Topics in Timed Automata

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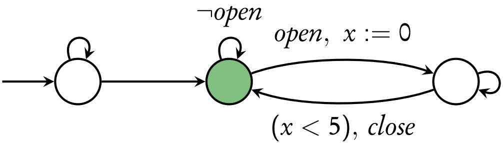
Software modeling and Verification group

Model-Checking Real-Time Systems

Correctness: Safety + **Liveness** + **Fairness**



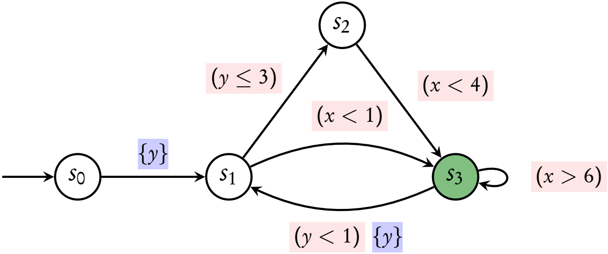
"Infinitely often, the gate is open for at least 5 s."

Realistic counter-examples: infinite **non-Zeno** runs

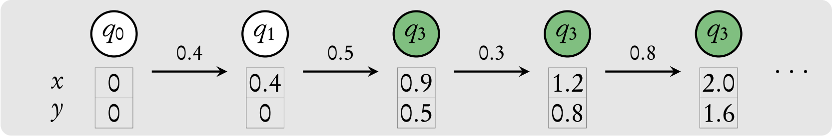
Lecture 8:

Non-Zenoness

Timed Büchi automata



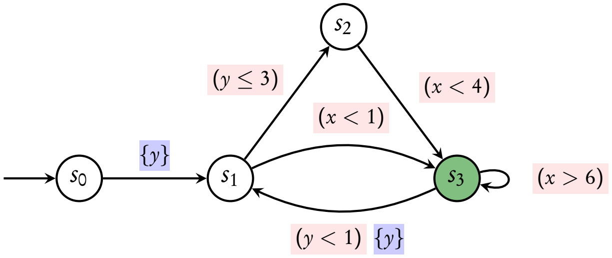
Run: infinite sequence of transitions



- **accepting** if infinitely often green state
- **non-Zeno** if time diverges ($\sum_{i \geq 0} \delta_i \to \infty$)

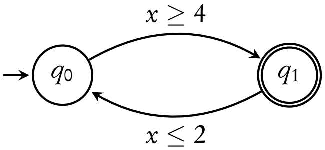
Büchi non-emptiness problem

Given a TBA, does it **have** a **non-Zeno** accepting run

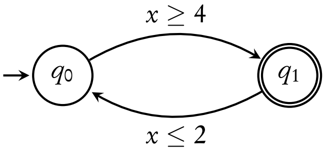


Theorem [AD94]

This problem is **PSPACE-complete**



No infinite run



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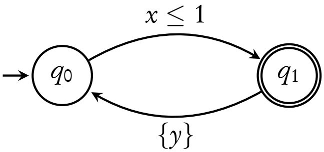
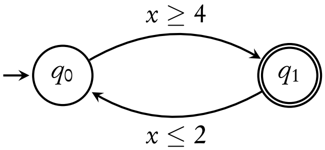


No non-Zeno run



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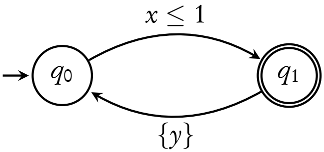
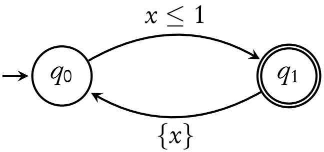


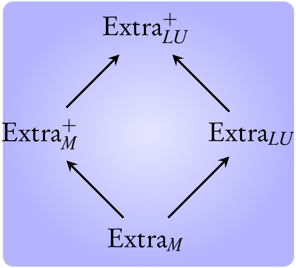
No non-Zeno run



Non-Zeno run $\surd$

How do we detect **infinite non-Zeno** runs given an automaton?

