Topics in Timed Automata

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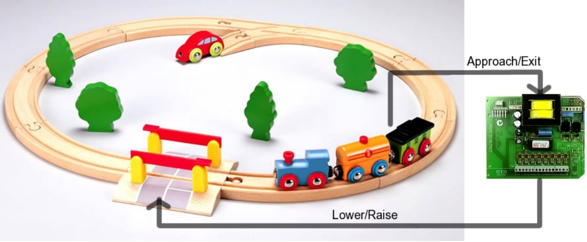
RWTH-Aachen

Software modeling and Verification group



Reachability: Does something **bad** happen?

"The gate is still open when the train is 2 minutes away from the crossing"

This problem is PSPACE-complete

A theory of timed automata

R. Alur and D.L. Dill, *TCS'94*

Tools

- UPPAAL:

  Uppsaala university *(Sweden)*, Aalborg university *(Denmark)*

- KRONOS:

  Verimag *(France)*

- RED

  National Taiwan University *(Taiwan)*

- Rabbit

  Brandenburg TU Cottbus *(Germany)*

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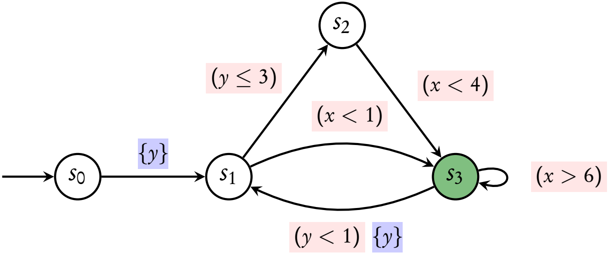
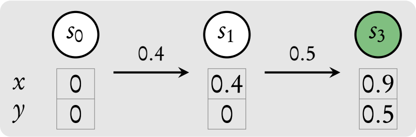
and still research on for efficient algorithms . . .

Lecture 6:
Reachability

Timed Automata



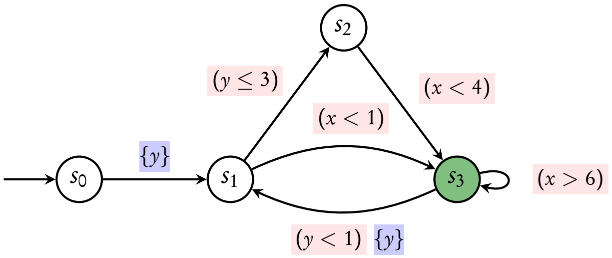
Run: finite sequence of transitions



- **accepting** if ends in green state

Reachability problem

Given a TA, does it **have** an **accepting** run

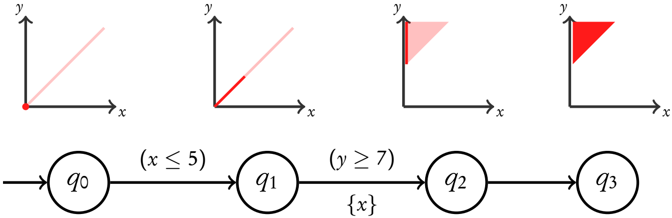


Theorem [AD94]

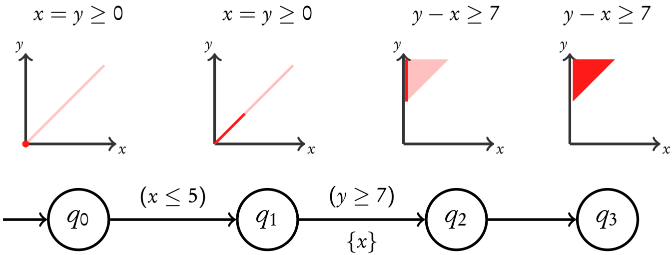
This problem is **PSPACE-complete**

first solution based on Regions

Key idea: Maintain **sets of valuations** reachable along a path

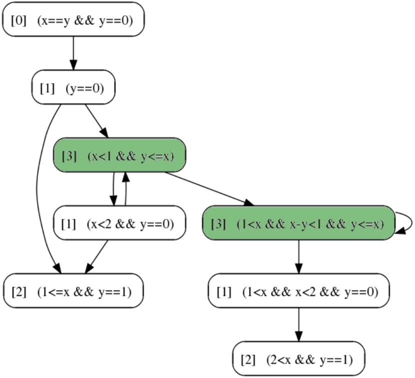


Key idea: Maintain **sets of valuations** reachable along a path



Easy to describe **convex** sets

Zones and zone graph



- Zone: set of valuations defined by conjunctions of constraints:

$$x \sim c$$
$$x - y \sim c$$

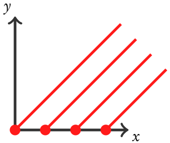
e.g. $(x - y \geq 1) \wedge (y < 2)$

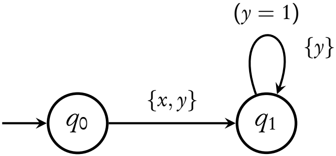
- Representation: by DBM [Dil89]

Sound and complete [DT98]

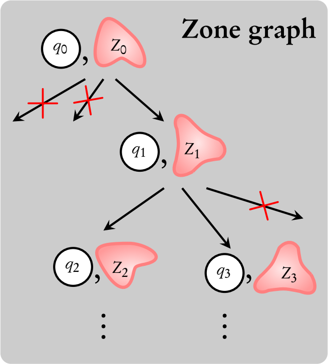
Zone graph preserves state **reachability**

Problem of non-termination



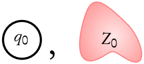


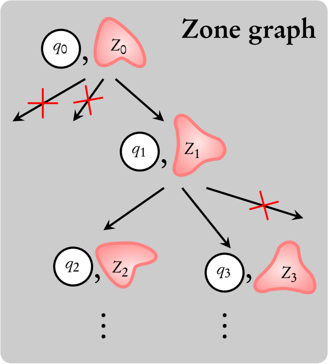
Abstractions



potentially infinite...

Abstractions





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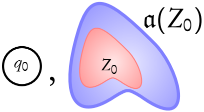

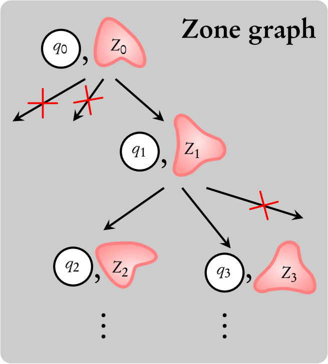

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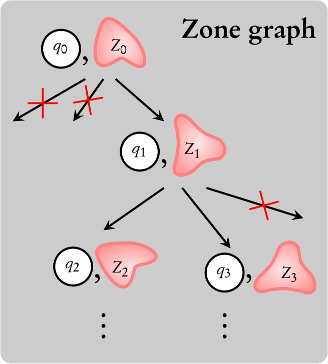

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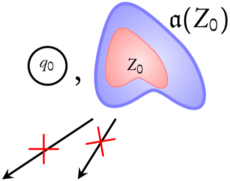

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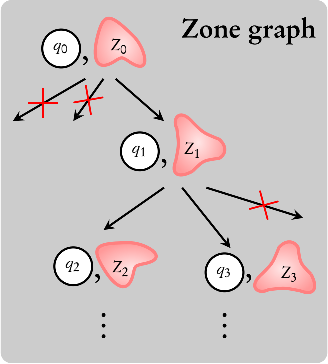

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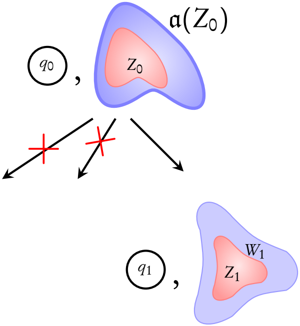

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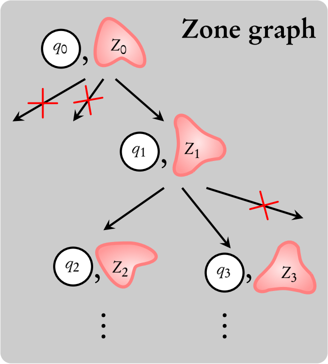

