by #parties

Open Problems

Talk Outline

Introduction

Preliminaries

High-level methodology

W[2]-hardness parameterized by #parties

Open Problems

The Parameterized Paradigm

Beyond Worst-Case

Classical complexity: measure the performance of an algorithm as a function of the input size.

The Parameterized Paradigm

Beyond Worst-Case

Parameterized complexity: acknowledge the presence of additional structure, which manifests as a secondary measurement — a **parameter.**

The Parameterized Paradigm

Beyond Worst-Case

Parameterized complexity: acknowledge the presence of additional structure, which manifests as a secondary measurement — a **parameter.**

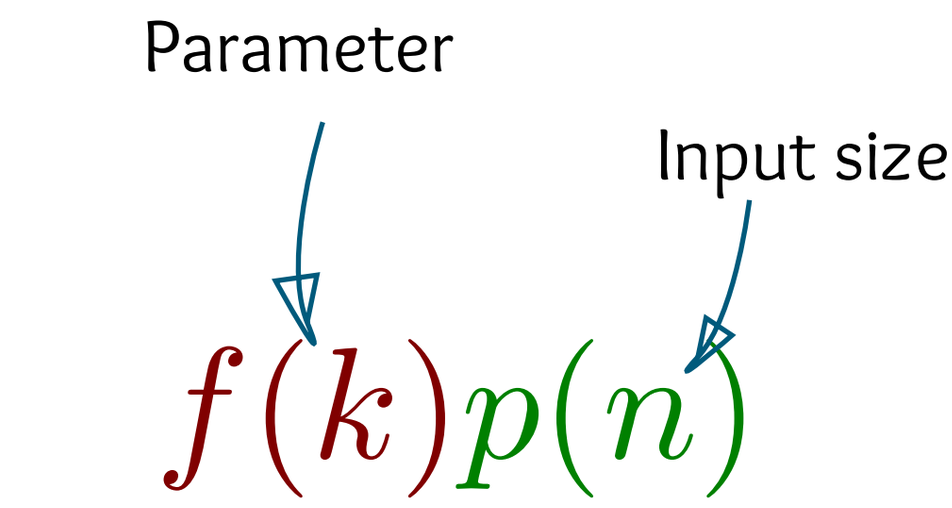
🎯 Design algorithms that **restrict the combinatorial explosion** to a function of the parameter.

The Parameterized Paradigm

Beyond Worst-Case

Parameterized complexity: acknowledge the presence of additional structure, which manifests as a secondary measurement — a **parameter.**

🎯 Design algorithms that **restrict the combinatorial explosion** to a function of the parameter.

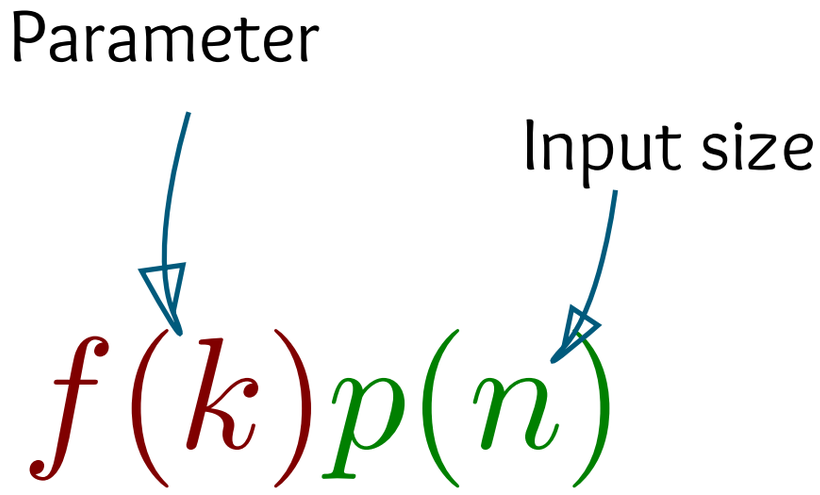


fixed-parameter tractability

The Parameterized Paradigm

Beyond Worst-Case

⚠️ W-hardness: a framework for arguing the
likely non-existence of FPT algorithms
for parameterized problems

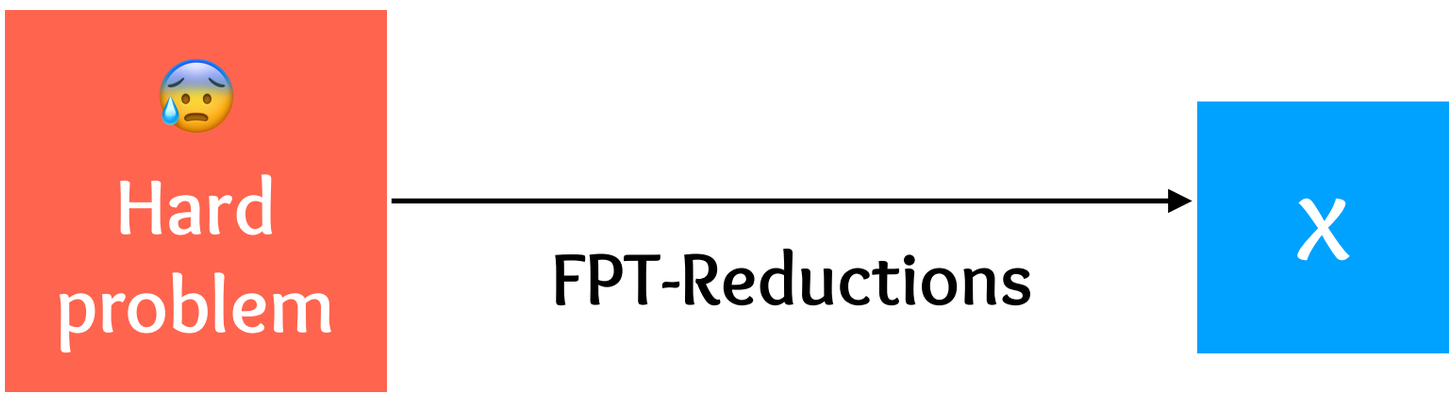


fixed-parameter tractability

The Parameterized Paradigm

Beyond Worst-Case

⚠️ W-hardness: a framework for arguing the
likely non-existence of FPT algorithms
for parameterized problems