Abstract zone graphs again



$$ZG^{\mathfrak{a}}(\mathcal{A}): \quad (q_0, Z_0) \rightarrow (q_1, Z_1) \rightarrow (q_2, Z_2) \longrightarrow \cdots$$

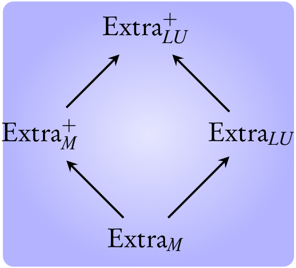
$$\qquad\qquad \Cup \qquad\qquad \Cup \qquad\qquad \Cup$$

$$\mathcal{A}: \quad (q_0, v_0) \rightarrow (q_1, v_1) \rightarrow (q_2, v_2) \longrightarrow \cdots$$

Sound and complete [Tri09, Li09]

All the above abstractions preserve **repeated state reachability**

Abstract zone graphs again



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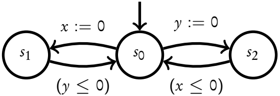
Sound and complete [Tri09, Li09]

All the above abstractions preserve **repeated state reachability**

What about **non-Zenoness**?

Time progress criterion [AD94]

$\bigwedge_{x \in X} \text{unbounded}(x) \vee \text{fluctuating}(x)$



Region graph:

$(s_1, 0 = x < y)$ $\quad$ $(s_2, 0 = y < x)$

$(s_0, 0 = x = y) \longrightarrow (s_1, 0 = x = y) \longrightarrow (s_0, 0 = x = y) \longrightarrow (s_2, 0 = y = x) \dashrightarrow$

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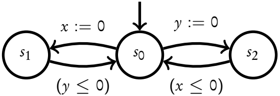

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Zone graph with Extra^+_M:

$$(s_0, 0 \leq x = y) \longrightarrow (s_1, 0 \leq x \leq y) \longrightarrow (s_0, 0 \leq x = y) \longrightarrow (s_2, 0 \leq y \leq x) \dashrightarrow$$

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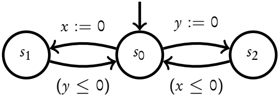

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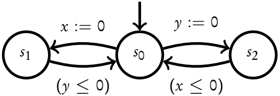
Zone graph with Extra_{LU}^+:

$(s_0, \top) \longrightarrow (s_1, \top) \longrightarrow (s_0, \top) \longrightarrow (s_2, \top) \dashrightarrow$

Time progress criterion [AD94]

$\bigwedge_{x\in X}$ unbounded$(x) \vee$ fluctuating(x)



Region graph:

$(s_1, 0 = x < y)$ $\qquad$ $(s_2, 0 = y < x)$

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Zone graph with Extra_M^+:

$(s_0, 0 \leq x = y) \longrightarrow (s_1, 0 \leq x \leq y) \longrightarrow (s_0, 0 \leq x = y) \longrightarrow (s_2, 0 \leq y \leq x) \dashrightarrow$

Zone graph with Extra_{LU}^+:

$(s_0, \top) \longrightarrow (s_1, \top) \longrightarrow (s_0, \top) \longrightarrow (s_2, \top) \dashrightarrow$

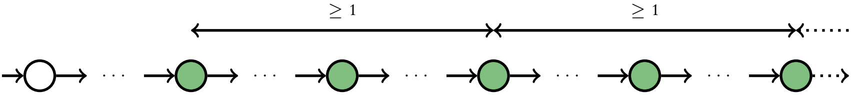
The **time progress** criterion is **not sound** on **zones**

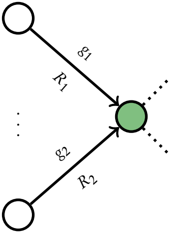
Coming next...

Strongly non-Zeno construction [TYB05]

From TBA to Strongly non-Zeno TBA

Key Idea : reduce non-Zenoness to Büchi acceptation

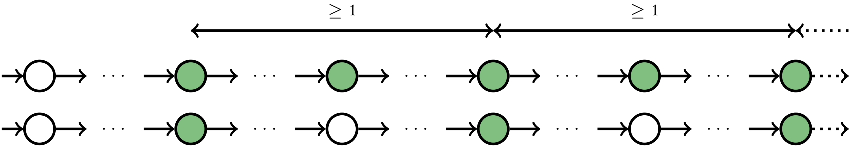


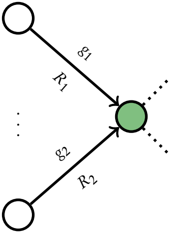


A

From TBA to Strongly non-Zeno TBA

Key Idea : reduce non-Zenoness to Büchi acceptation





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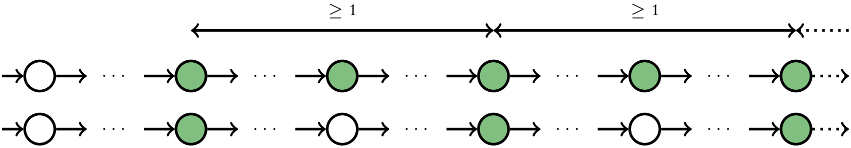