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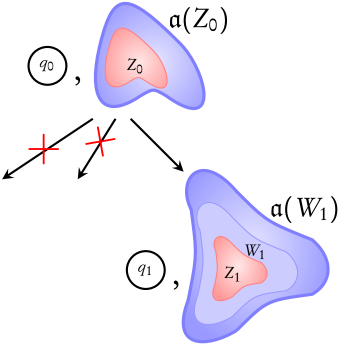

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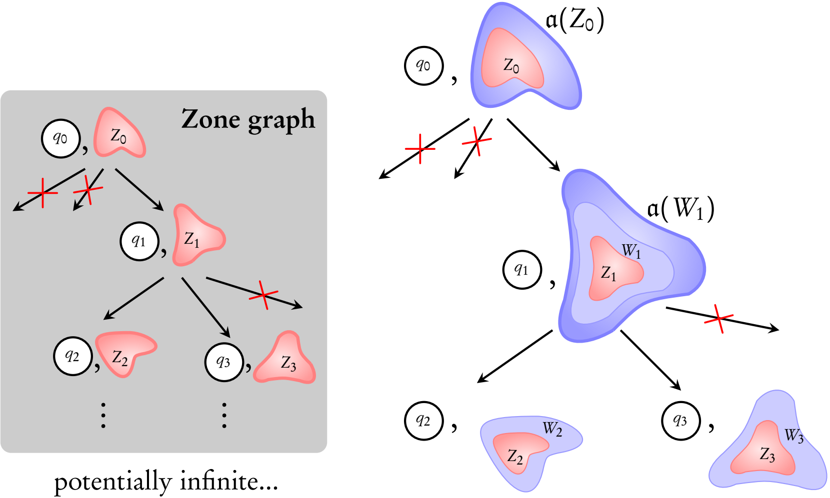

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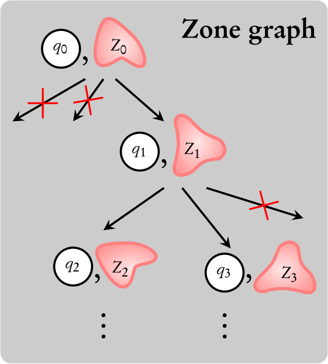

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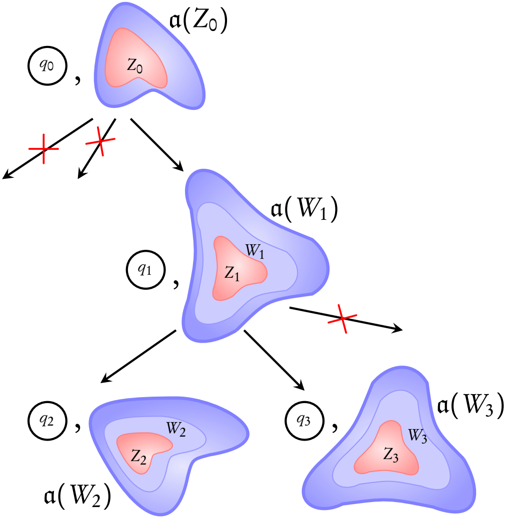

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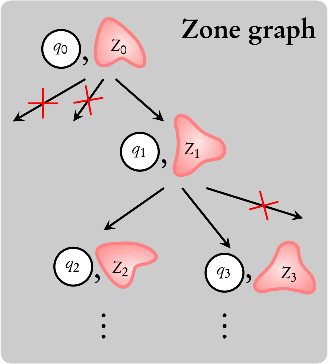

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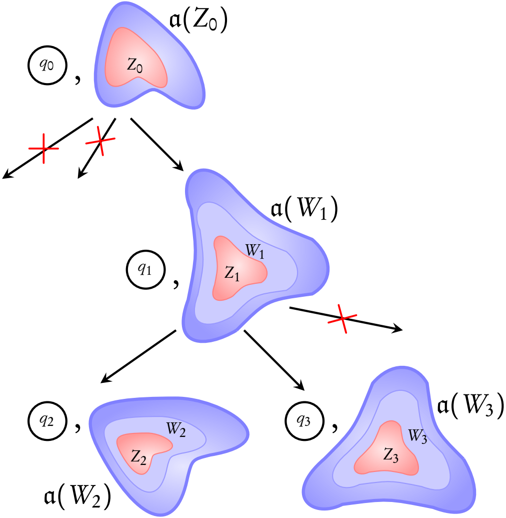
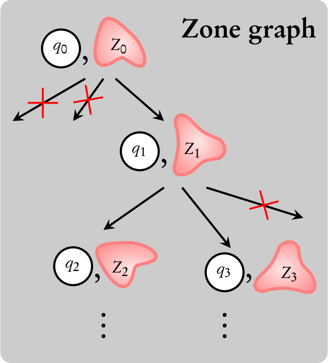


Find $\mathfrak{a}$ such that number of **abstracted** sets is **finite**

Abstractions



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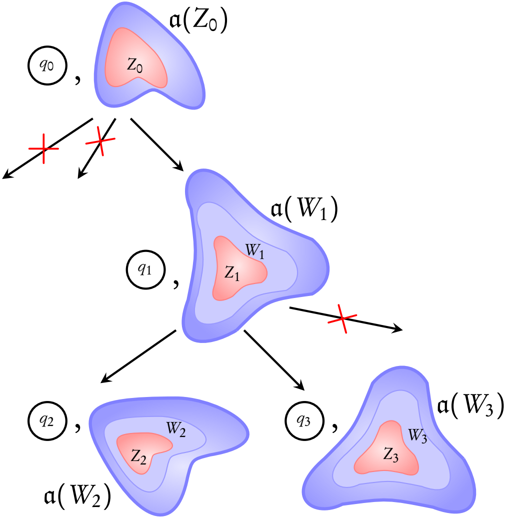
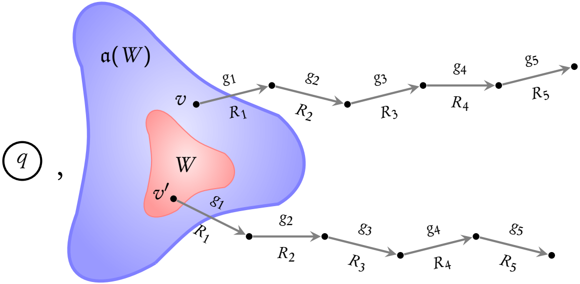


Coarser the abstraction, **smaller** the abstracted graph

Condition 1: Abstractions should have **finite range**

Condition 2: Abstractions should be sound ⇒ $\mathfrak{a}(W)$ can contain only valuations **simulated** by W



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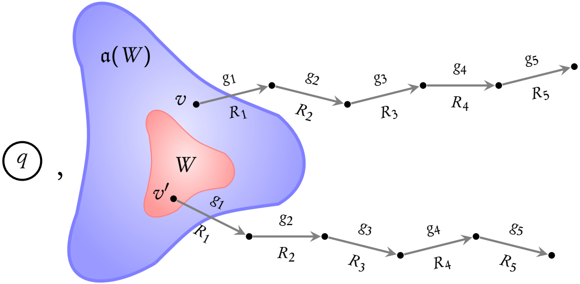
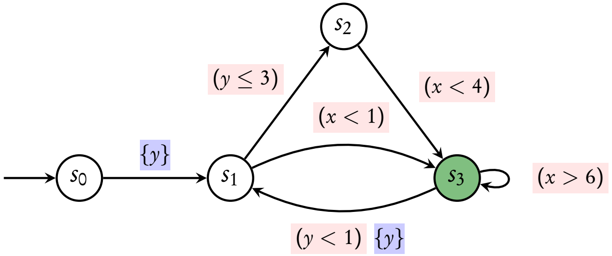


Question: Why not add **all** the valuations **simulated** by W?