The Parameterized Paradigm

Beyond Worst-Case

⚠️ W-hardness: a framework for arguing the
likely non-existence of FPT algorithms
for parameterized problems

Runs in FPT time • Preserves the parameter • Maintains equivalence

Talk Outline

Introduction

Preliminaries

High-level methodology

W[2]-hardness parameterized by #parties

Open Problems

High Level Methodology

Possible President

NP-complete even when the size of the largest party is two, *and* the profiles are 1D-Euclidean.

XP and **W[2]-hard** parameterized by the **number of parties**.

FPT parameterized by **number of parties** on **1D-Euclidean profiles.**

High Level Methodology

Possible President

NP-complete even when the size of the largest party is two,

and the profiles are 1D-Euclidean.

XP and **W[2]-hard** parameterized by the **number of parties**.

FPT parameterized by **number of parties** on **1D-Euclidean profiles.**

Reduction from "Linear" SAT aka LSAT
(a structured variation of SAT,
originally used in the context of geometric problems*)

---

* Esther M. Arkin, Aritra Banik, Paz Carmi, Gui Citovsky, Matthew J. Katz, Joseph S. B. Mitchell, Marina Simakov. Choice is Hard, ISAAC 2015

High Level Methodology

Possible President

NP-complete even when the size of the largest party is two, *and* the profiles are 1D-Euclidean.

XP and **W[2]-hard** parameterized by the **number of parties**.

FPT parameterized by **number of parties** on **1D-Euclidean profiles.**

Brute-force
(guess the nominee from each party)

High Level Methodology

Possible President

NP-complete even when the size of the largest party is two,

and the profiles are 1D-Euclidean.

XP and **W[2]-hard** parameterized by the **number of parties**.

FPT parameterized by **number of parties** on **1D-Euclidean profiles.**

FPT-reduction
(from a variant of Dominating Set,
also coming up in this talk)

High Level Methodology

Possible President

NP-complete even when the size of the largest party is two, *and* the profiles are 1D-Euclidean.

XP and **W[2]-hard** parameterized by the **number of parties**.

FPT parameterized by **number of parties** on **1D-Euclidean profiles.**

Dynamic Programming
(updates along the 1D-Euclidean axis,
also appeals to "SP and SC aspects" of 1D-Euclidean profiles)

High Level Methodology

Possible President

NP-complete even when the size of the largest party is two, *and* the profiles are 1D-Euclidean.

XP and **W[2]-hard** parameterized by the **number of parties**.

FPT parameterized by **number of parties** on **1D-Euclidean profiles.**

Necessary President

polynomial-time when the profiles are single-crossing.

High Level Methodology

Possible President

NP-complete *even* when the size of the largest party is two, *and* the profiles are 1D-Euclidean.

XP and **W[2]-hard** parameterized by the **number of parties**.

FPT parameterized by **number of parties** on **1D-Euclidean profiles.**

Necessary President

polynomial-time when the profiles are single-crossing.

Adversarial approach: guess a nominee + a rival candidate
(use a "block property" and reduce to a structured Hitting Set instance)