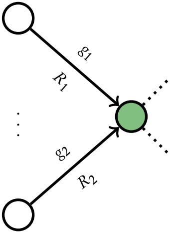

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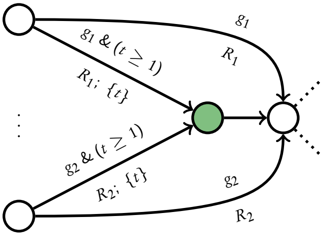

A′

Adding a clock for non-Zenoness [TYB05]

$\mathbf{A}'$: strongly non-Zeno TBA
$|X| + 1$ clocks and at most $2 \cdot |Q|$ states

Theorem [TYB05]

$\mathbf{A}$ has a non-Zeno accepting run iff $ZG^{\mathfrak{a}}(\mathbf{A}')$ has an **accepting** run

Adding a clock for non-Zenoness [TYB05]

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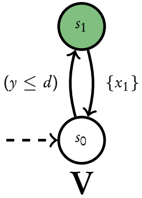
Theorem [TYB05]

$\mathbf{A}$ has a non-Zeno accepting run iff $ZG^{\mathfrak{a}}(\mathbf{A}')$ has an **accepting** run

Question: Is this good enough?

Adding one clock leads to an **exponential blowup** in the zone graph! [HSW12]

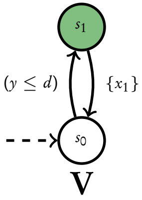
Guard $t \geq 1$ Allows to Count...

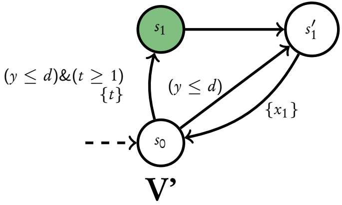


Run of **V**: 2 different zones in s_0

$$\cdots (s_0, y \leq x_1 \leq x_2) \xrightarrow{y \leq d} (s_1, y \leq x_1 \leq x_2 \,\&\, y \leq d) \xrightarrow{\{x_1\}}$$
$$(s_0, 0 = x_1 \leq y \leq x_2) \xrightarrow{y \leq d} (s_1, x_1 \leq y \leq x_2 \,\&\, y \leq d) \xrightarrow{\{x_1\}}$$
$$(s_0, 0 = x_1 \leq y \leq x_2) \cdots$$

Guard $t \geq 1$ Allows to Count...





Run of **V'**: $d + 2$ different zones in s_0

$$\cdots (s_0, y \leq x_1 \leq x_2 \leq t) \xrightarrow{(y \leq d)\&(t \geq 1),\, t:=0} \rightarrow \xrightarrow{\{x_1\}}$$

$$(s_0, 0 = x_1 \leq t \leq y \leq x_2 \& \mathbf{y} - \mathbf{t} \geq 0) \xrightarrow{(y \leq d)\&(t \geq 1),\, t:=0} \rightarrow \xrightarrow{\{x_1\}}$$

$$(s_0, 0 = x_1 \leq t \leq y \leq x_2 \& \mathbf{y} - \mathbf{t} \geq \mathbf{1}) \xrightarrow{(y \leq d)\&(t \geq 1),\, t:=0} \rightarrow \xrightarrow{\{x_1\}}$$

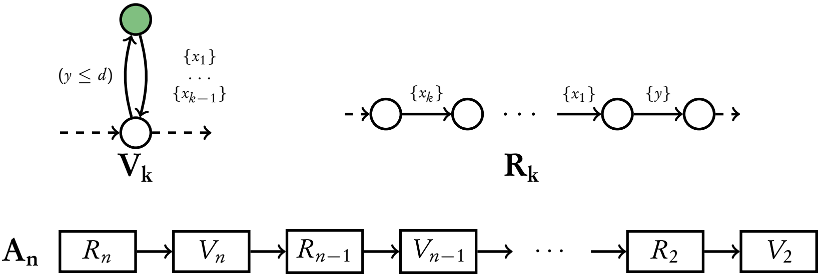
$$(s_0, 0 = x_1 \leq t \leq y \leq x_2 \& \mathbf{y} - \mathbf{t} \geq \mathbf{2}) \xrightarrow{(y \leq d)\&(t \geq 1),\, t:=0} \rightarrow \xrightarrow{\{x_1\}}$$

$$\cdots$$

$$(s_0, 0 = x_1 \leq t \leq y \leq x_2 \& \mathbf{y} - \mathbf{t} \geq \mathbf{d})$$

Remark: $y - t \geq c$ implies $x_2 - x_1 \geq c$

...and Leads to a Combinatorial Explosion

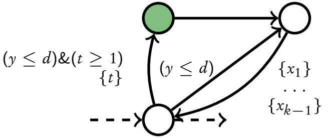


Lemma

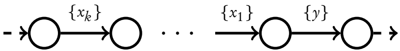
$ZG^{\mathfrak{a}}(A_n)$ has linear size in n

Key Idea: at V_k only two possible zones that **collapse** to the same zone after R_{k-1}.

...and Leads to a Combinatorial Explosion

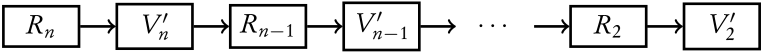


$\mathbf{V'_k}$



$\mathbf{R_k}$

$\mathbf{A'_n}$



Lemma

$ZG^{\mathfrak{a}}(A'_n)$ has size exponential in n

Key Idea: at V'_k, $\bigwedge_{i \in [k;n]} x_i - x_{i-1} \geq c_i$ with $c_i \in [0; d]$ chosen **non-deterministically**

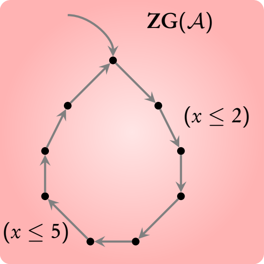
What we have:

- $ZG^{\mathfrak{a}}(A_n)$ has size $\mathcal{O}(n)$
- $ZG^{\mathfrak{a}}(A'_n)$ has size $\mathcal{O}(2^n)$

Coming next:

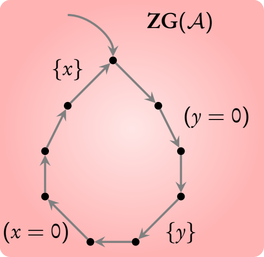
A $|ZG^{\mathfrak{a}}(A_n)|.\mathcal{O}(|X|^2)$ algorithm [HSW12]

When does a path in $ZG(\mathcal{A})$ yield only Zeno runs?



Blocking clocks

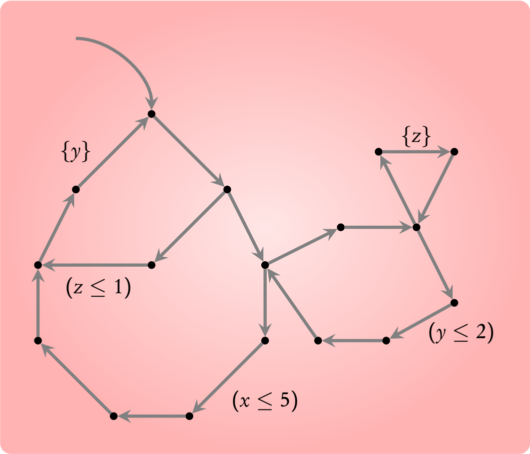
x never reset but checked for upper bound