Bounds and abstractions

Theorem [LS00]

Coarsest simulation relation is **EXPTIME-hard**



Bounds and abstractions

Theorem [LS00]

Coarsest simulation relation is **EXPTIME-hard**

$(y \leq 3)$ $(x < 4)$

$(x < 1)$

$(x > 6)$

$(y < 1)$

Bounds and abstractions

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Coarsest simulation relation is **EXPTIME-hard**

$(y \leq 3)$

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$(x > 6)$

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M-bounds [AD94]

$$M(x) = 6, \ M(y) = 3$$

$$v \preccurlyeq_M v'$$

Bounds and abstractions

Theorem [LS00]

Coarsest simulation relation is **EXPTIME-hard**

$(y \leq 3)$

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$(x > 6)$

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M-bounds [AD94]

$$M(x) = 6, \; M(y) = 3$$

$$v \preccurlyeq_M v'$$

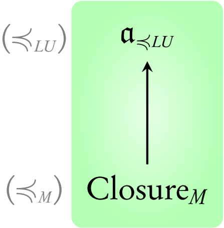
LU-bounds [BBLP04]

$$L(x) = 6, \; L(y) = -\infty$$

$$U(x) = 4, \; U(y) = 3$$

$$v \preccurlyeq_{LU} v'$$

Abstractions in literature [BBLP04, Bou04]



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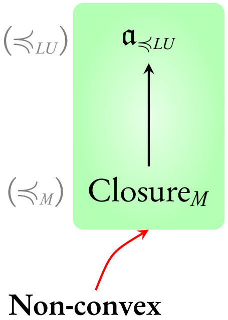

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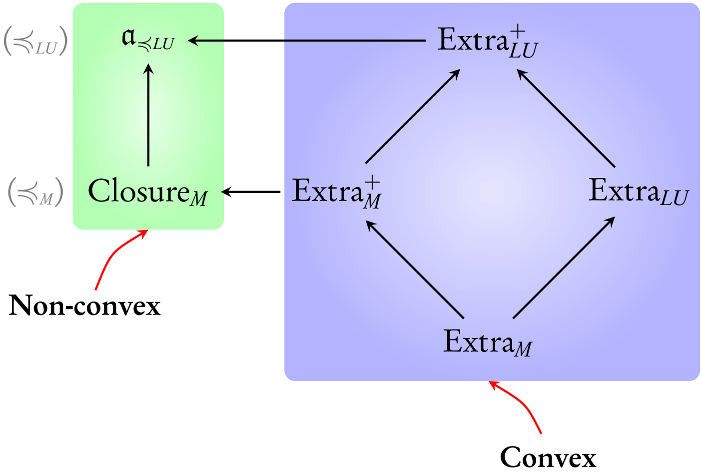


Only convex abstractions used in **implementations!**

Timed automata

↓

Zone graph

↓

Problem of non-termination → Use finite abstractions

Zones
are
efficient

↓

Bounds as parameters

↓

Restriction to convex abstractions

Non-convex abstr.
are
coarser

Timed automata

↓

Zone graph

↓

Problem of non-termination ⟶ Use finite abstractions

↓

Bounds as parameters

↓

Restriction to convex abstractions

Zones are **efficient**

Non-convex abstr. are **coarser**

Question: Can we benefit from both together?

In this lecture...

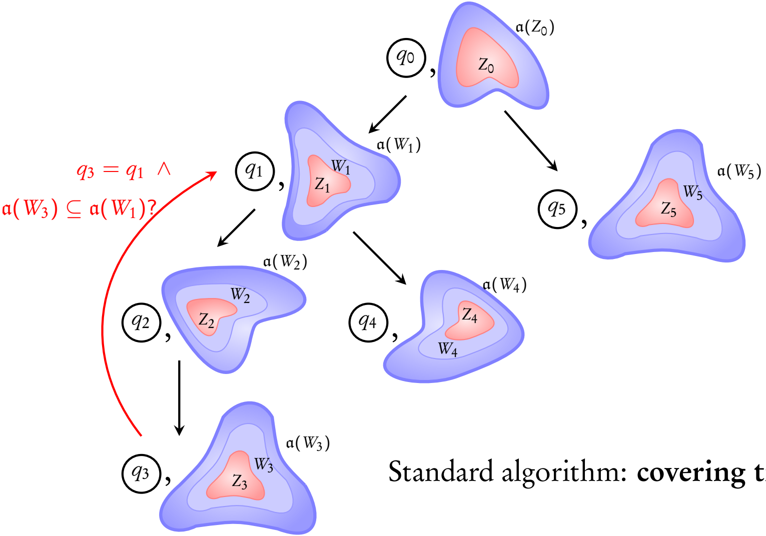
Efficient use of the **non-convex** Closure approximation

Using non-convex approximations for efficient analysis of timed automata

F. Herbreteau, D. Kini, B. Srivathsan, I. Walukiewicz. *FSTTCS'11*

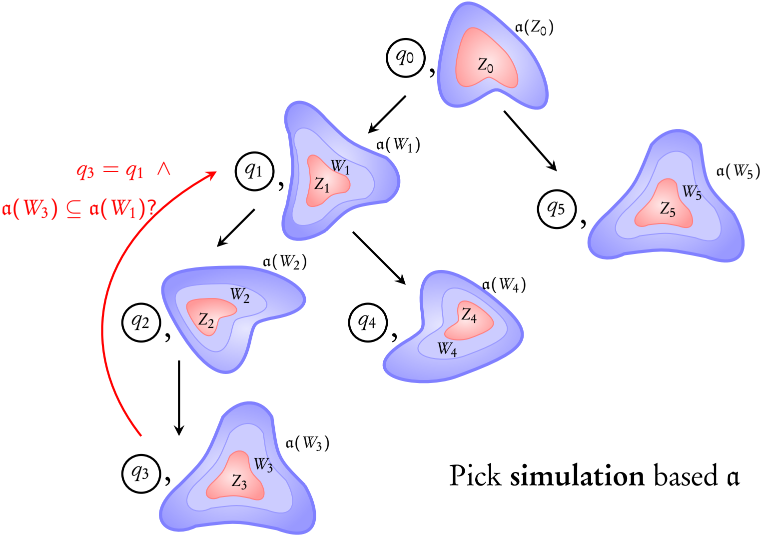
Observation 1: We can use abstractions **without storing** them

Using non-convex abstractions



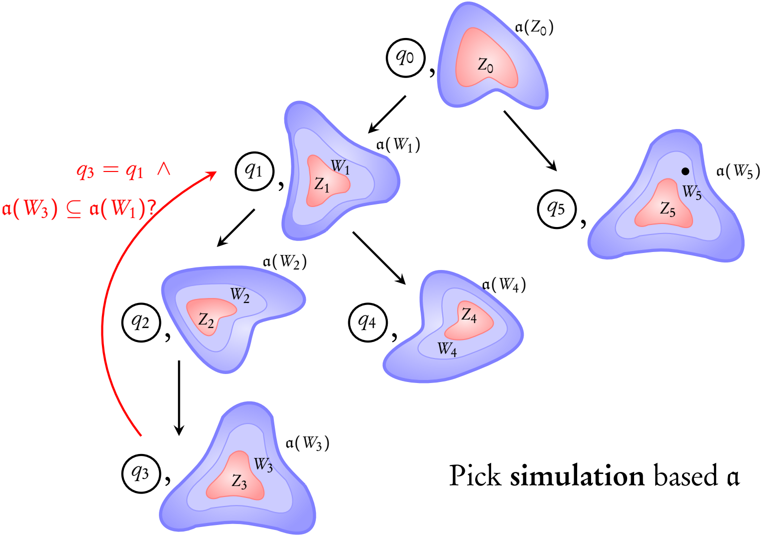
Standard algorithm: **covering tree**

Using non-convex abstractions



Pick **simulation** based $\mathfrak{a}$

Using non-convex abstractions



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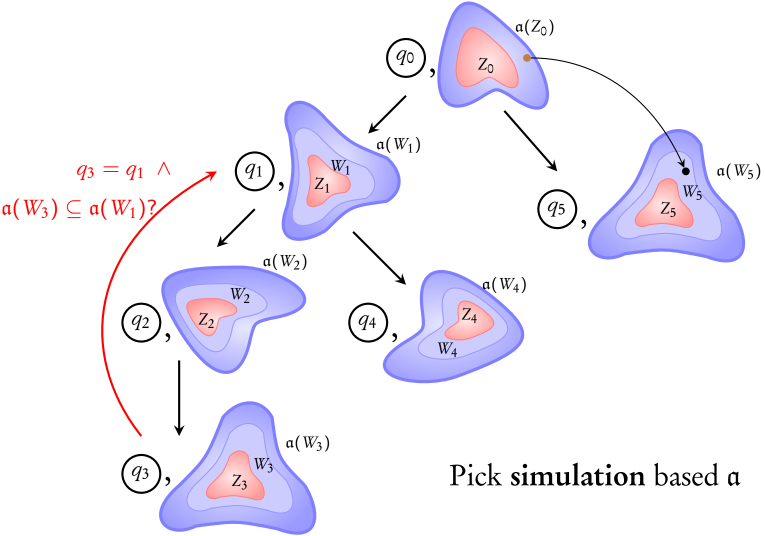

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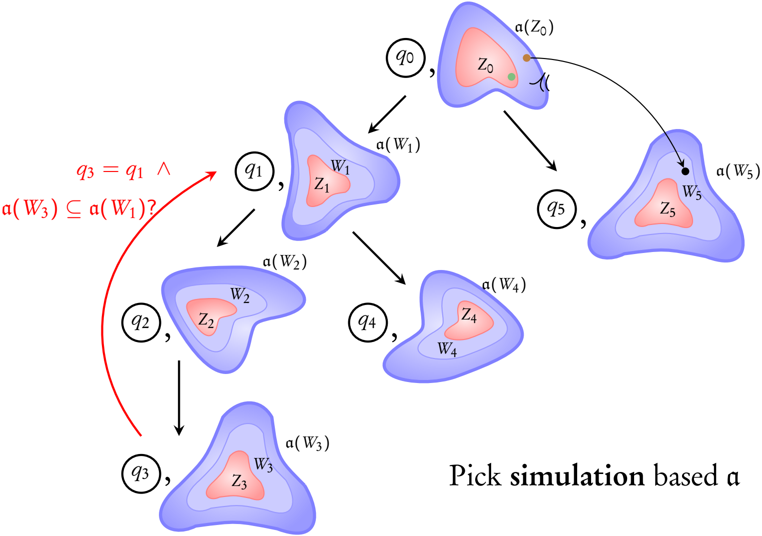

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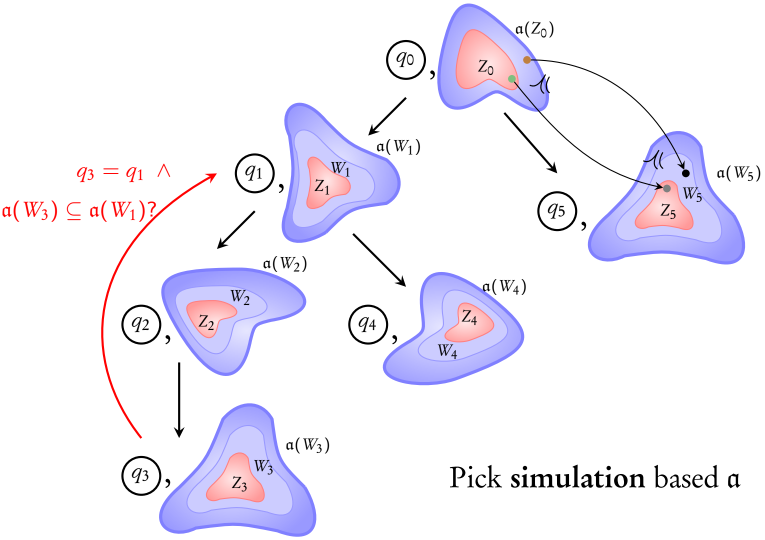

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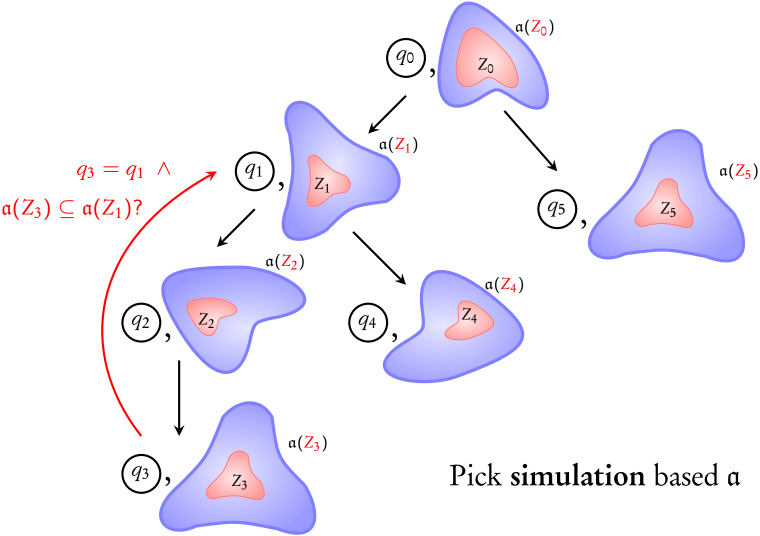

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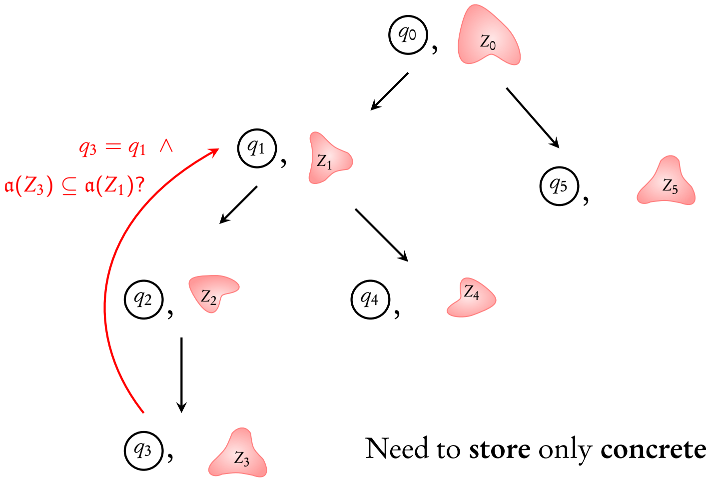
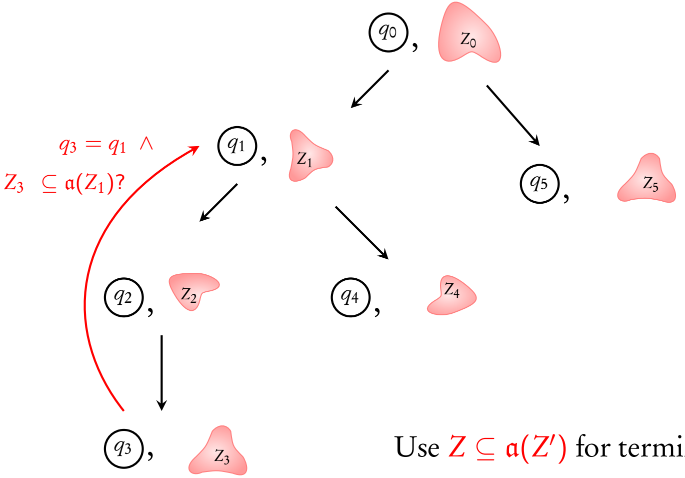


Need to **store** only **concrete** semantics

Using non-convex abstractions



Use $Z \subseteq \mathfrak{a}(Z')$ for termination

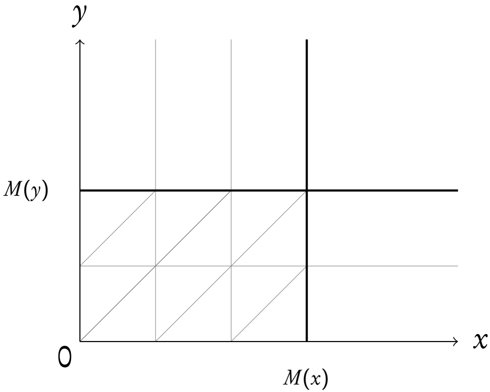
Observation 1: We can use abstractions **without storing** them

Observation 2: We can do the **inclusion** test **efficiently**

Coming next...

The inclusion test $Z \subseteq \text{Closure}_M(Z')$

What is Closure$_M$?



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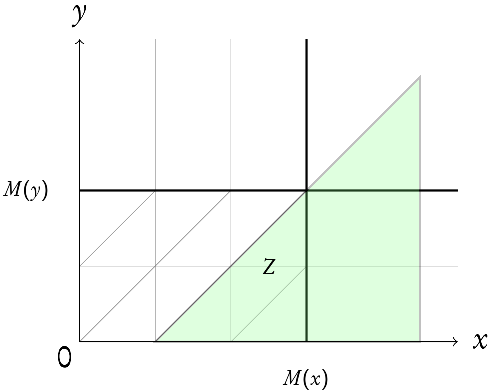

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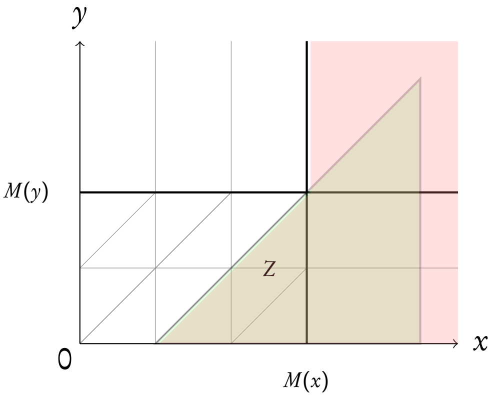
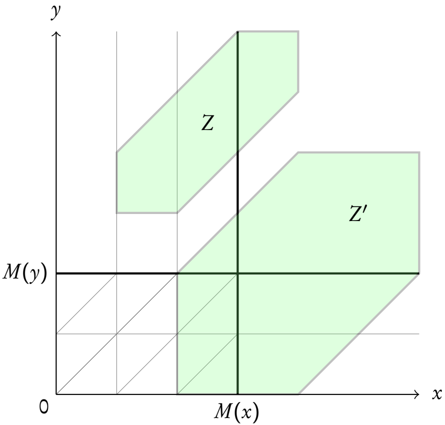


Closure$_M$(Z): set of regions that Z intersects

$Z \subseteq \text{Closure}_M(Z')$?



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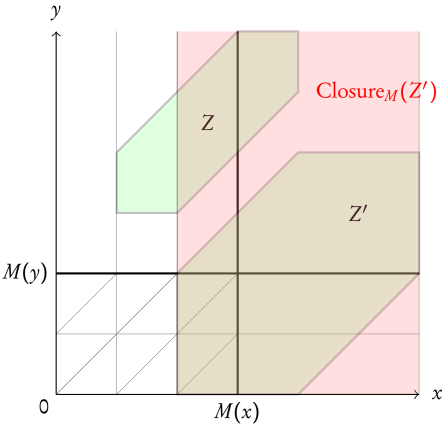

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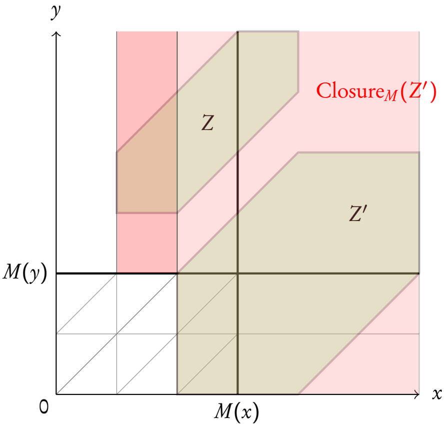
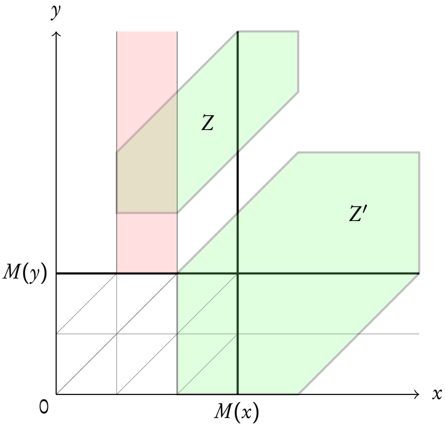
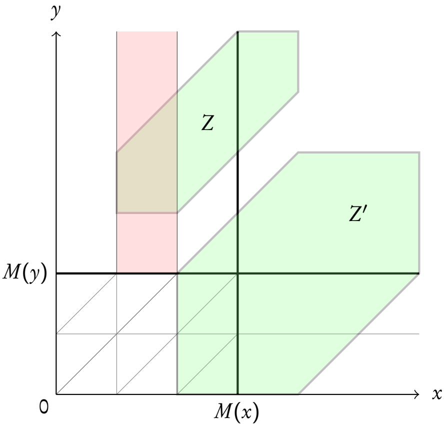


Z ⊆ $\text{Closure}_M(Z')$?



$Z \not\subseteq \text{Closure}_M(Z') \Leftrightarrow \exists R.$ R **intersects Z**, R does **not intersect Z′**

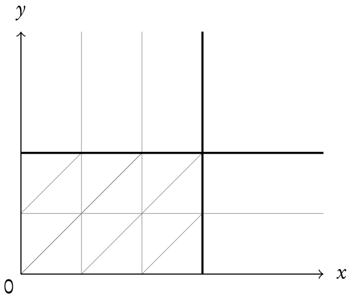
$Z \subseteq \text{Closure}_M(Z')$?



$Z \not\subseteq \text{Closure}_M(Z') \Leftrightarrow \exists R.\ R$ **intersects Z**, R does **not intersect Z′**

Coming next: Steps to the **efficient algorithm** for $Z \not\subseteq \text{Closure}_M(Z')$

Step 1: Representing regions and zones



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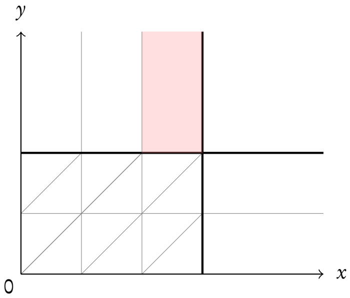
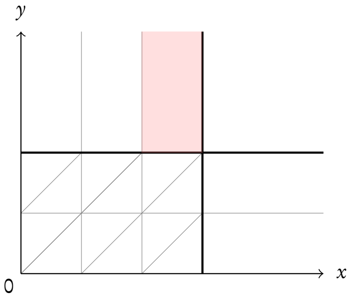


$x < 3$ $\quad y < \infty$

$x > 2$ $\quad y > 2$

Step 1: Representing regions and zones



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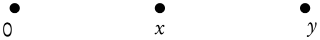

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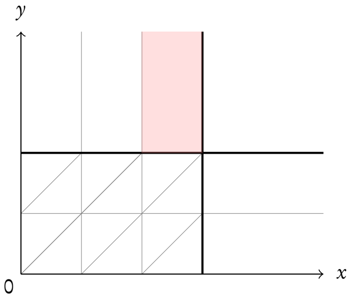

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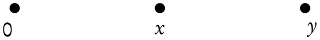

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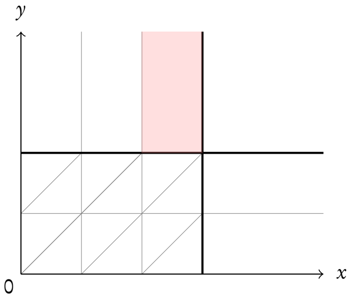

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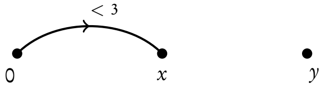
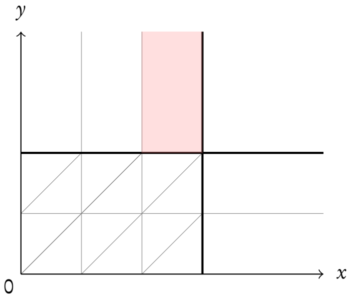
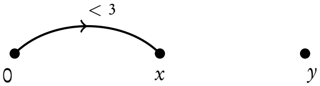


Step 1: Representing regions and zones



$x - 0 < 3$ $\quad$ $y < \infty$

$x > 2$ $\quad$ $y > 2$



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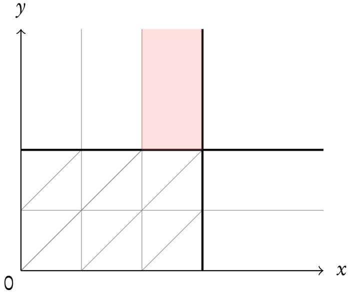

$$x - 0 < 3 \qquad y < \infty$$

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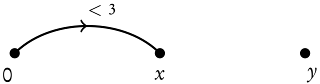

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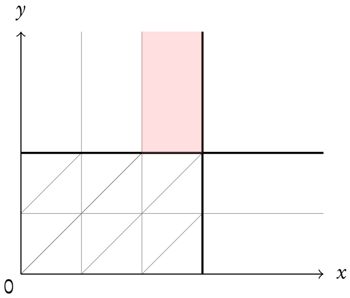

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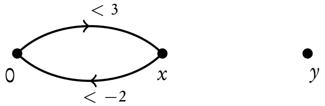

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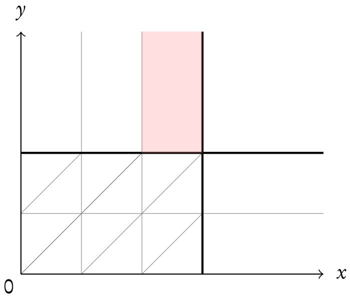

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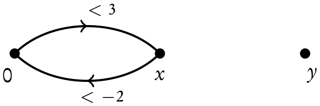

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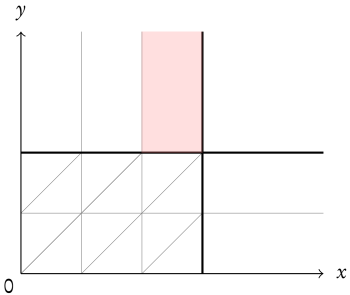

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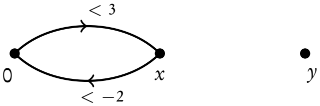

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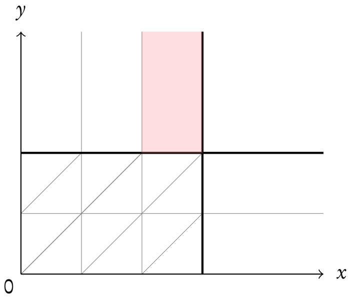

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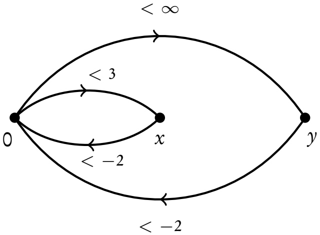

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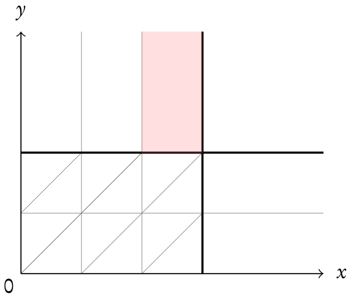

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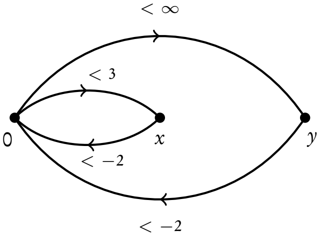

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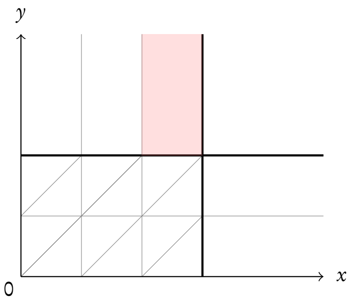

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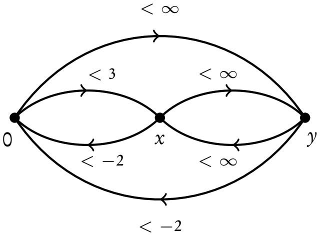

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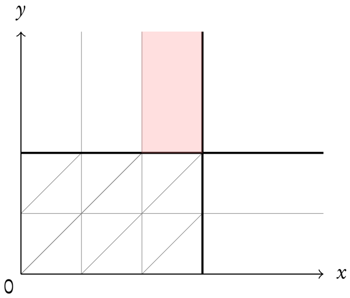

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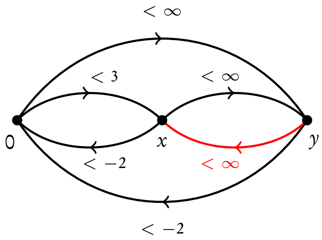
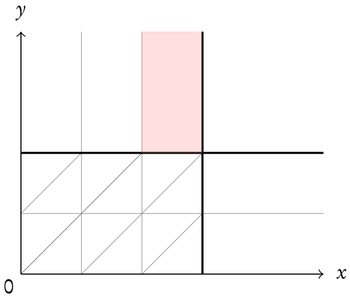


Need a **canonical** representation

Step 1: Representing regions and zones



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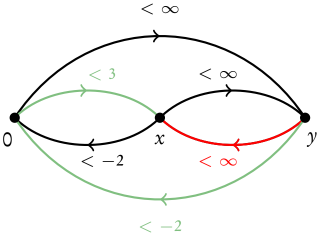
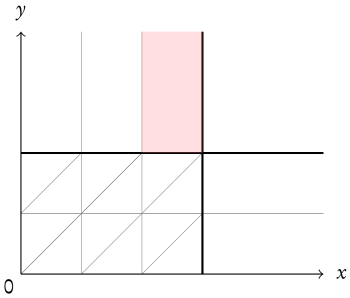


Shortest path should be given by the **direct edge**

Step 1: Representing regions and zones



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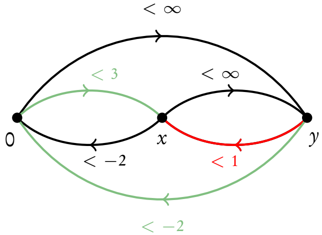

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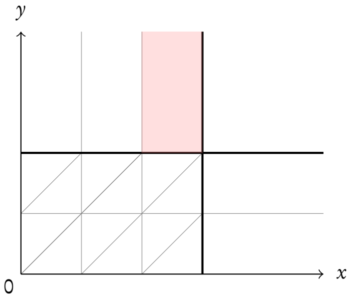

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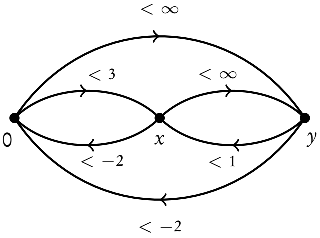

For every zone Z, **canonical distance graph** G_Z

Step 2: When is $R \cap Z'$ empty?

Inspired by an observation made in [Bou04]

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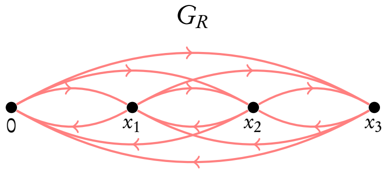

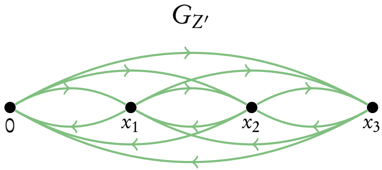

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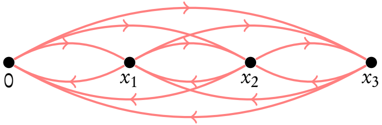
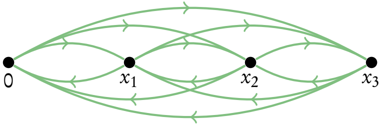
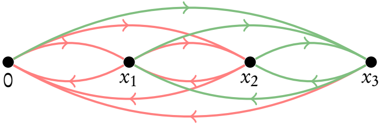
G_R



$G_{Z'}$



$\min(G_R, G_{Z'})$



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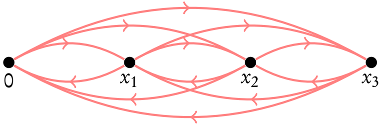