Talk Outline

Introduction

Preliminaries

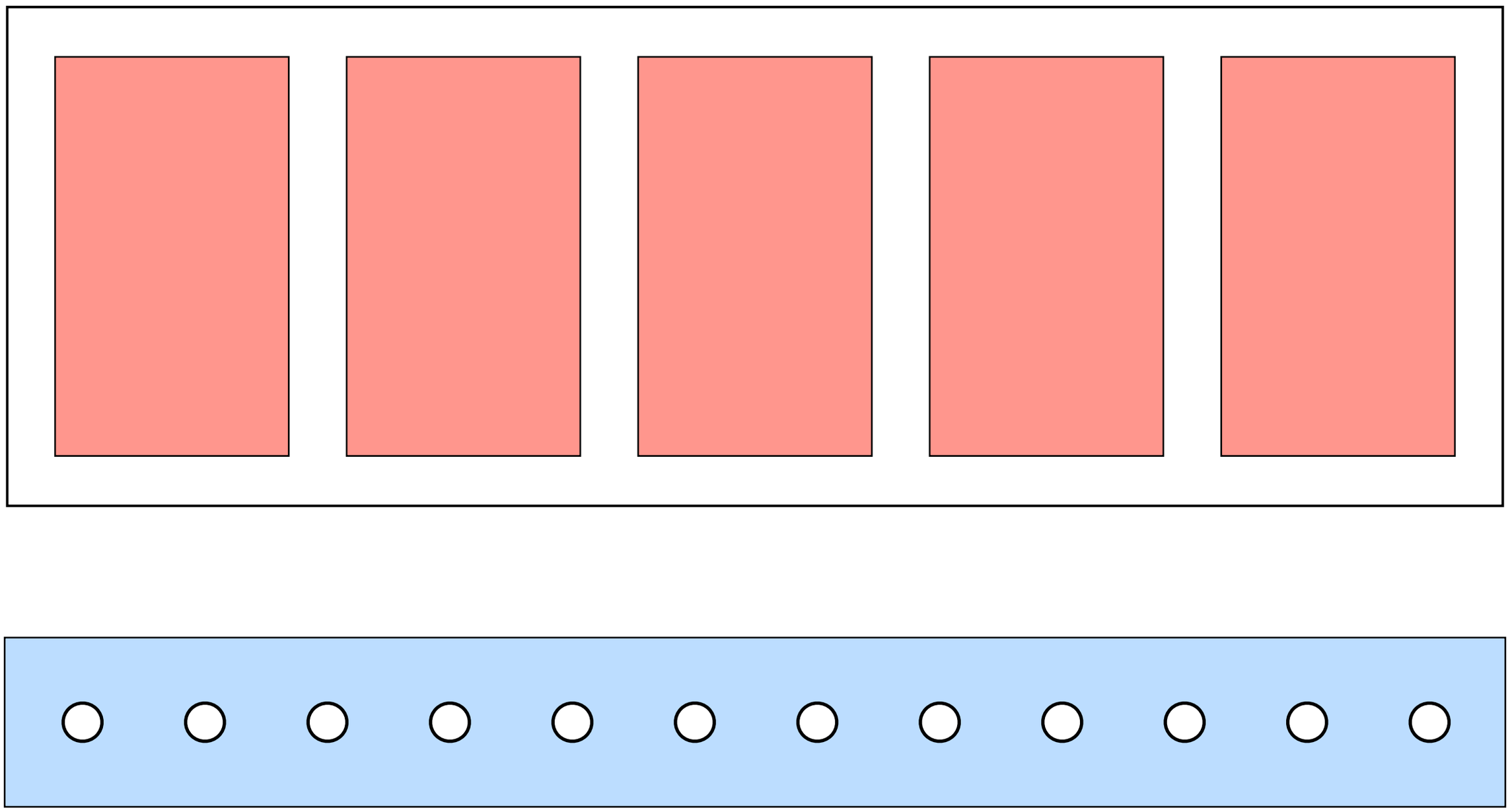
High-level methodology

W[2]-hardness parameterized by #parties

Open Problems

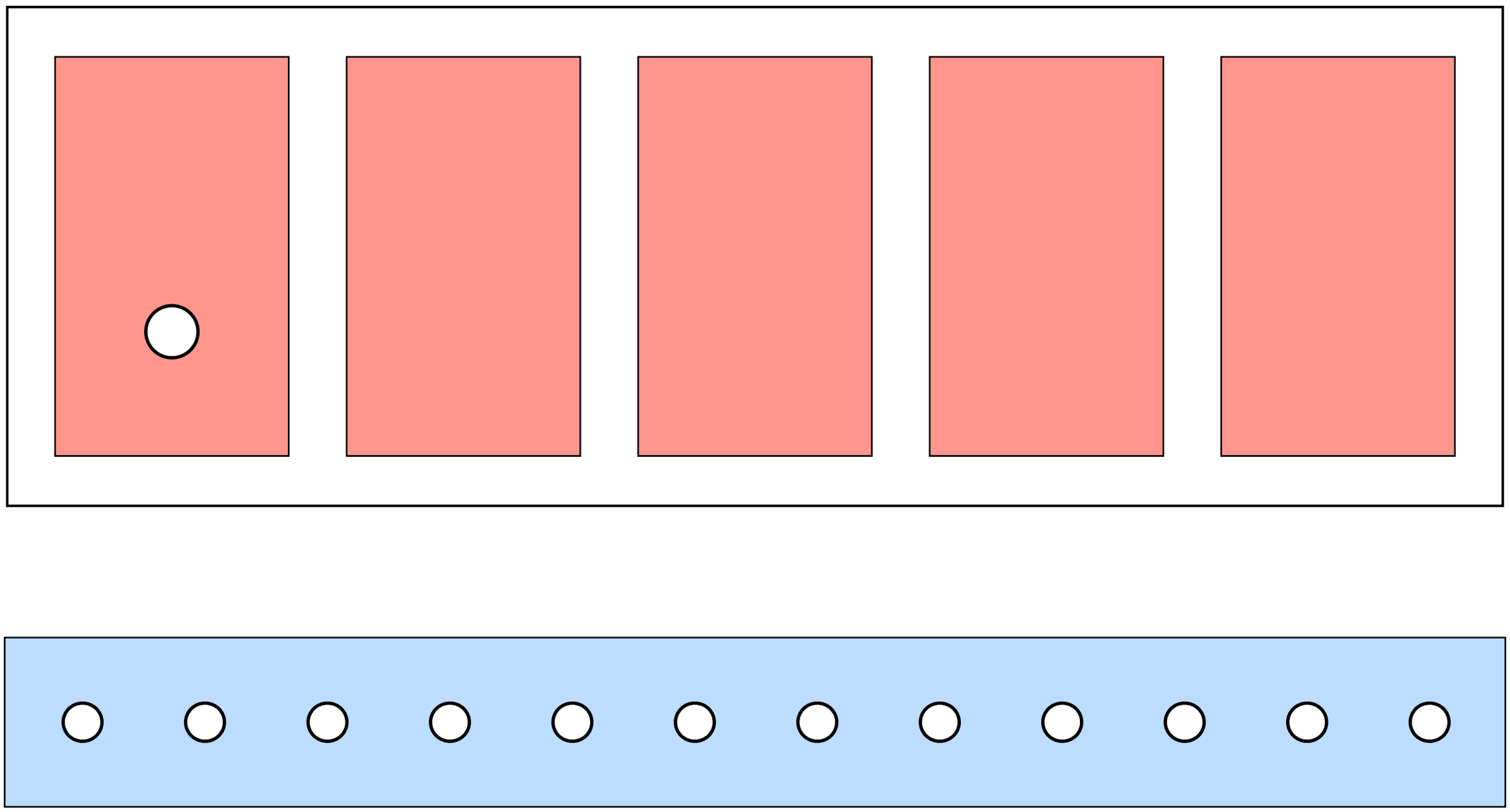
Colourful Red-Blue Dominating Set

— hard parameterized by the “solution size”



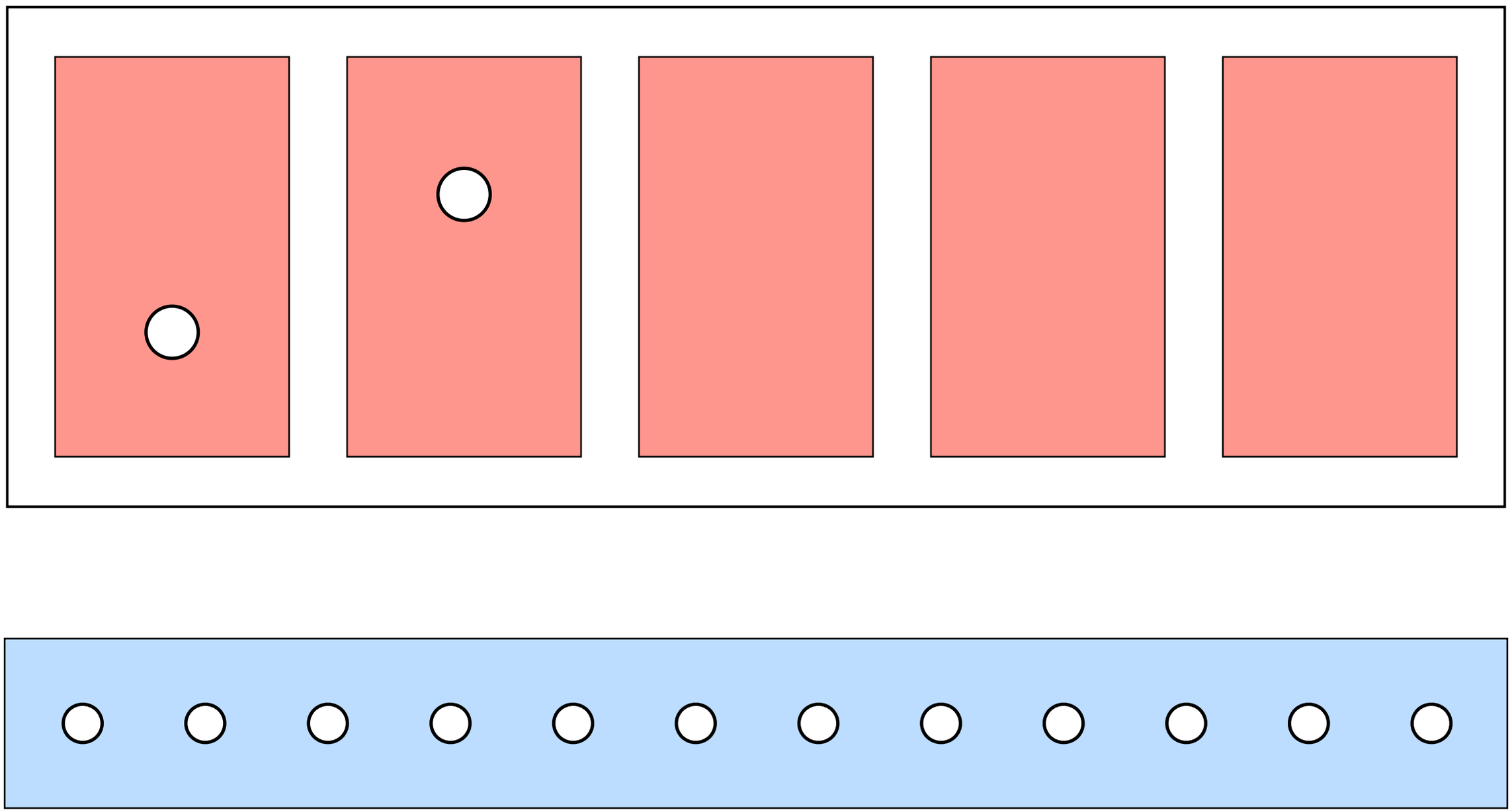
Colourful Red-Blue Dominating Set

— hard parameterized by the “solution size”



