

1. Give an example of a Turing Machine that loops on all inputs. What is the language of this machine?
2. Construct a T.M. which on every input $w \in \{a, b\}^*$ first erases w and then accepts it. Give the formal description.
3. Construct a T.M. which *shifts* the input to the right as shown in the picture below: if $\vdash w\sqcup^\omega$ is the initial tape content, the machine changes it to $\vdash\vdash w\sqcup^\omega$ and then goes to the accept state. Give its formal description.



4. Construct a T.M. that informally does the following on every input:



It first *doubles* the input in the way shown in the picture and then goes to accept state.

Note that $\begin{matrix} a \\ a \end{matrix}$ is just another way of writing (a, a) .

What is the tape alphabet for the above T.M.? Give the formal description of the T.M. that achieves the above. Is your T.M. total?

5. Define a *stay-put* T.M. to be a T.M. whose tape head can stay at the same cell it reads, in addition to moving either left or right. If $M = (Q, \Sigma, \Gamma, \delta, s, t, r, \vdash, \sqcup)$ is a stay-put T.M., the transition function δ is as follows:

$$\delta : Q \times \Gamma \mapsto Q \times \Gamma \times \{L, R, S\}$$

If $\delta(q, a) = (p, b, S)$, it means that the T.M. in state q on reading an a moves to state p , overwrites a with b and the tape head stays at the same cell.

Show that for every stay-put T.M. M , there is a T.M. N s.t. $L(M) = L(N)$.

6. Let M be a T.M. equipped with a two-way infinite tape. Provide a T.M. N with a semi-infinite tape that can simulate M , that is $L(N) = L(M)$.
7. Let D be an NPDA. Show that there is an NPDA D_2 which has only two stack symbols s.t. $L(D_2) = L(D)$.

8. Construct a T.M. that does the following. It first erases the input. Then it starts enumerating the natural numbers one after the other in the standard ordering. After writing down each number, it goes back to the start marker before writing the next number. We give the schema for a part of the computation in the following figure:

