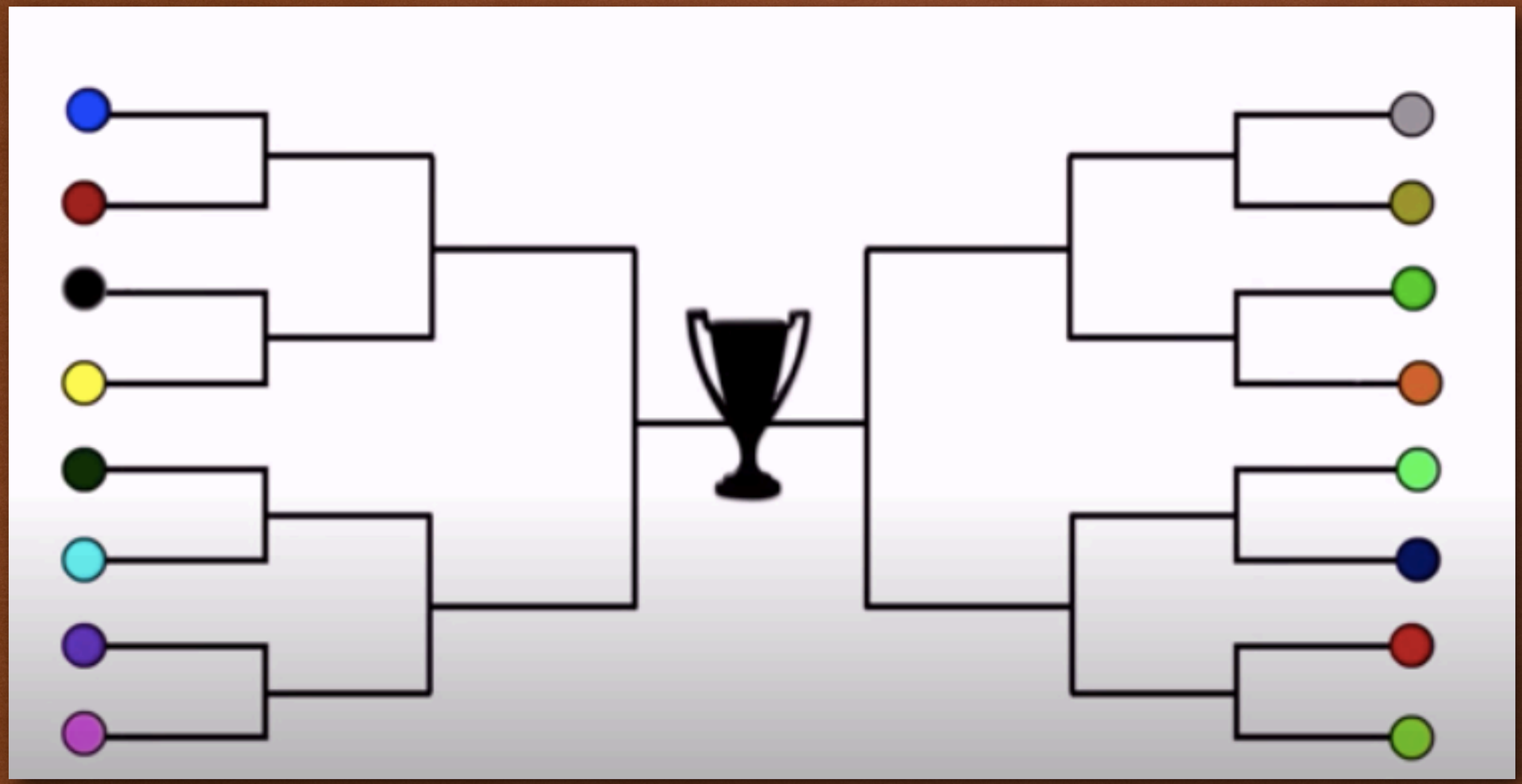
TO FIX A TOURNAMENT: A PARAMETERIZED COMPLEXITY PERSPECTIVE



SUSHMITA GUPTA, IMSC

TYPES OF TOURNAMENTS

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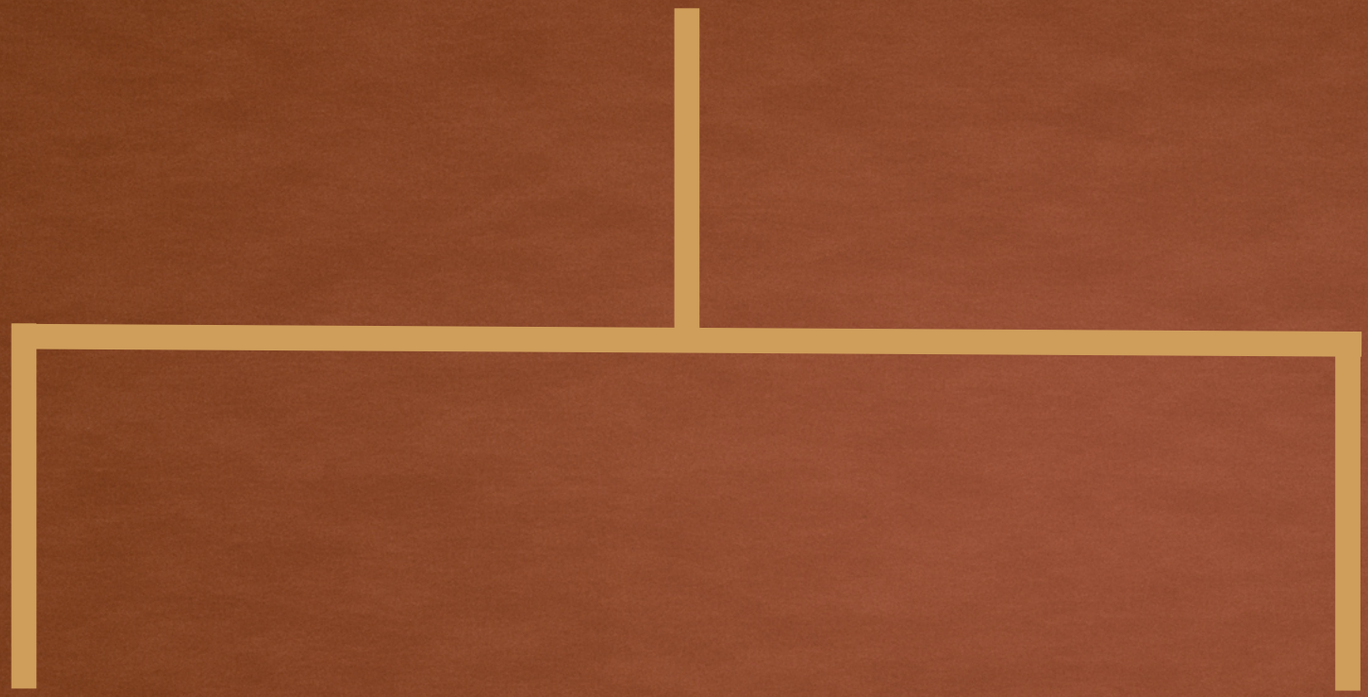

TYPES OF TOURNAMENTS

ROUND-ROBIN

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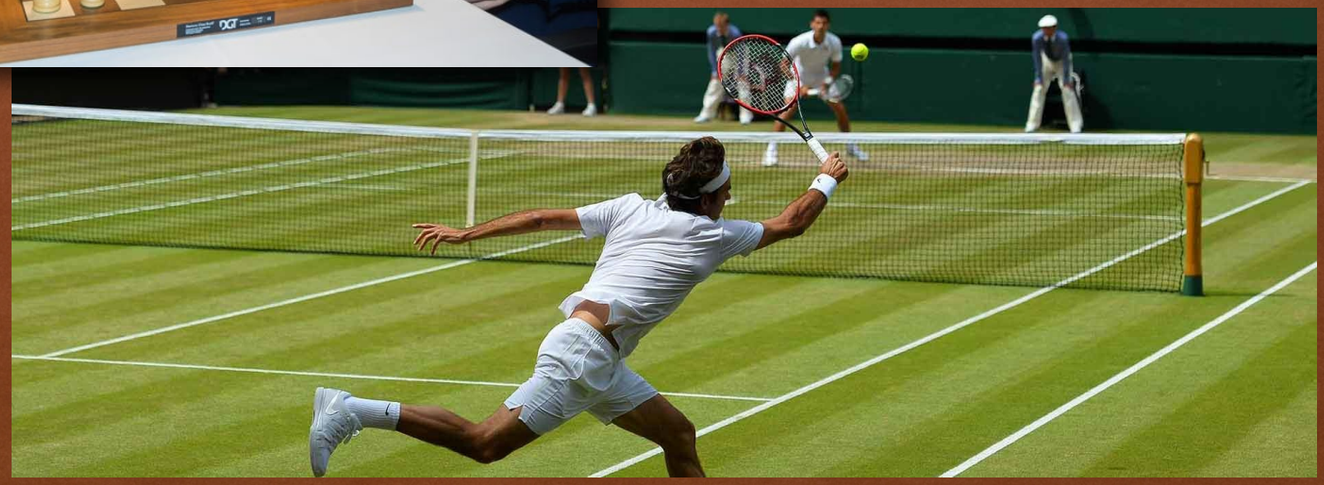
ROUND-ROBIN

KNOCKOUT

TOURNAMENTS: APPLICATIONS



MANIPULATION

TYPES OF MANIPULATIONS

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- CONSTRUCTIVE MANIPULATION
  - Make a favourite team win

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TYPES OF MANIPULATIONS

- CONSTRUCTIVE MANIPULATION
  - Make a favourite team win
- DESTRUCTIVE MANIPULATION
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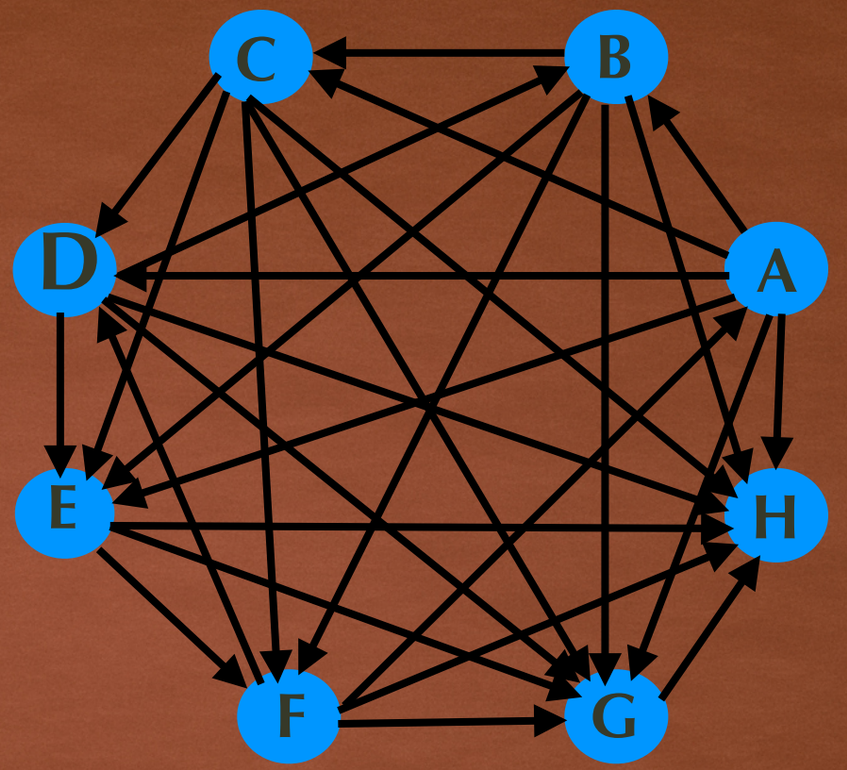
- OTHER CONSIDERATIONS
  - Specific matchups, Location, Playing time, Practice Time, etc

The Math Behind the Games...ROUND-ROBIN

The Math Behind the Games...ROUND-ROBIN

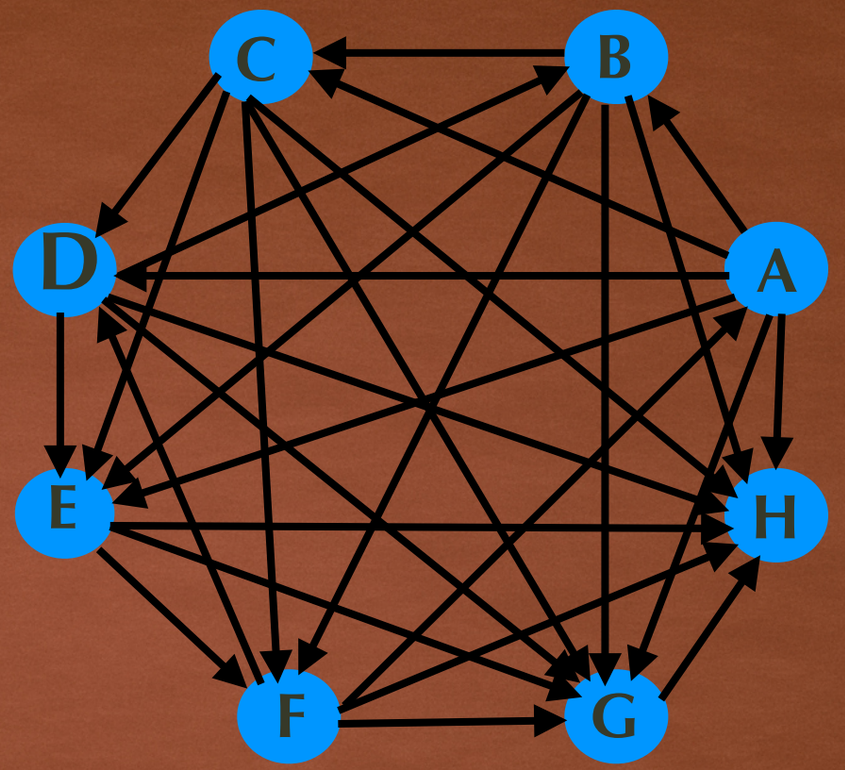
TOURNAMENT GRAPH



The Math Behind the Games...ROUND-ROBIN

TOURNAMENT GRAPH

No. Of vertices =No. Of teams



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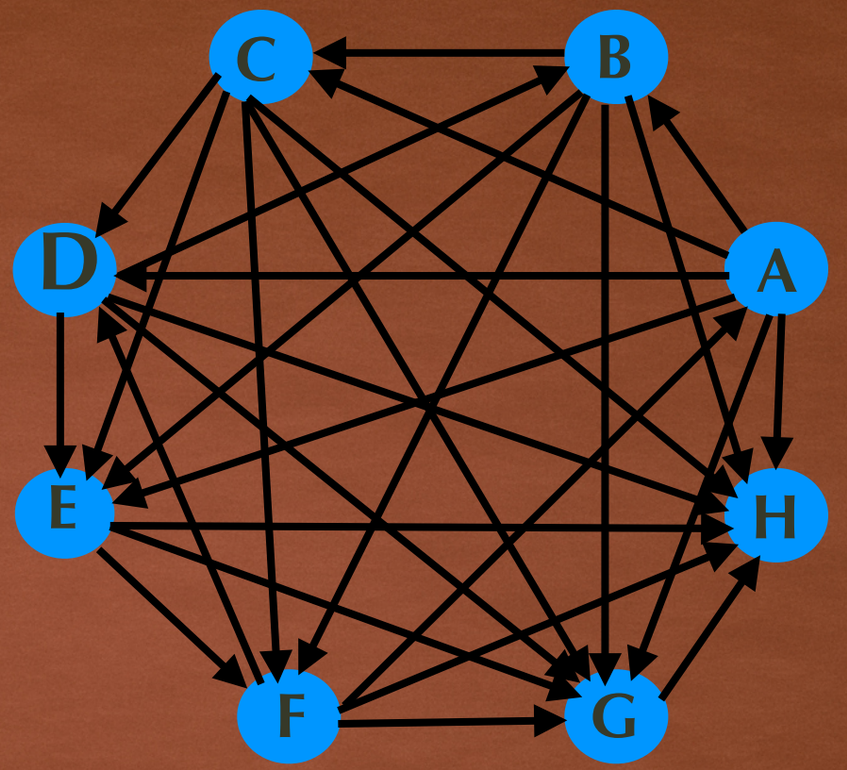

Direction of edge denotes who wins matchup

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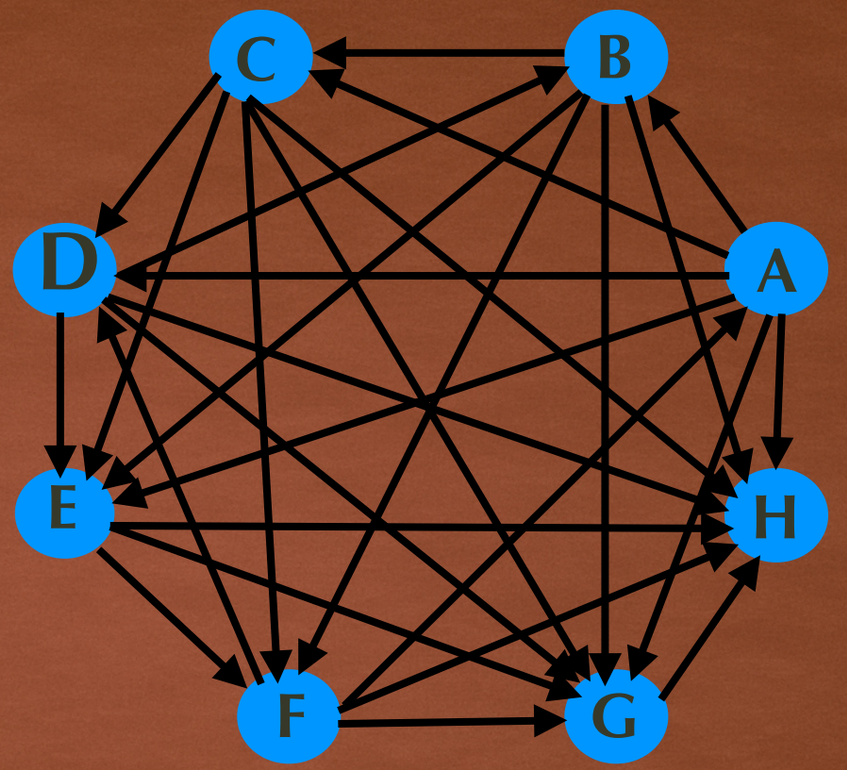

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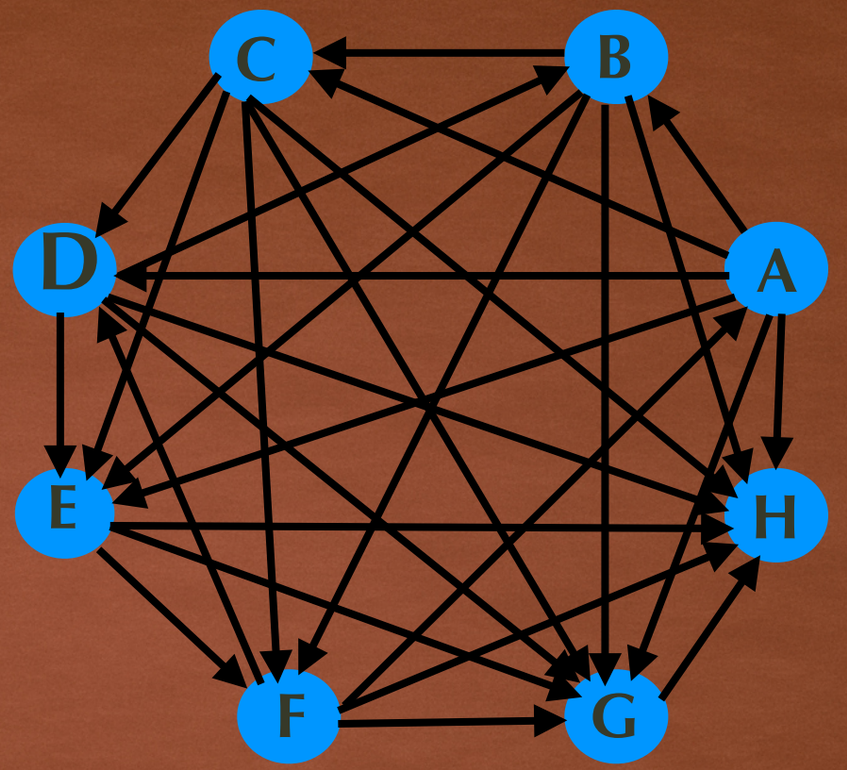

No. Of vertices =No. Of teams

Direction of edge denotes who wins matchup

Weights on edges may denote points awarded

Manipulation is reversing direction of the edge

The Math Behind the Games...