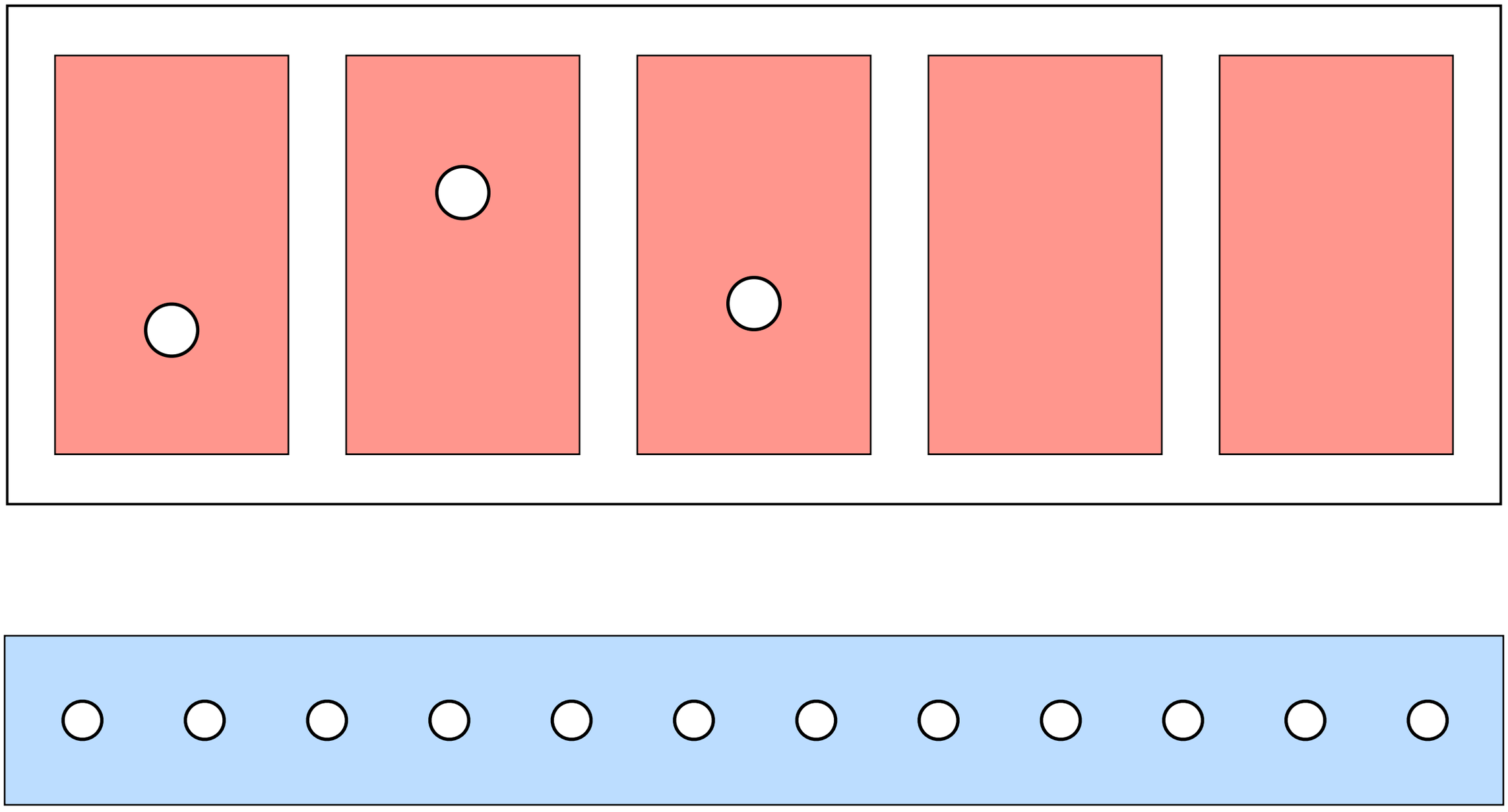
Colourful Red-Blue Dominating Set

— hard parameterized by the “solution size”