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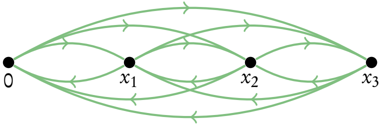

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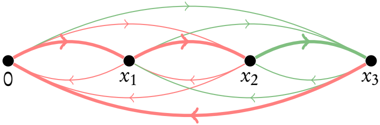

Lemma

$R \cap Z'$ is **empty** $\quad \Leftrightarrow \quad$ $\min(G_R, G_{Z'})$ has a **negative cycle**

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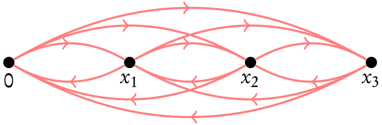

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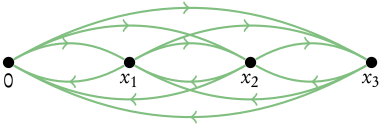

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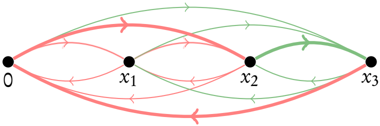

Lemma

$R \cap Z'$ is **empty** $\Leftrightarrow$ $\min(G_R, G_{Z'})$ has a **negative cycle** involving **at most 2 clocks!**

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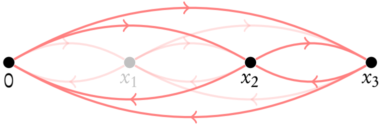

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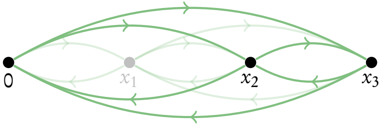

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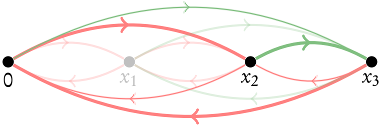

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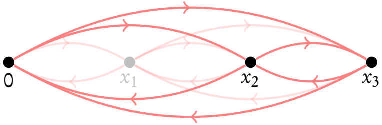

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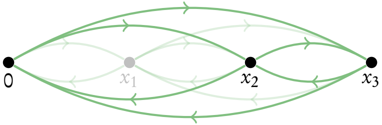

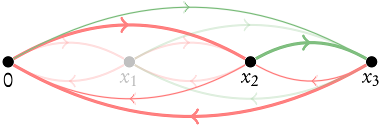

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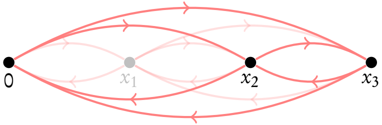

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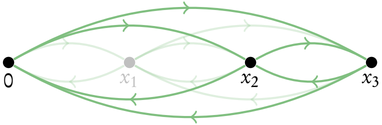
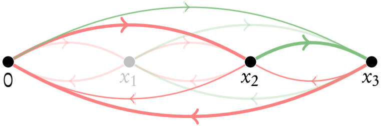


$\min(G_{\mathsf{Proj}_{x_2x_3}(R)}, G_{\mathsf{Proj}_{x_2x_3}(Z')})$



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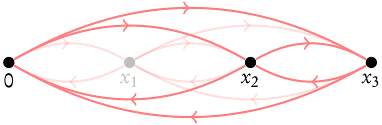

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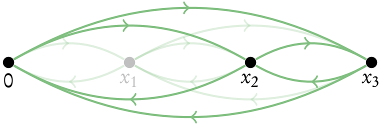

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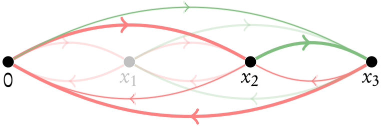

Lemma

$R \cap Z'$ is **empty** $\Leftrightarrow$ $\exists\, x, y.\ \mathsf{Proj}_{\mathbf{xy}}(R) \cap \mathsf{Proj}_{\mathbf{xy}}(Z')$ is **empty**

Step 3: Reduction to two clocks

Recall: $Z \not\subseteq \text{Closure}_M(Z') \Leftrightarrow \exists R.\ R$ intersects Z, R does not intersect Z'

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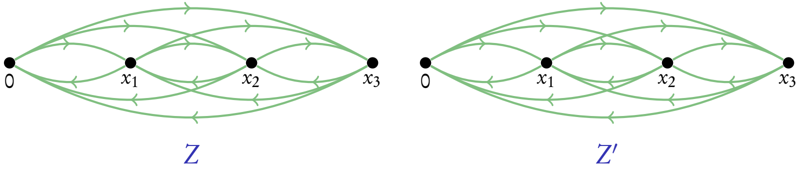

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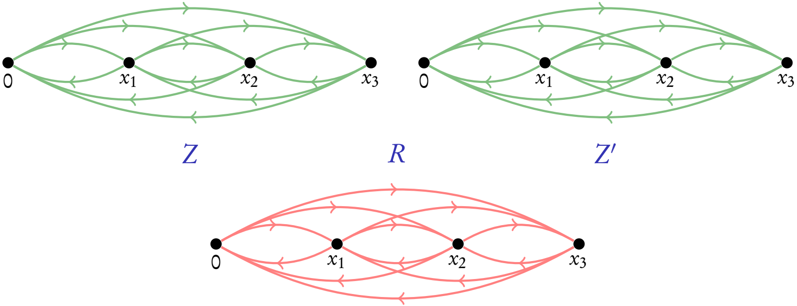

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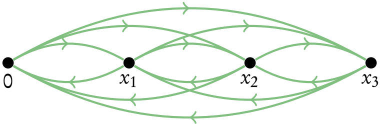
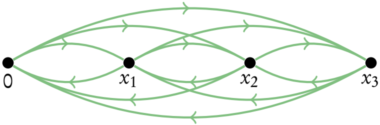
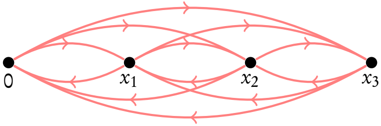




$$Z \quad \cap \quad R \quad \not\cap \quad Z'$$



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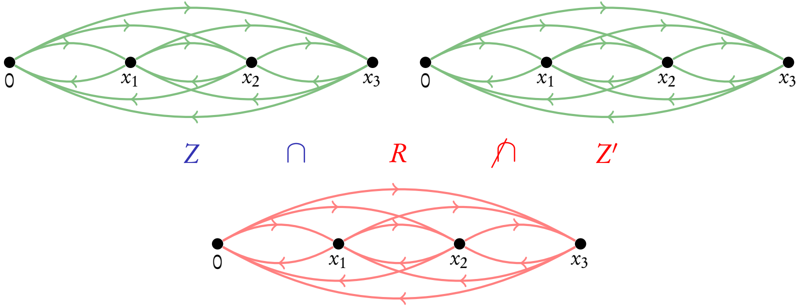

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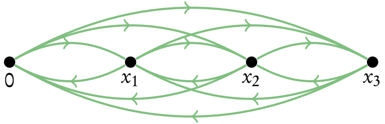
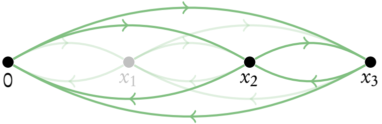
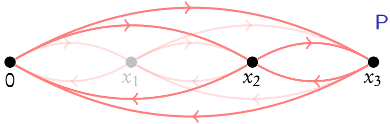




$Z \quad \cap \quad R \quad \not\cap \quad Z'$



$\text{Proj}_{x_2 x_3}(R) \not\cap \text{Proj}_{x_2 x_3}(Z')$

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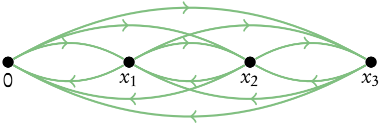

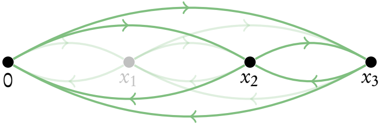

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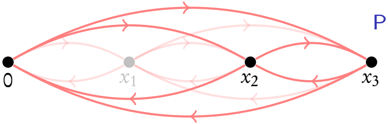