How to ensure our **favorite** team wins a knockout tournament?

How to ensure that favorite wins a knockout tournament?

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- ☑ We have predictive information about various match-ups

How to ensure that favorite wins a knockout tournament?

☑ We have predictive information about various match-ups

What if favorite didn't have to play those it can't beat....?

FIFA WORLD CUP'18
KNOCKOUT STAGES

Round of 16	Quarterfinals	Semifinals

FT	Match: 49
Uruguay	2
Portugal	1

FT	Match: 50
France	4
Argentina	3

FT	Match: 53
Brazil	2
Mexico	0

FT	Match: 54
Belgium	3
Japan	2

FT - PENS	Match: 51
Spain	1 (3)
Russia	1 (4)

FT - PENS	Match: 52
Croatia	1 (3)
Denmark	1 (2)

FT	Match: 55
Sweden	1
Switzerland	0

FT - PENS	Match: 56
Colombia	1 (3)
England	1 (4)

FT	Match: 57
Uruguay	0
France	2

FT	Match: 58
Brazil	1
Belgium	2

FT - PENS	Match: 59
Russia	2 (3)
Croatia	2 (4)

FT	Match: 60
Sweden	0
England	2

FT	Match: 61
France	1
Belgium	0

AET	Match: 62
Croatia	2
England	1

Seeding in a Tournament

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- **Preliminary ranking for the purpose of draw/bracket.**
  - Originally used in Tennis

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  - Originally used in Tennis
- **It describes a player's path to the final and potential opponents in each round.**
  - Specific to the tournament.

SEEDING

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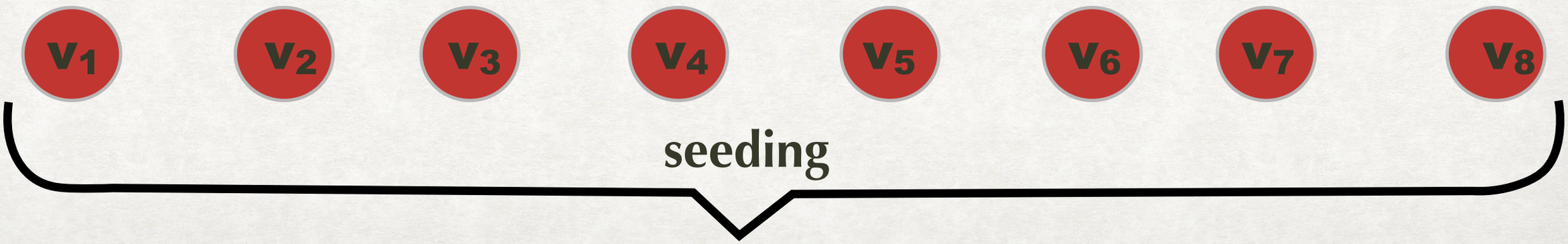

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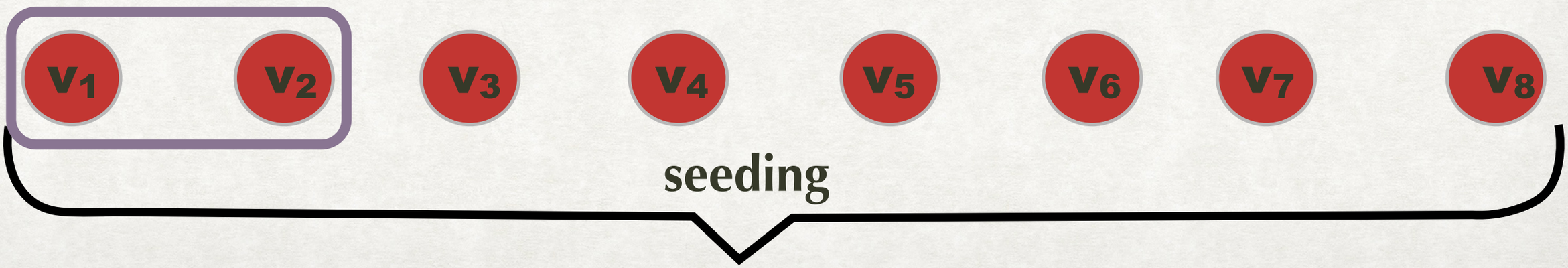

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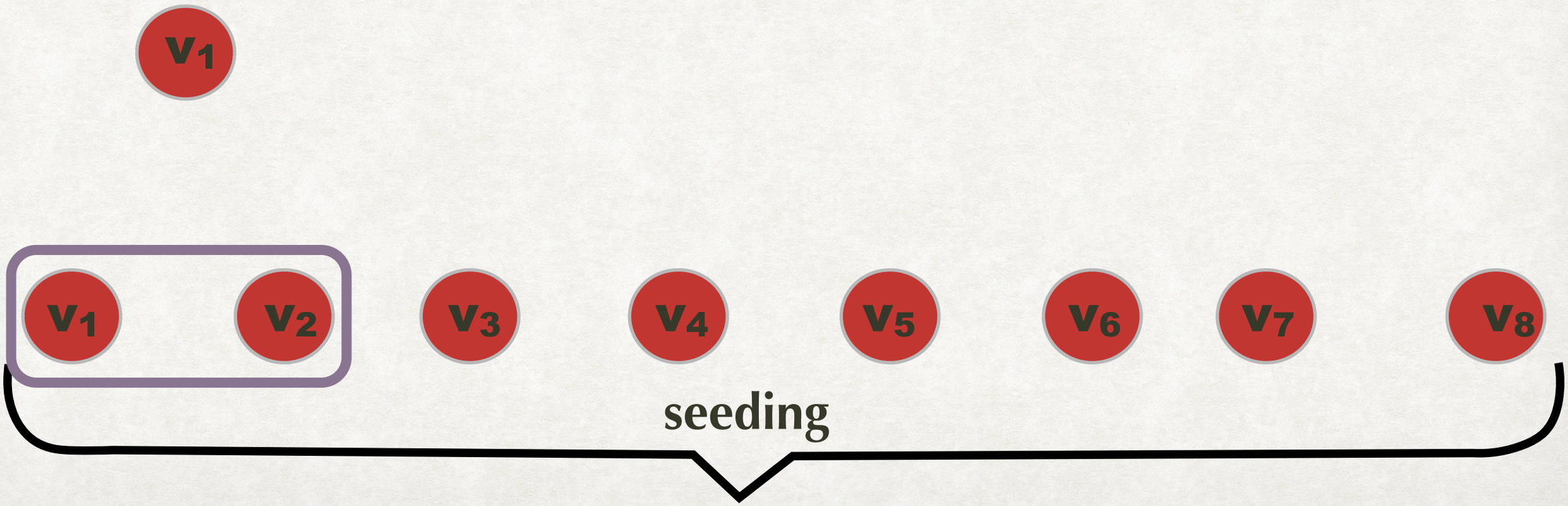

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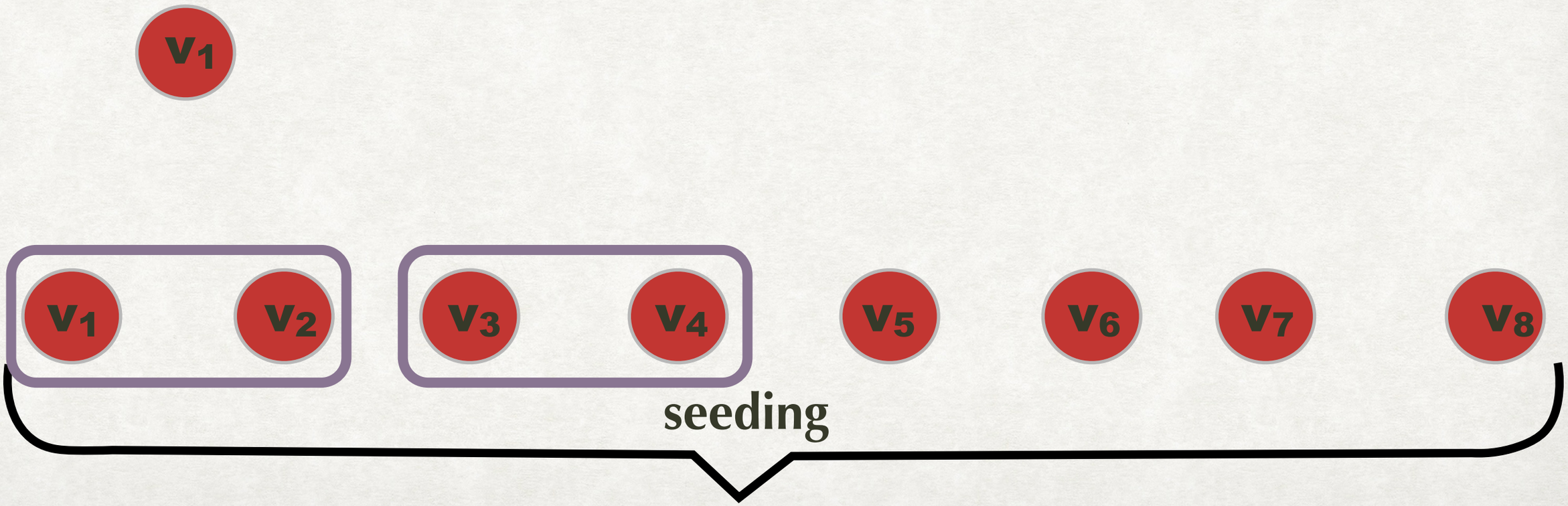