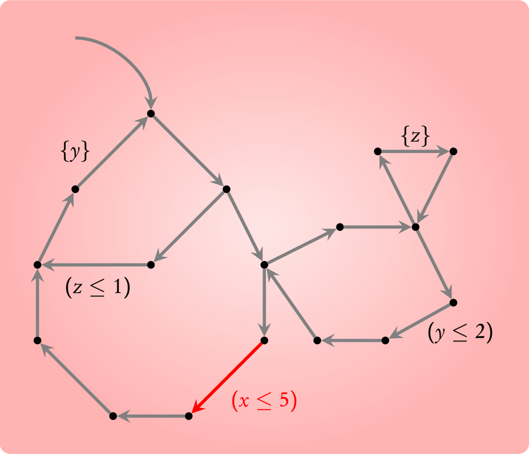
Zero-checks

x and y should be 0 all along the path

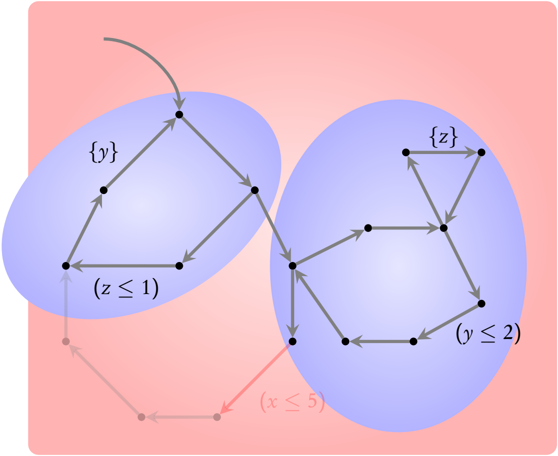
Blocking clocks



Blocking clocks



Blocking clocks



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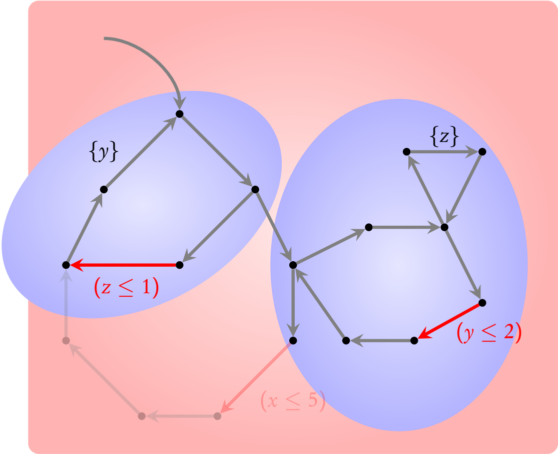

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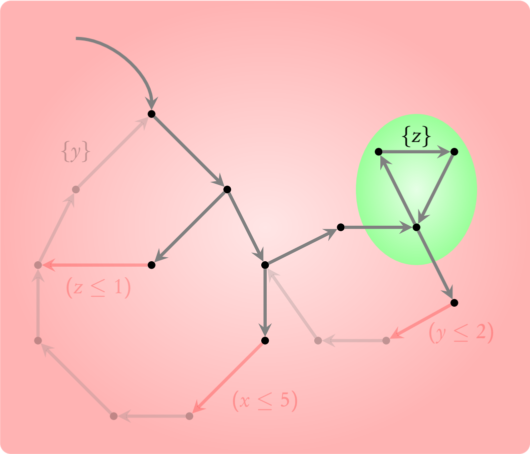
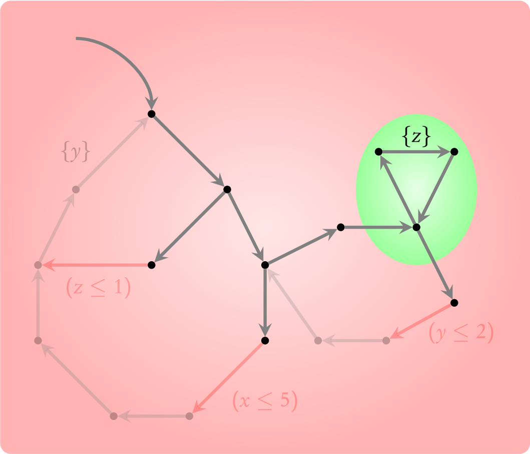


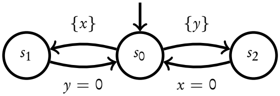
Blocking clocks



Theorem

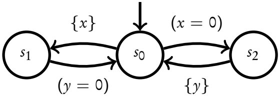
Blocking clocks can be detected in $|\mathrm{ZG}^{\mathfrak{a}}(\mathcal{A})| \cdot (|X| + 1)$ time

The case of zero checks



$$s_0 \xrightarrow{\{x\}} s_1 \xrightarrow{(y=0)} s_0 \xrightarrow{\{y\}} s_2 \xrightarrow{(x=0)} s_0$$

All states are in the scope of a zero check!



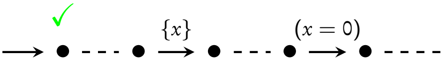
$$s_0 \xrightarrow{\{x\}} s_1 \xrightarrow{(y=0)} s_0 \xrightarrow{(x=0)} s_2 \xrightarrow{\{y\}} s_0$$

State s_2 is **clear**: all zero-checks are **preceded** by resets!

Zero-checks

Can time elapse here?

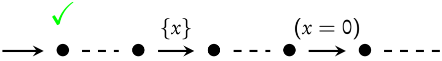
Zero-checks



Time can elapse at a node if

every zero-check is **preceded** by a reset

Zero-checks



Time can elapse at a node if

every zero-check is **preceded** by a reset

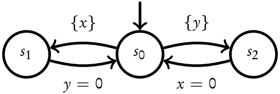
Guessing Zone Graph ($GZG^{\mathfrak{a}}(\mathcal{A})$) :

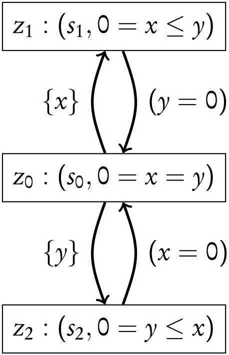
$$(q, Z, Y) \xrightarrow{\{x\}} (q', Z', Y \cup \{x\})$$

$$(q, Z, Y) \xrightarrow{(x=0)} \text{enabled only if } x \in Y$$

$$(q, Z, Y) \xrightarrow{\tau} (q, Z, \emptyset)$$

Zero checks (1st example)





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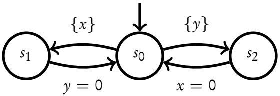
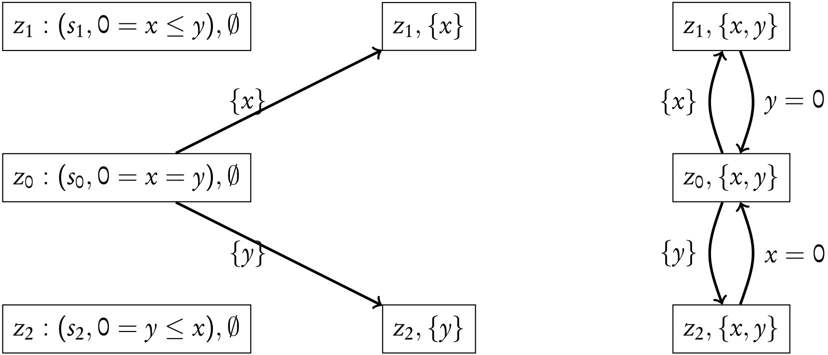
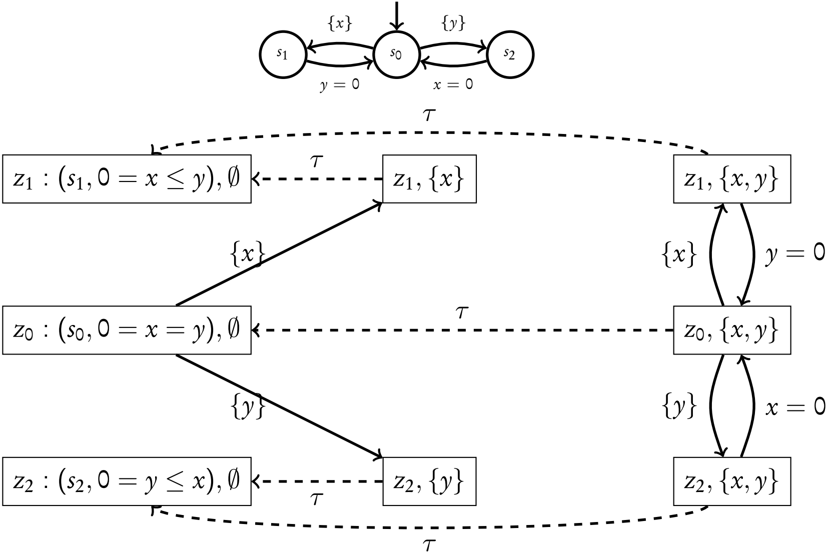
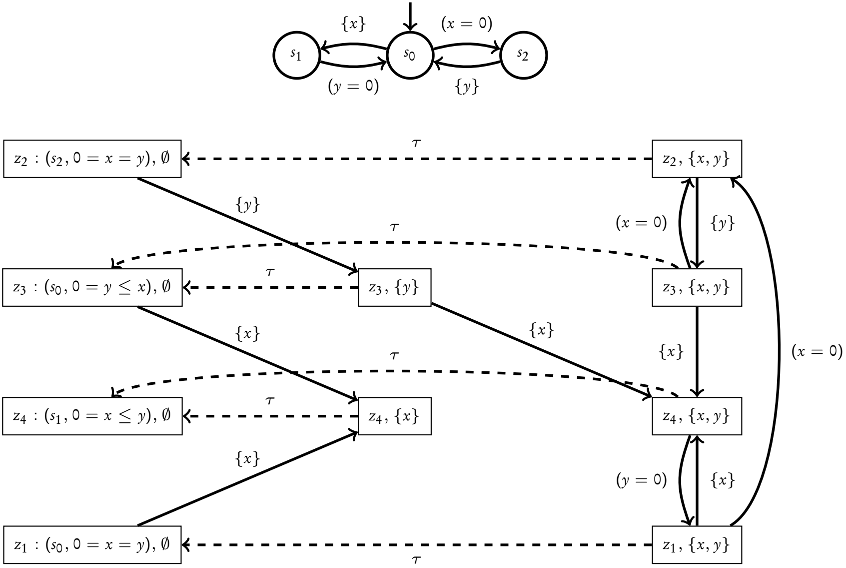