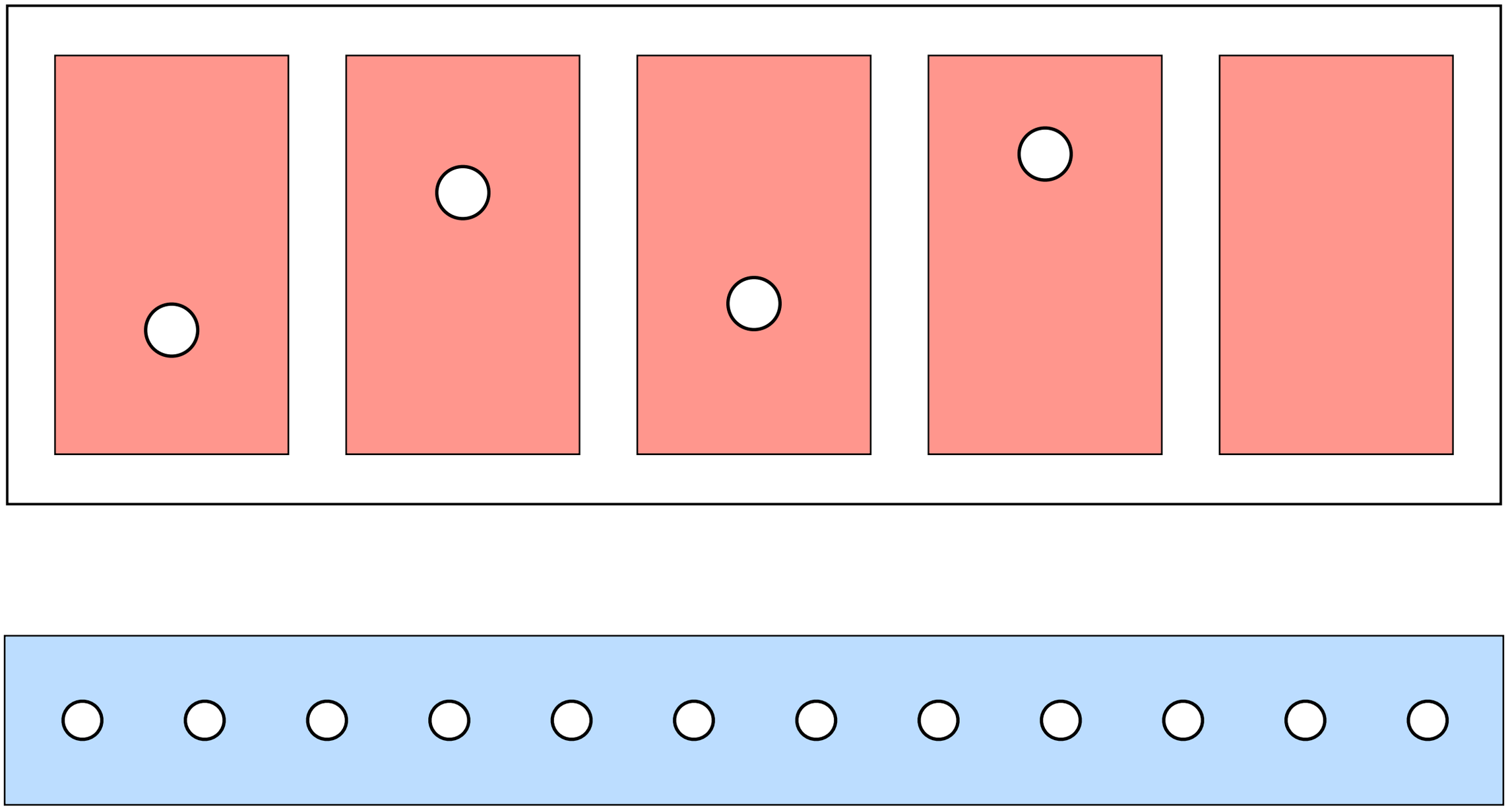
Colourful Red-Blue Dominating Set

— hard parameterized by the “solution size”



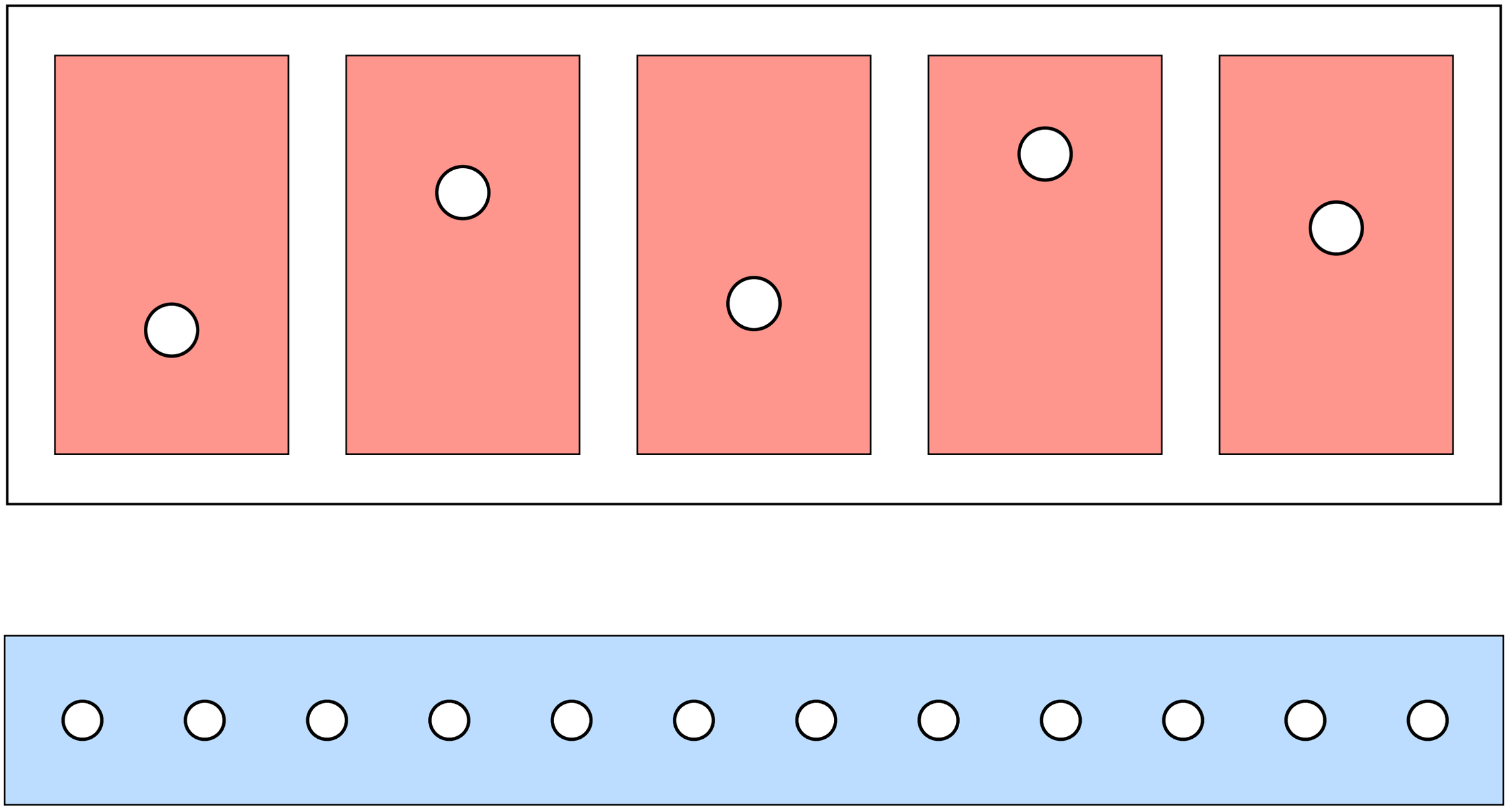
Colourful Red-Blue Dominating Set

— hard parameterized by the “solution size”

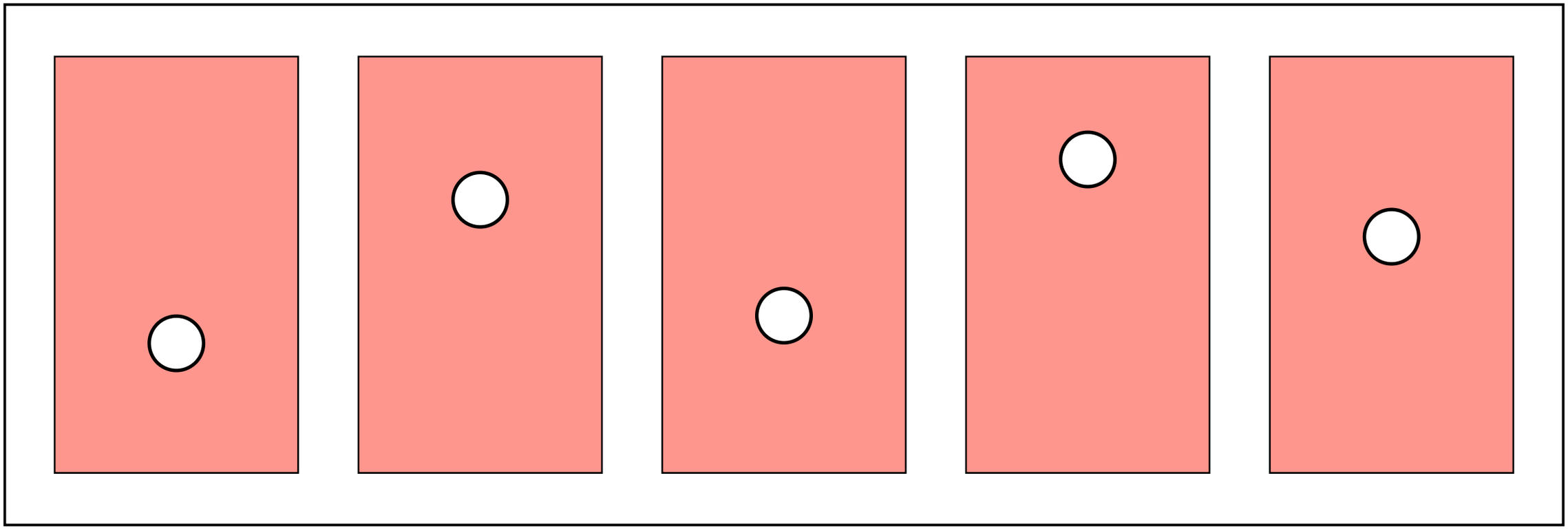


Colourful Red-Blue Dominating Set

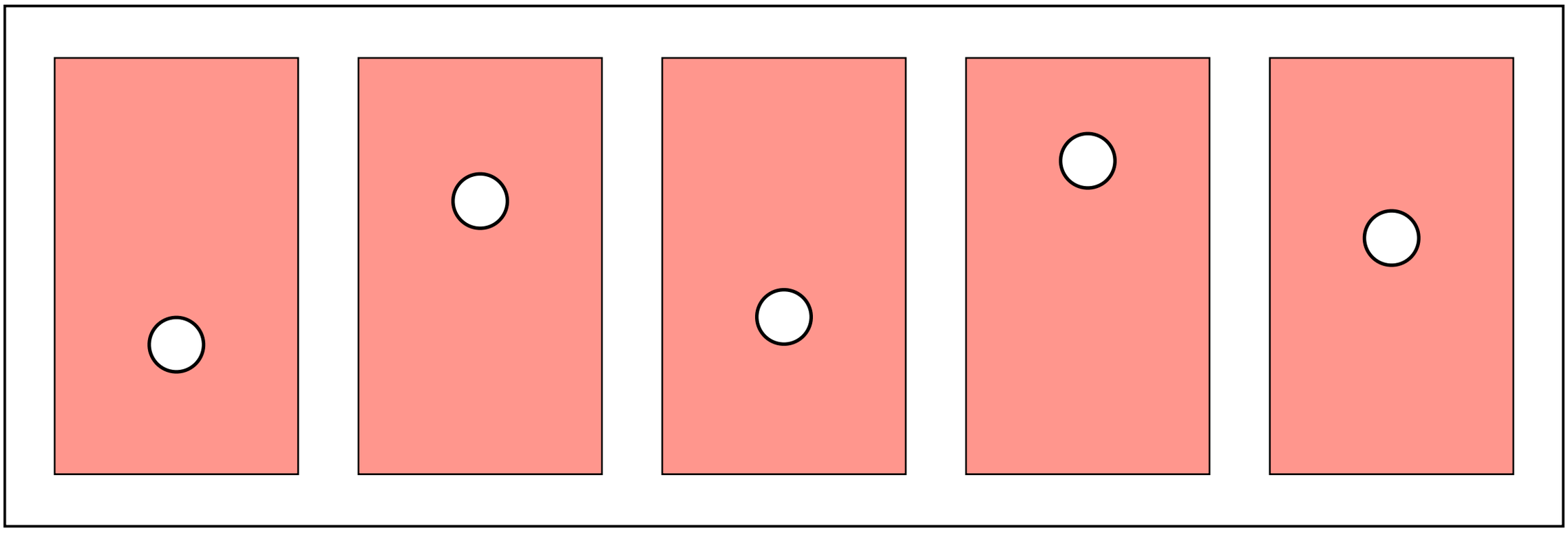
— hard parameterized by the “solution size”



W[2]-hardness of Possible President (parameterized by #parties)

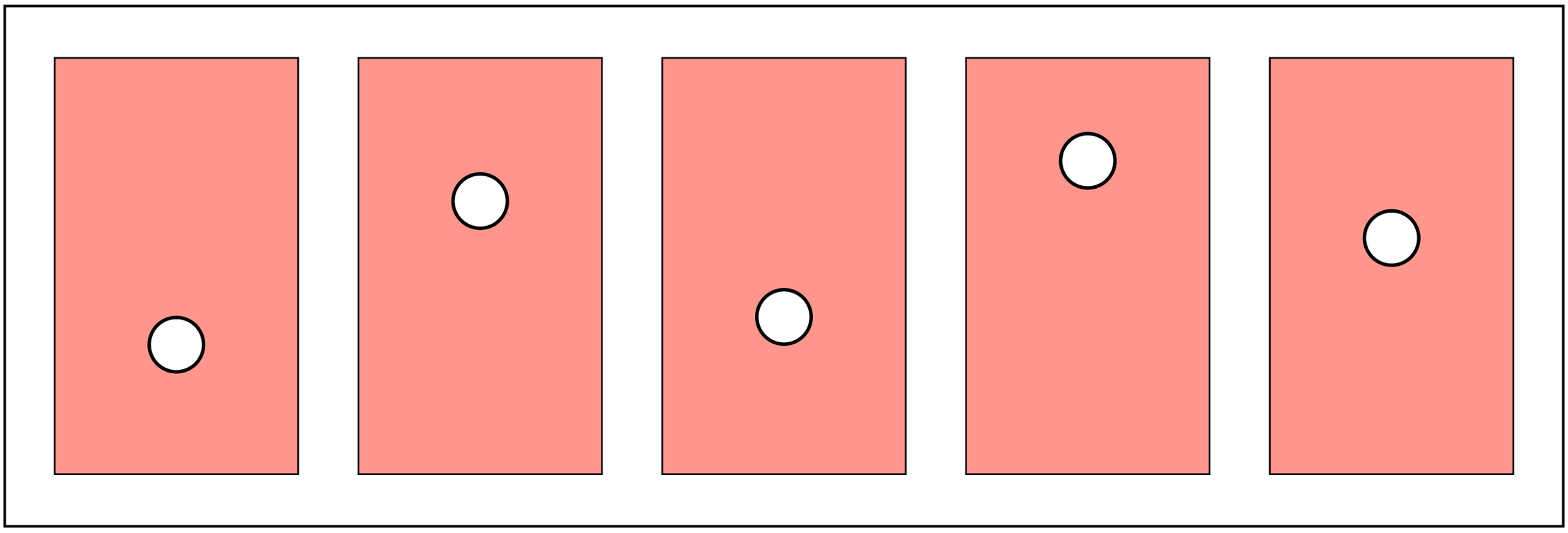


W[2]-hardness of Possible President (parameterized by #parties)



Introduce a candidate for *every* red vertex;
and **two special candidates p and q.**

W[2]-hardness of Possible President (parameterized by #parties)



Introduce a candidate for every red vertex;
and **two special candidates p and q.**

Parties. p,q are singletons.
The other parties correspond to color classes of the CRBDS instance.

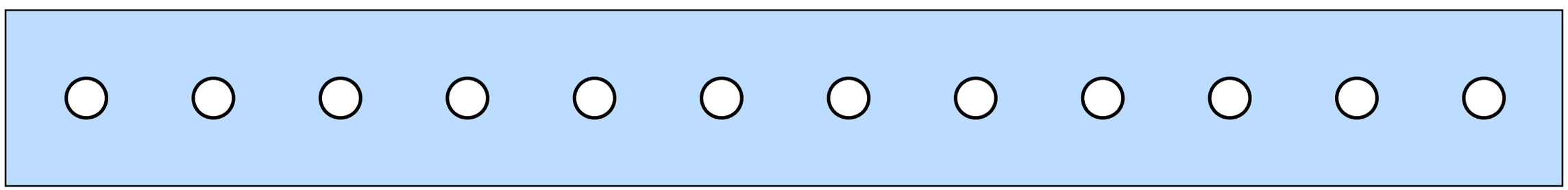
W[2]-hardness of Possible President (parameterized by #parties)

Introduce a vote for every blue vertex with the ordering:

non-neighbours

$$v_k : \overrightarrow{S_k} \succ q \succ \overrightarrow{C \setminus S_k} \succ p.$$

neighbours



W[2]-hardness of Possible President (parameterized by #parties)

W[2]-hardness of Possible President (parameterized by #parties)

Also introduce n copies of two special votes:

W[2]-hardness of Possible President (parameterized by #parties)

Also introduce n copies of two special votes:

$$g_p : p \succ q \succ \overrightarrow{C} \text{ and } g_q : q \succ p \succ \overrightarrow{C}.$$

W[2]-hardness of Possible President (parameterized by #parties)

Also introduce n copies of two special votes:

$$g_p : p \succ q \succ \overrightarrow{C} \text{ and } g_q : q \succ p \succ \overrightarrow{C}.$$

Ask if **p** is a possible president.

W[2]-hardness of Possible President (parameterized by #parties)

Also introduce n copies of two special votes:

$$g_p : p \succ q \succ \overrightarrow{C} \text{ and } g_q : q \succ p \succ \overrightarrow{C}.$$

Ask if **p** is a possible president.

Answer: Yes if and only if the "other nominees" correspond to a colourful red-blue dominating set.

W[2]-hardness of Possible President (parameterized by #parties)

Also introduce n copies of two special votes:

$$g_p : p \succ q \succ \overrightarrow{C} \text{ and } g_q : q \succ p \succ \overrightarrow{C}.$$

Ask if **p** is a possible president.

To begin with, p and q tie at a score of n each.

p's score is "locked in" at n.

Nominees from a dominating set
"block" q from acquiring any additional score.

Talk Outline

Introduction

Preliminaries

High-level methodology

W[2]-hardness parameterized by #parties

Open Problems

Open Problems

Is Possible President parameterized by the number of parties FPT on single-peaked or single-crossing domains?

Parameterized complexity when parameterized by the number of voters?

Open Problems

Intermediate notions of incomplete information.

What if we have partial information about the other nominees,
served either in a stochastic fashion or
as a fixed fraction of the number of parties?

Thank You!