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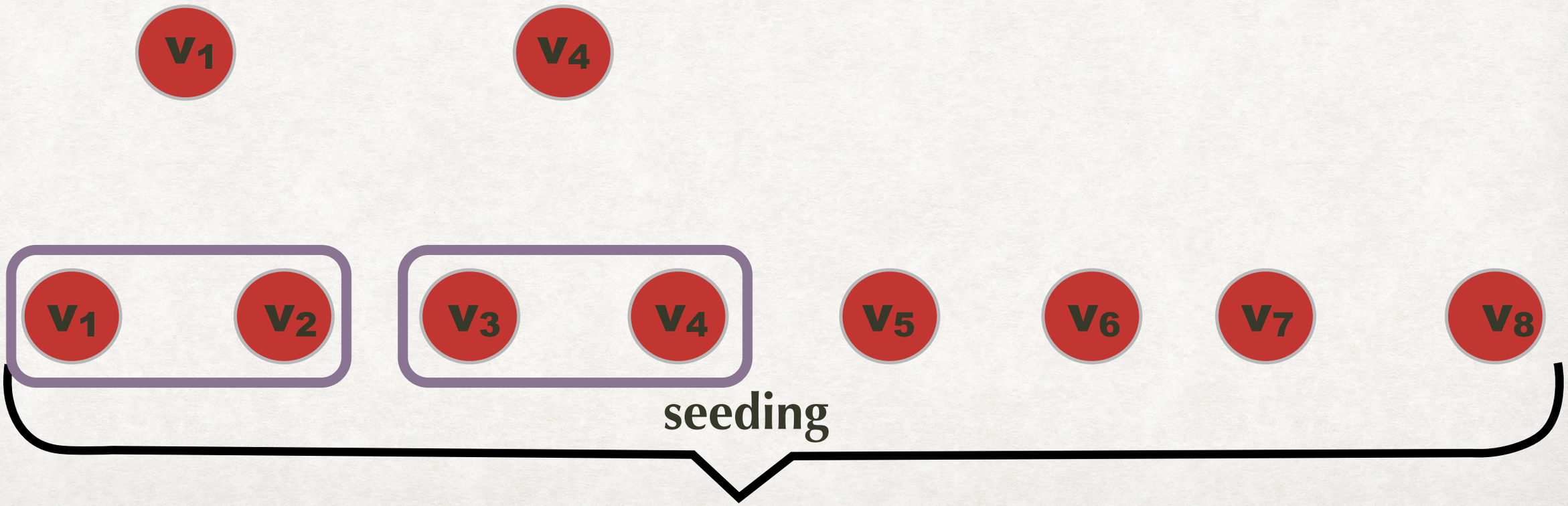

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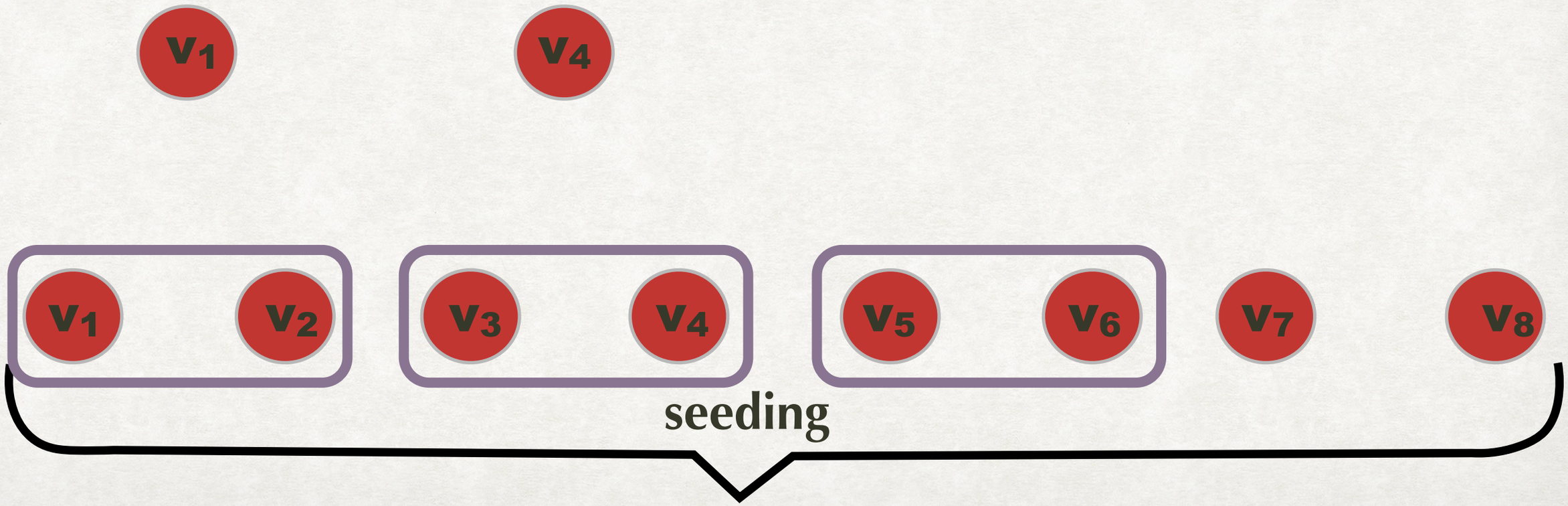

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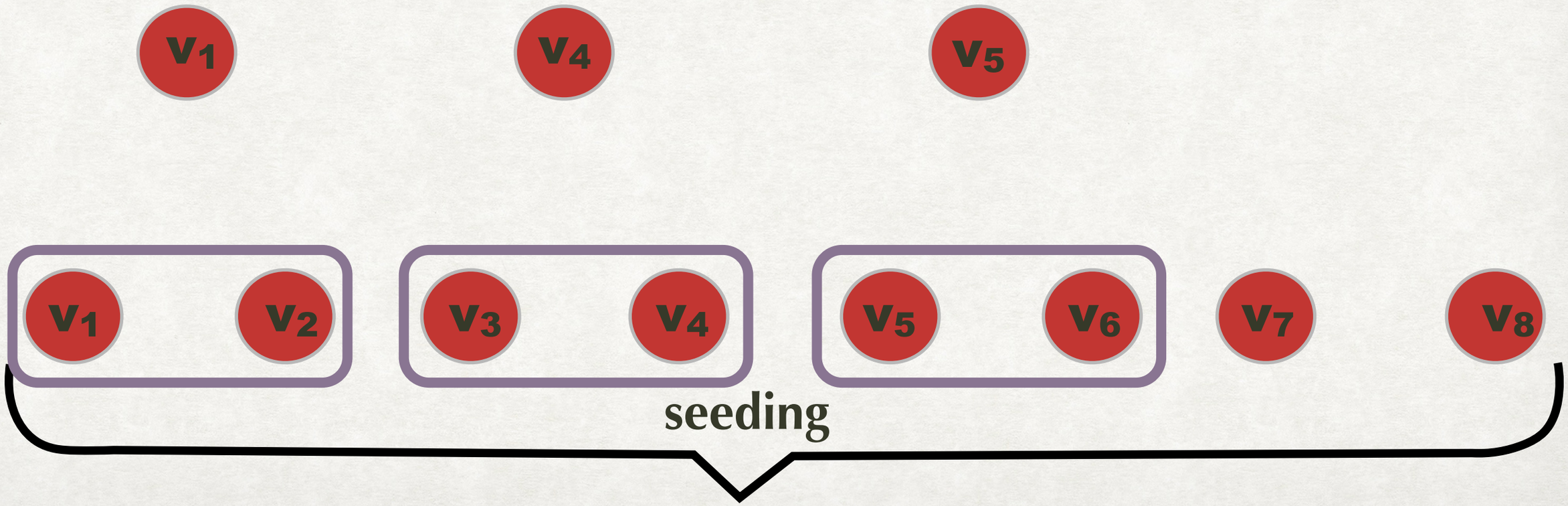

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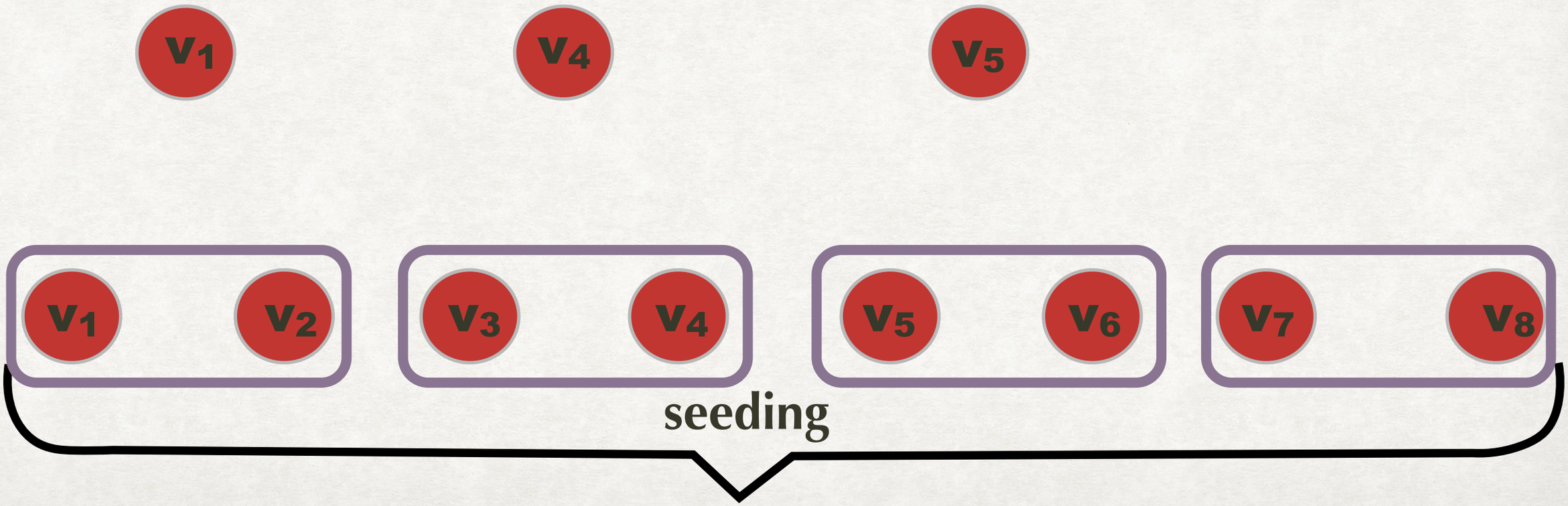