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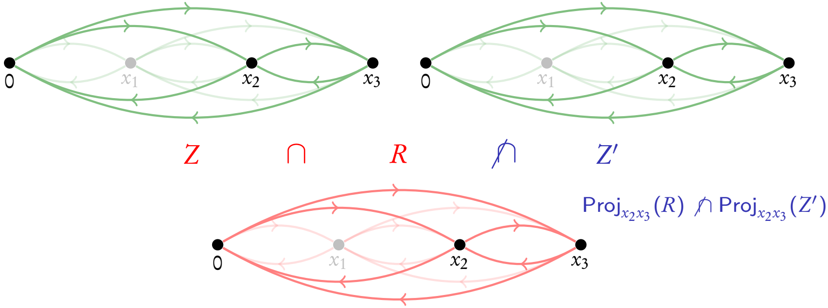
Step 3: Reduction to two clocks

Recall: $Z \not\subseteq \text{Closure}_M(Z') \Leftrightarrow \exists R.\ R$ intersects Z, R does not intersect Z'



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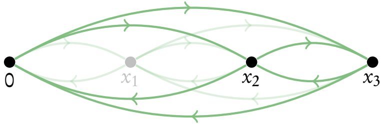
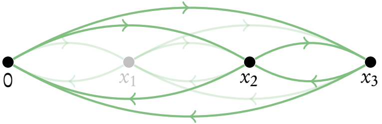
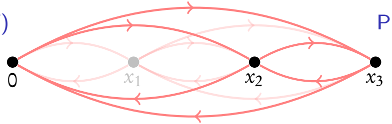




$Z \quad \cap \quad R \quad \not\cap \quad Z'$

$\text{Proj}_{x_2x_3}(R) \cap \text{Proj}_{x_2x_3}(Z)$



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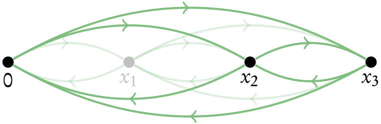

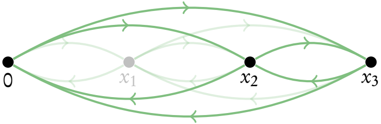

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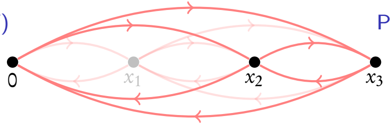


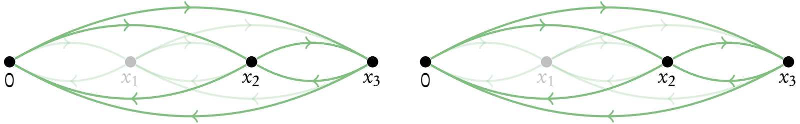
$\text{Proj}_{x_2x_3}(R) \not\cap \text{Proj}_{x_2x_3}(Z')$

Theorem

$Z \not\subseteq \text{Closure}_\alpha(Z')$ if and only if there **exist 2 clocks** x, y s.t.

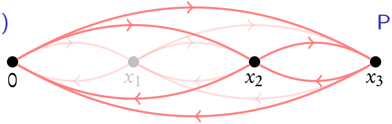
$$\mathbf{Proj_{xy}}(Z) \not\subseteq \text{Closure}_M(\mathbf{Proj}_{xy}(Z'))$$

Step 3: Reduction to two clocks



$Z \quad \cap \quad R \quad \not\cap \quad Z'$

$\mathsf{Proj}_{x_2x_3}(R) \cap \mathsf{Proj}_{x_2x_3}(Z)$



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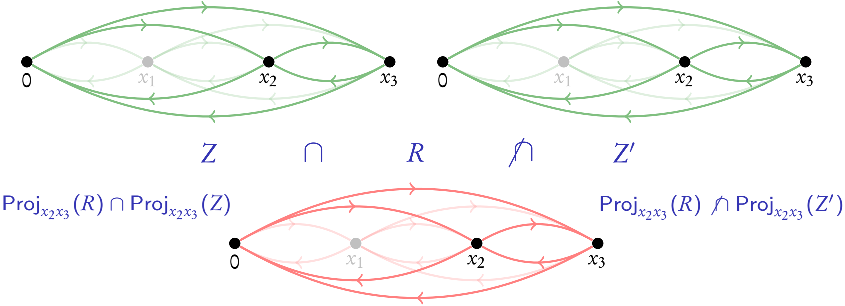
Slightly **modified edge-edge comparison** is enough

Step 3: Reduction to two clocks



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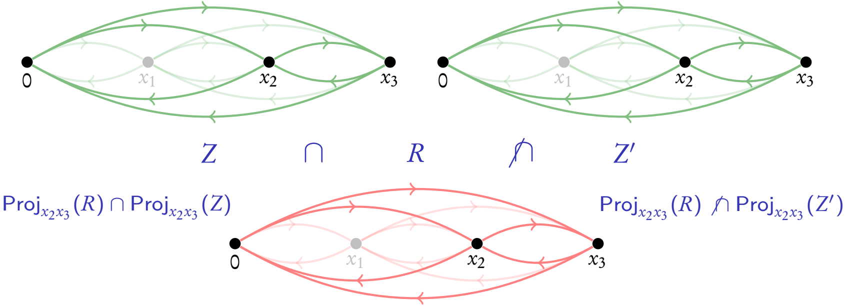
Complexity: $\mathcal{O}(|X|^2)$, where X is the set of clocks

Step 3: Reduction to two clocks



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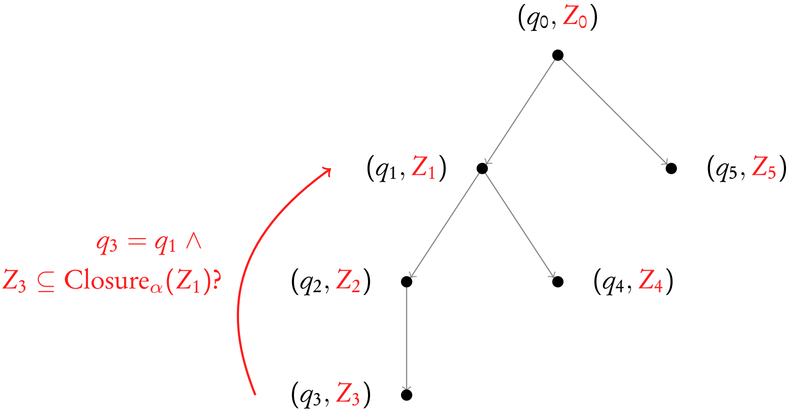
Same complexity as $Z \subseteq Z'$!

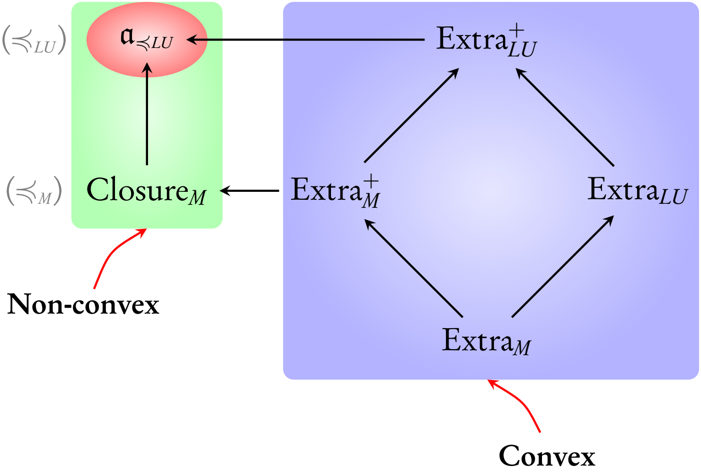
So what do we have now...



Efficient algorithm for $Z \subseteq \text{Closure}_\alpha(Z')$

Overall algorithm

- **Store** concrete semantics : zones
- Compute $ZG(\mathcal{A})$: $Z \subseteq \text{Closure}_{\alpha'}(Z')$ for **termination**



Next lecture: $\mathfrak{a}_{\preccurlyeq LU}$, optimality and benchmarks

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