Zero checks (1st example)



Zero checks (2nd example)



Algorithm

Theorem [HSW12]

A has a non-Zeno run iff there is an **SCC** in $\mathrm{GZG}^{\mathfrak{a}}(A)$ that contains:

- an **accepting** node
- **no blocking** clocks
- a **clear** node $(q, Z, \emptyset)$

Complexity: $|\,\mathrm{GZG}^{\mathfrak{a}}(A)| \cdot (|X| + 1)$

$2^{|X|}$ **more nodes** in $\mathrm{GZG}^{\mathfrak{a}}(A)$ than in $\mathrm{ZG}^{\mathfrak{a}}(A)$ **due to** Y **sets?**

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Theorem

- For each reachable node (q, Z), Z entails a **total order** on X.
- Extra_M, Extra_M^+ **preserve the order**.
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Extra_{LU}, Extra_{LU}^+ **do not preserve order**

Theorem [HS11]

Non-Zenoness from LU-abstract zone graphs is **NP-complete**

Theorem [HS11]

A **slight weakening** of Extra_{LU}, Extra_{LU}^+ **preserves** order

Benchmarks

A	$ZG^{\mathfrak{a}}(A)$	$ZG^{\mathfrak{a}}(A')$		$GZG^{\mathfrak{a}}(A)$		
	size	size	otf	size	otf	opt
Train-Gate2 (mutex)	134	194	194	400	400	134
Train-Gate2 (bound. resp.)	988	227482	352	3840	1137	292
Train-Gate2 (liveness)	100	217	35	298	53	33
Fischer3 (mutex)	1837	3859	3859	7292	7292	1837
Fischer4 (mutex)	46129	96913	96913	229058	229058	46129
Fischer3 (liveness)	1315	4962	52	5222	64	40
Fischer4 (liveness)	33577	147167	223	166778	331	207
FDDI3 (liveness)	508	1305	44	3654	79	42
FDDI5 (liveness)	6006	15030	90	67819	169	88
FDDI3 (bound. resp.)	6252	41746	59	52242	114	60
CSMA/CD4 (collision)	4253	7588	7588	20146	20146	4253
CSMA/CD5 (collision)	45527	80776	80776	260026	260026	45527
CSMA/CD4 (liveness)	3038	9576	1480	14388	3075	832
CSMA/CD5 (liveness)	32751	120166	8437	186744	21038	4841

- Combinatorial explosion may **occur** in practice
- **Optimized** use of $GZG^{\mathfrak{a}}(A)$ gives best results

Conclusion

- Strongly non-Zeno construction can cause **exponential blowup**

- A **guessing zone graph** construction for non-Zenoness

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