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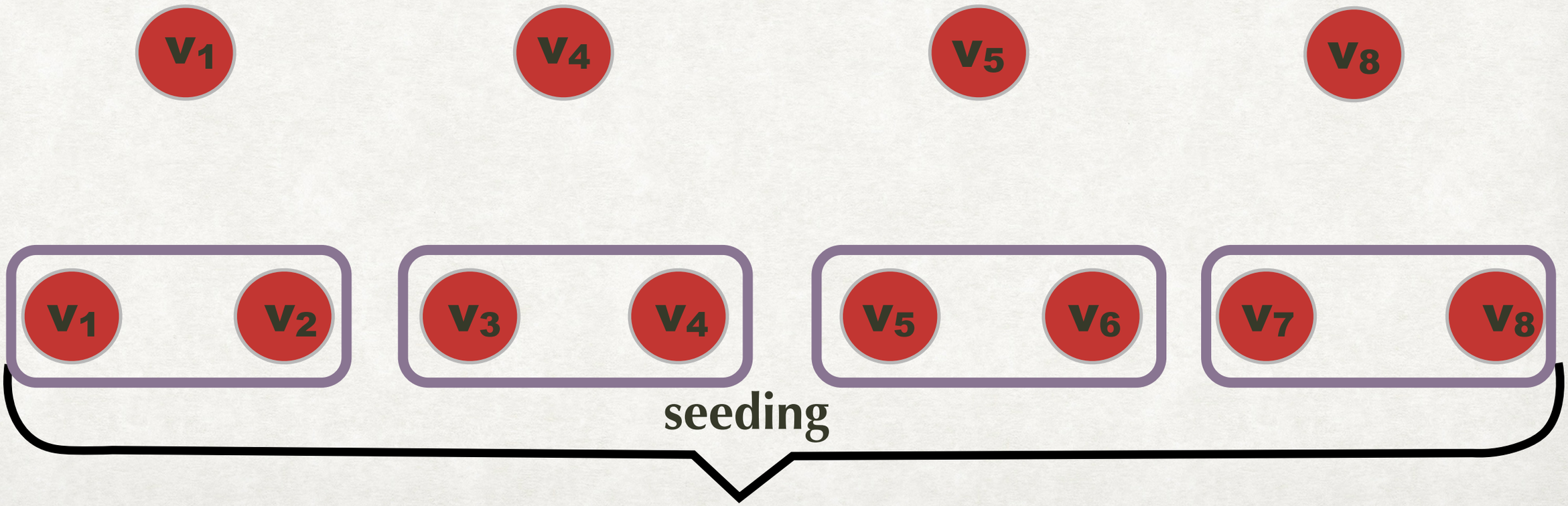

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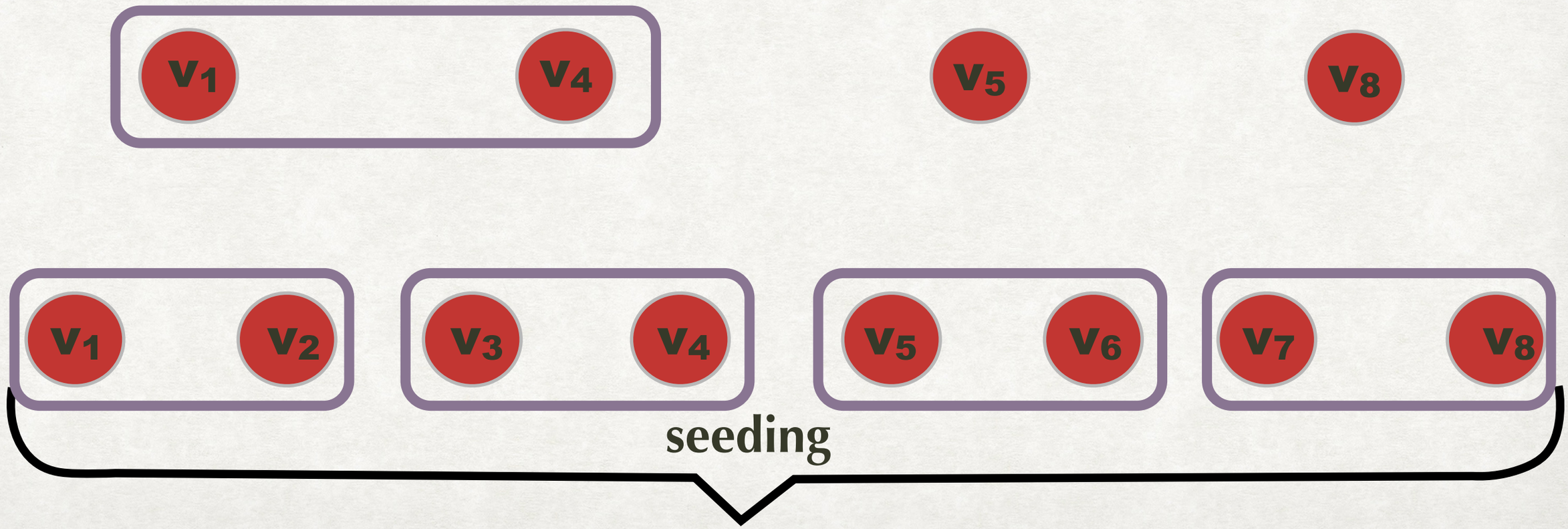

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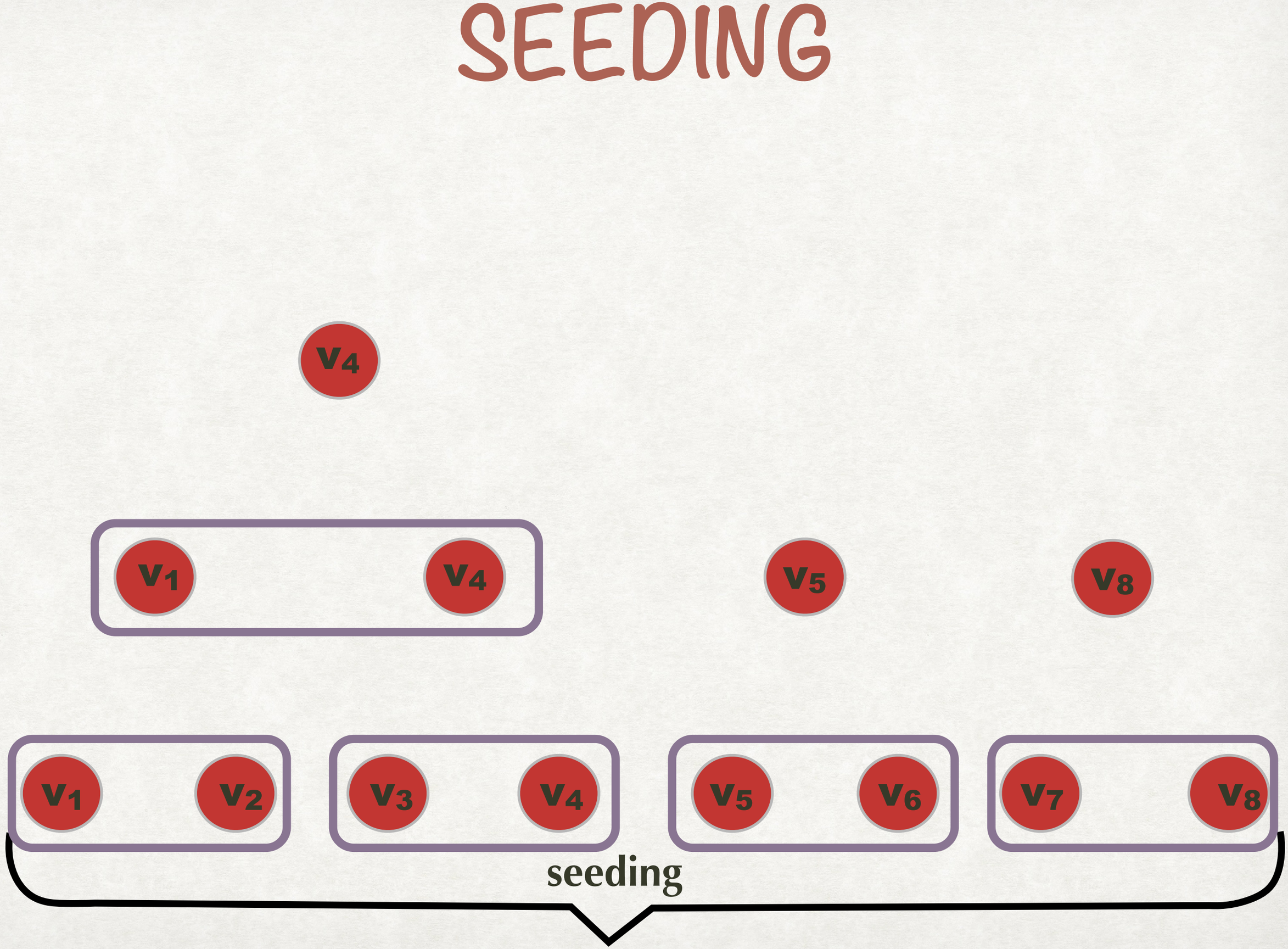

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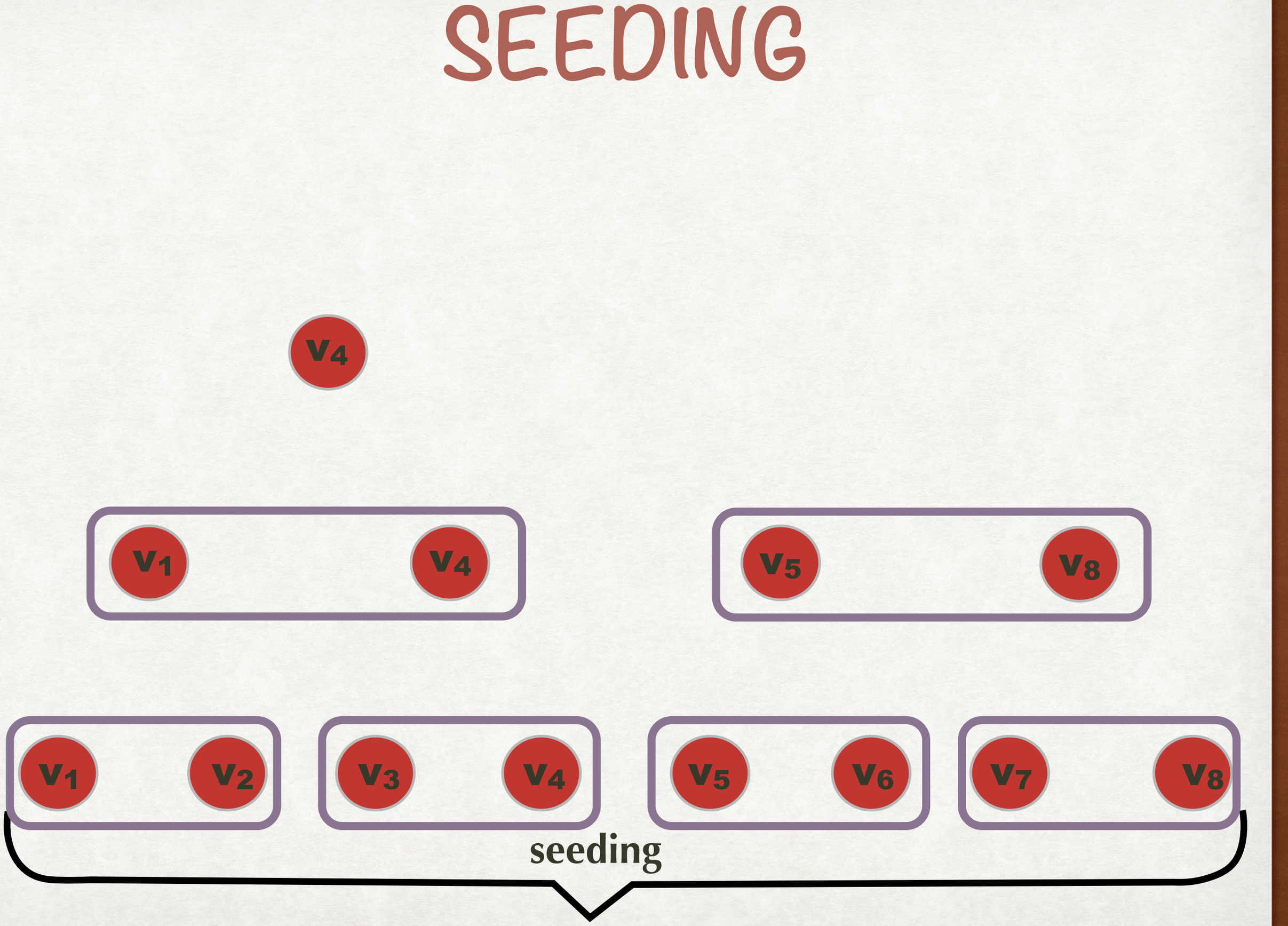

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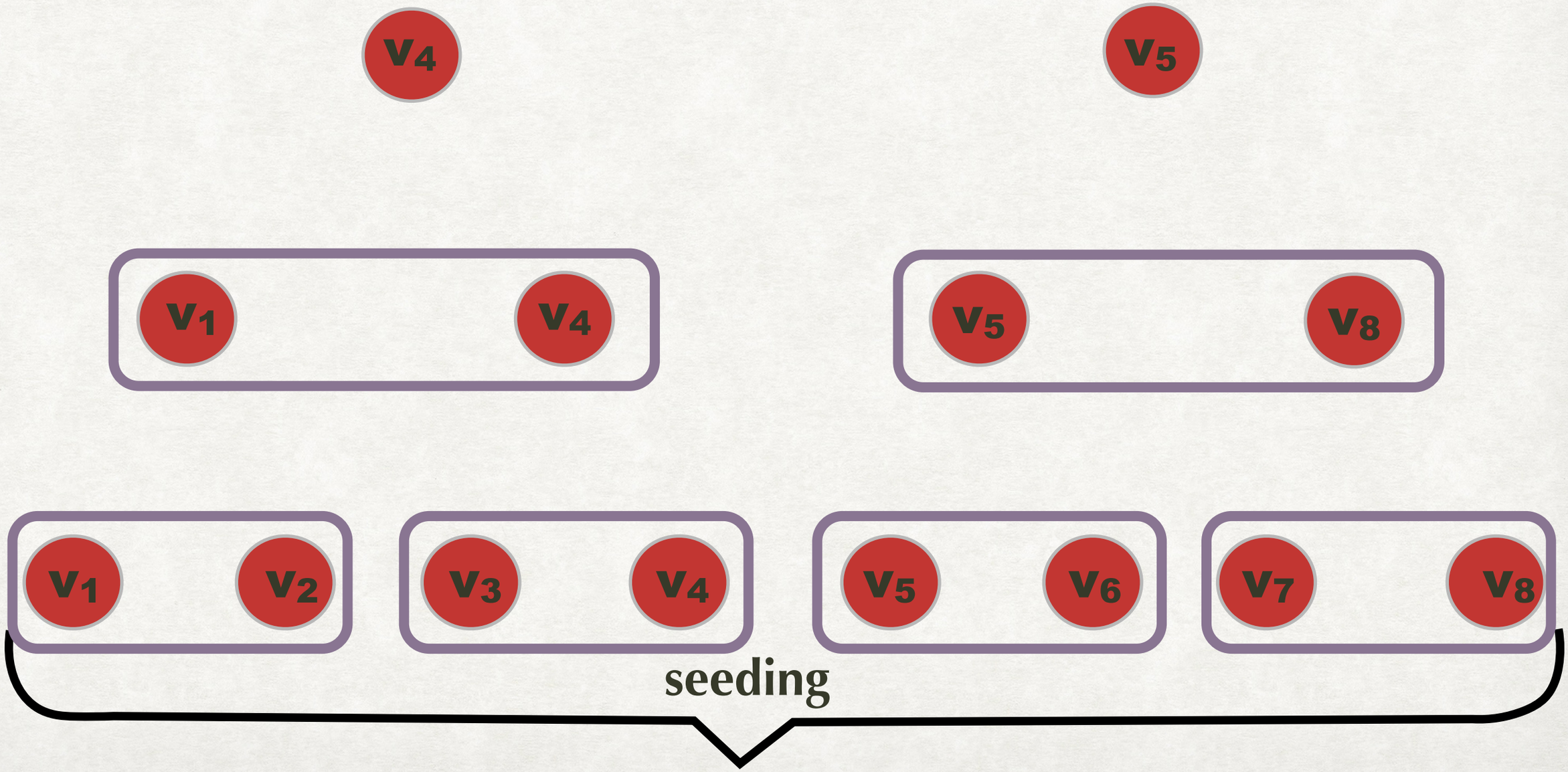

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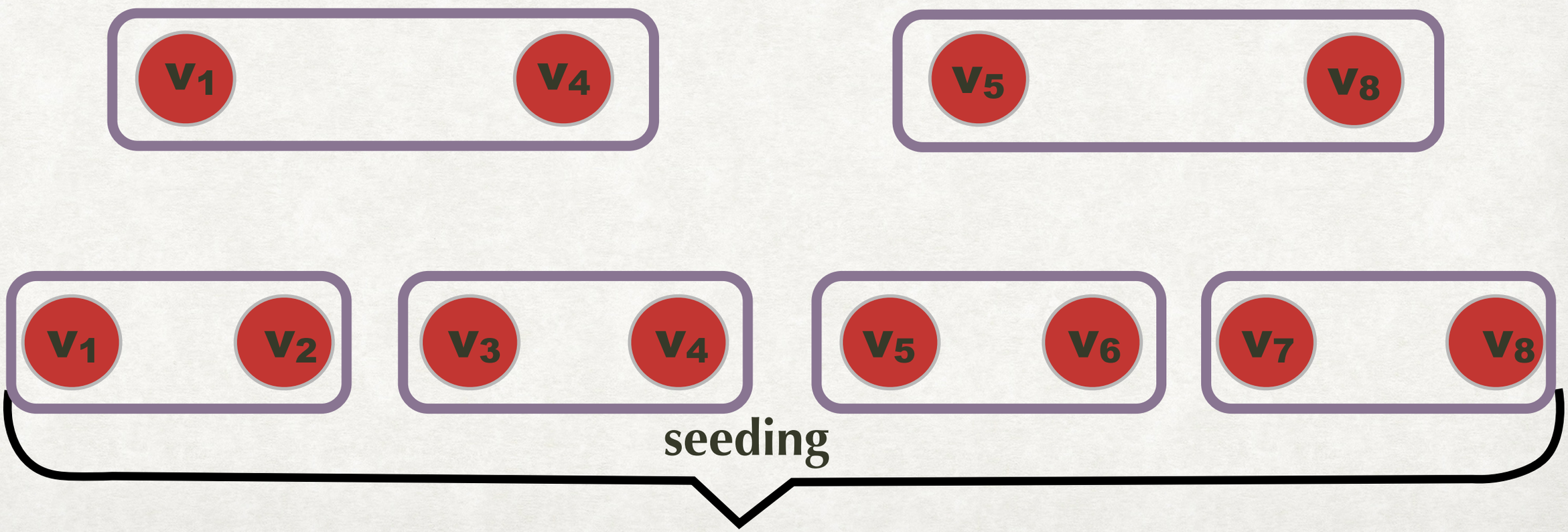

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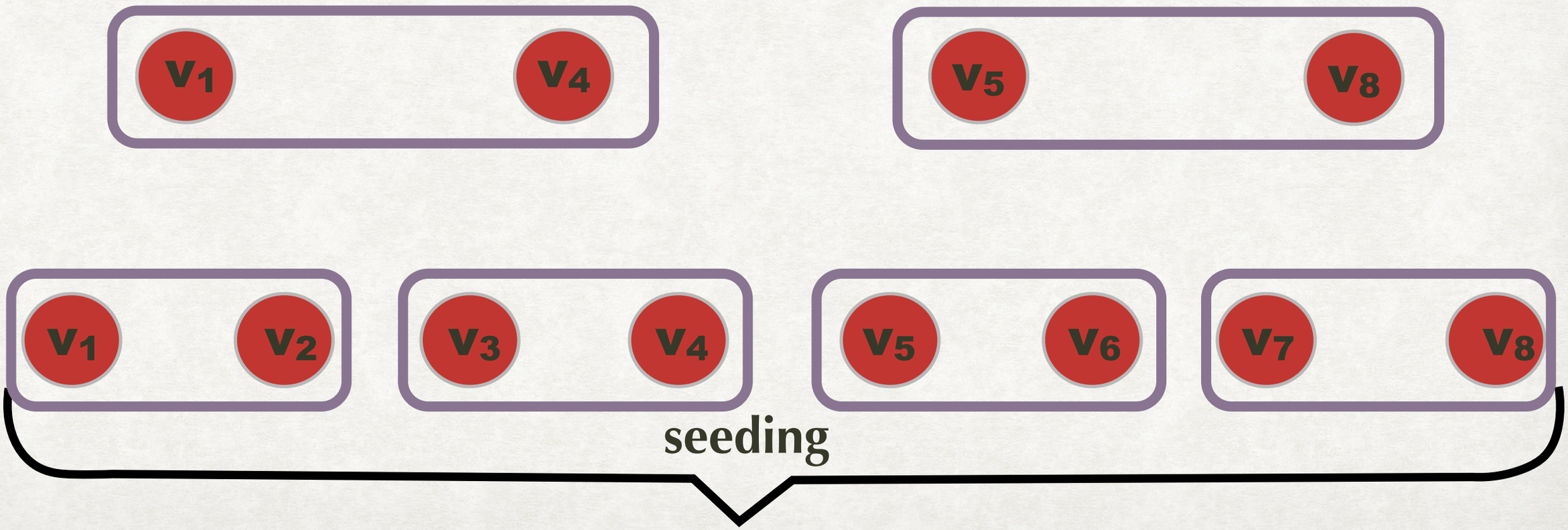
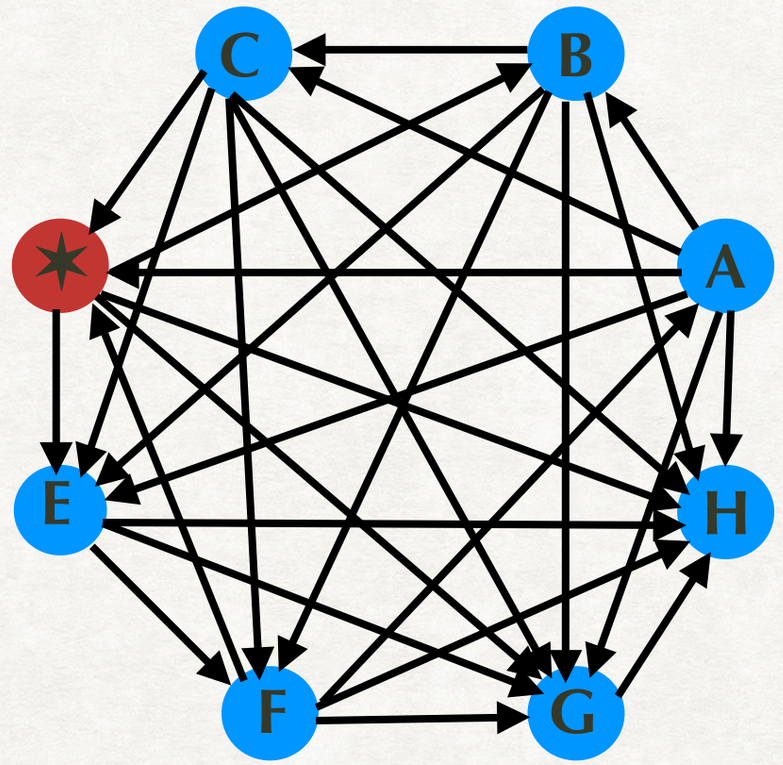


Is there a seeding to ensure that favorite wins the tournament ?

Our problem: Tournament Fixing

Our problem: TOURNAMENT FIXING

INPUT: Win-lose graph



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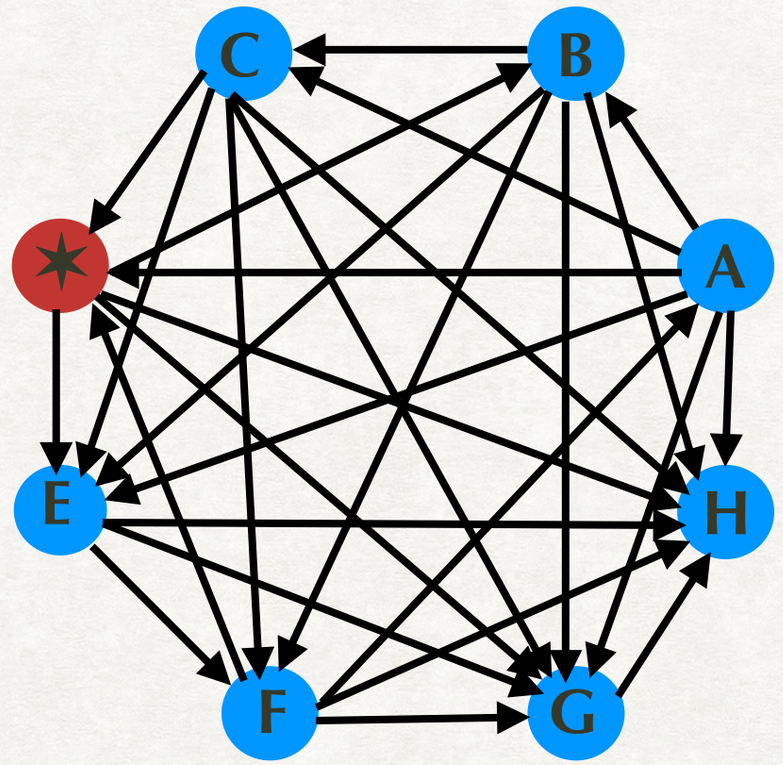

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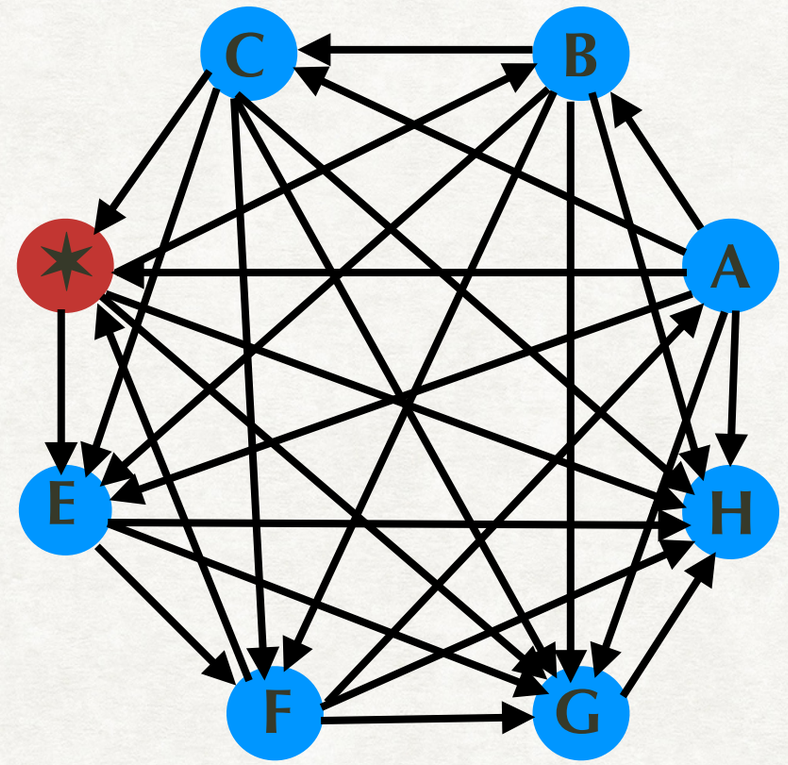

...introduced by Vu, Altman, Shoham AAMAS'09

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Easy to decide

- In general: NP-hard [Aziz et al. AAAI'14]

BRIEF DETOUR INTO PARAMETERIZED COMPLEXITY

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  - Input to the problem, $n \in \mathbb{Z}_{\geq 0}$
- In multivariate algorithms, a decision problem D=(Input Values, Question, Parameter)
  - Some secondary aspect of the input $k \in \mathbb{Z}_{\geq 0}$

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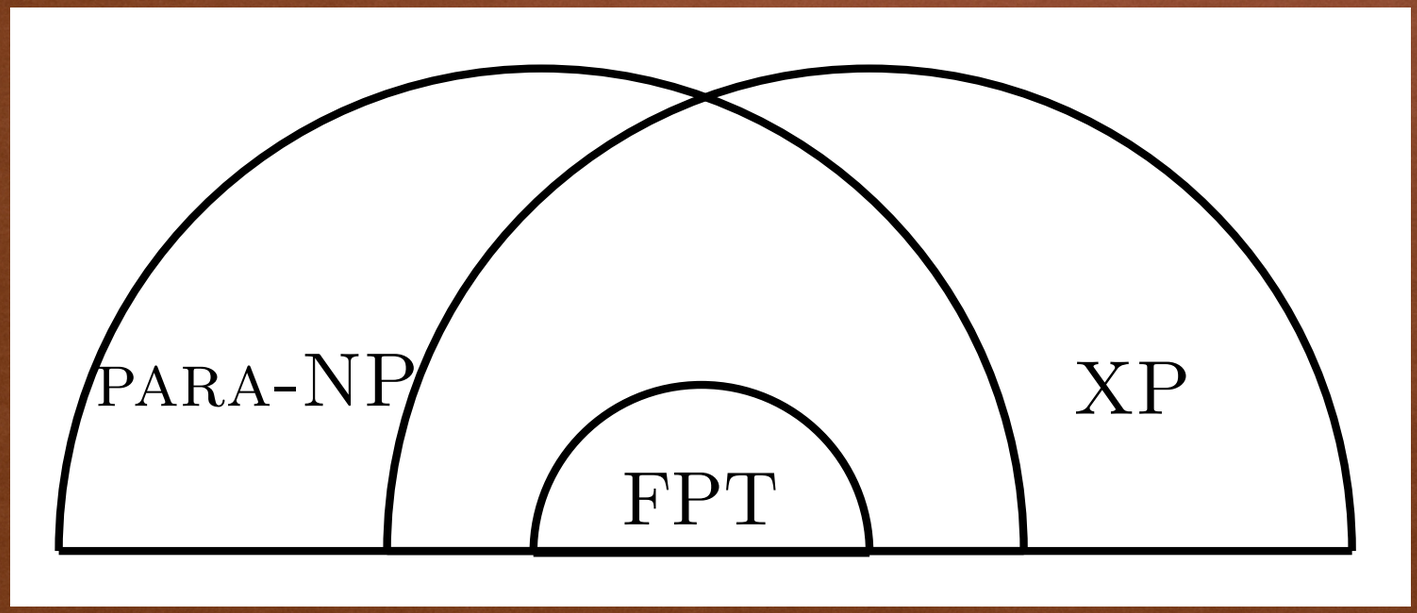

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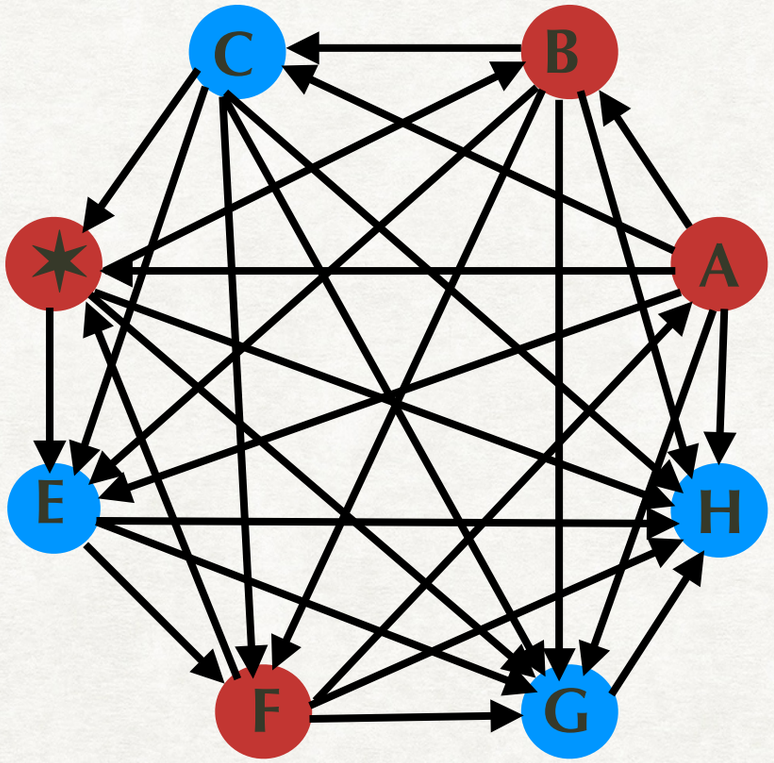
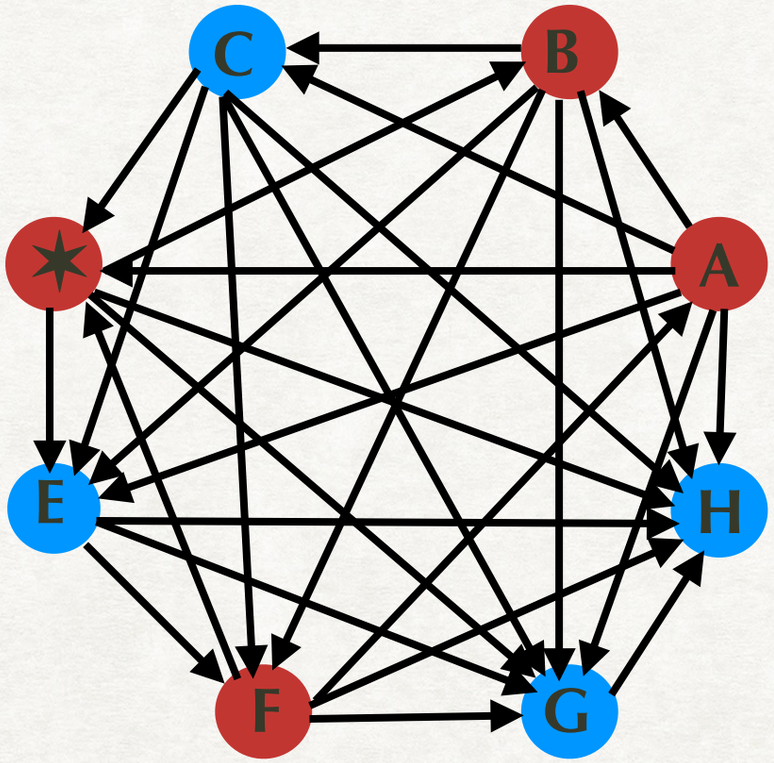
....But, how do we define better player ?

Ranking the players



We are given feedback arc set of the win-lose graph

Ranking the players



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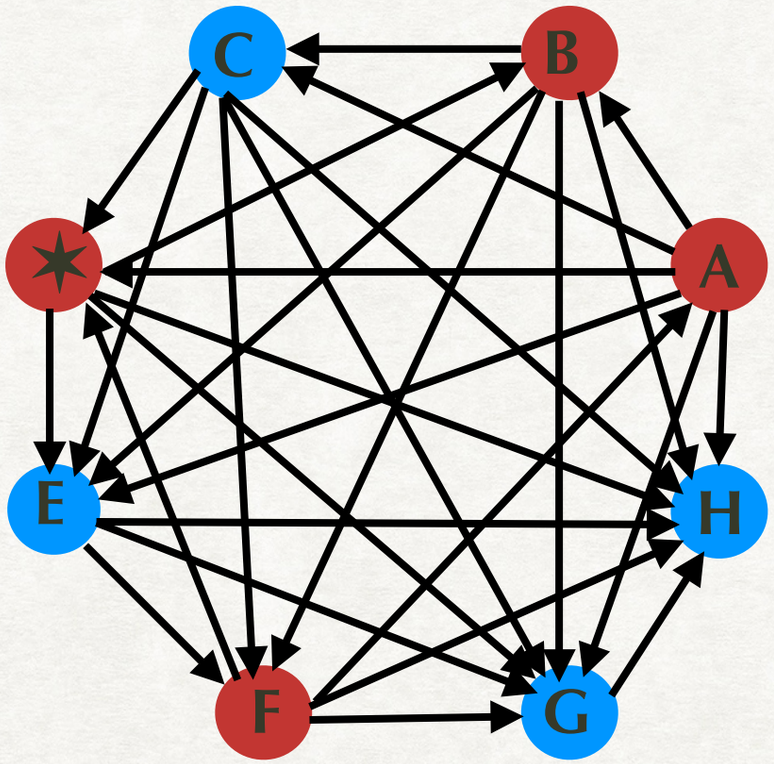

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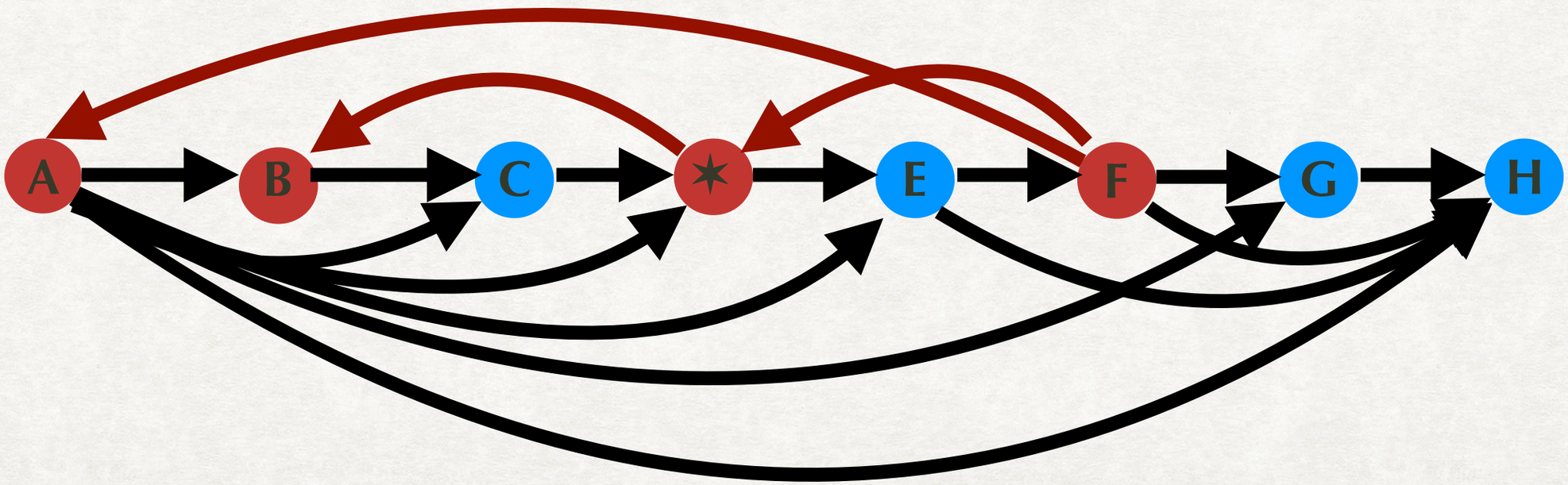

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(3) Best algorithm runs in time $2^{O(k \log k)}poly(n)$ [G, Roy, Saurabh, Zehavi, IJCAI'19] & polynomial-sized kernel

What if no favorable seeding exists for favorite ?

Can favorite win with bribery?

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INPUT: Win-lose graph

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QUESTION: Is it possible to fix some (say l) matches so that there is a seeding that enables favorite to win ?

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QUESTION: Is it possible to fix some (say l) matches so that there is a seeding that enables favorite to win ?

Can we reverse l arcs in the win-lose graph so that there will be a seeding that will enable favorite to win ?

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  - Uses our algorithm for TOURNAMENT FIXING

How can favorite win with bribery?

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Properties used by our algorithm.

What if we want to manipulate "minimally"

OPTIMAL MANIPULATION

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INPUT: Win-lose graph , initial seeding, integer d

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INPUT: Win-lose graph , initial seeding, integer d

QUESTION: Is it possible to find a seeding that enables favorite to win & is at most d Hamming Dist away from the initial seeding ?

MOTIVATION

Initial Seeding

$$S_0 = p_1\ p_2\ \ldots\ \ldots\ p_{n-1}\ p_n$$

MOTIVATION

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MOTIVATION

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[G, Saurabh, Zehavi]

In Conclusion

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- Interesting class of problems

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- Interesting structural properties

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- Interesting structural properties
- Many secondary ad tertiary parameters to consider

THANK YOU!