GOALS OF TODAY'S LECTURE

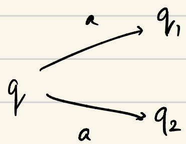
Event-clock Automata:

- a determinizable subclass of T.A.

- Alur, Fix, Henzinger

Problem with subset construction:

Untimed case:

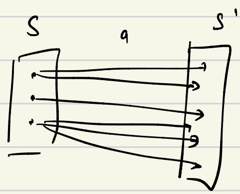


$\{q\} \xrightarrow{a} \{q_1, q_2\}$

$S \xrightarrow{a} S'$ $\quad S \subseteq Q$ states.

- 1. for every $q' \in S'$ $\exists q \in S$ s.t. $q \xrightarrow{a} q'$

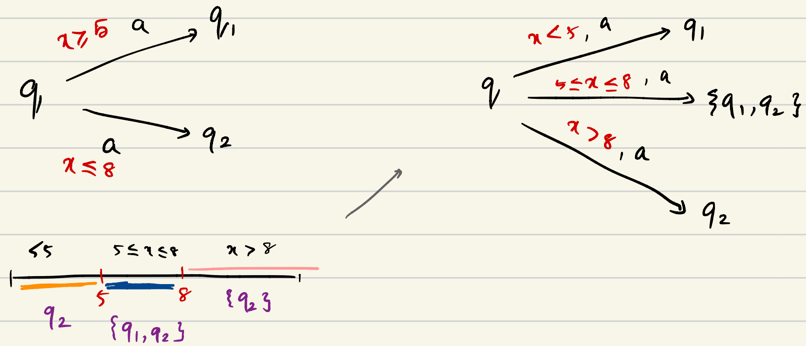
- 2. Moreover, $\forall q \in S$. if $q \xrightarrow{a} q'$ then $q' \in S'$.



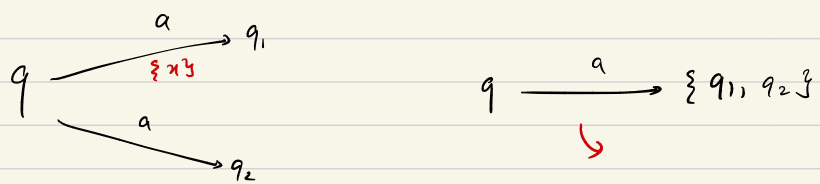
Timed case:

Guards a problem?

No.



Resets a problem?



Should x be reset or not?

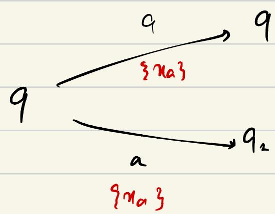
- If x is reset in $q \xrightarrow{a} \{q_1, q_2\}$ then there is potentially an incorrect run:

$$q \xrightarrow[\{x\}]{a} q_2 \longrightarrow \cdots$$

- If x is not reset, then there is an incorrect run:

$$q \xrightarrow{a} q_1 \longrightarrow$$

Main idea:



Whenever 'a' is on the transition
- x_a has to be reset.
- x_a cannot be reset in any other edge.

To do subset:

$$\{q\} \xrightarrow[\{x_c\}]{a} \{q_1, q_2\}$$

Summary:
- Problem with subset construction due to resets
- Can be circumvented by resetting a special clock 'x_a' at a, $\forall a$.

Event - recording Automata (ERA):

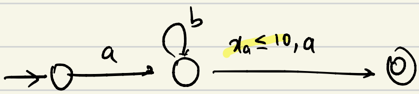
Σ: alphabet.

X_{Σ} : $\{ x_a \mid a \in \Sigma \}$

↳ Event - recording clocks.

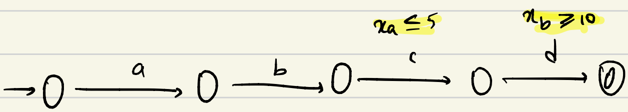
Example 1:

$\{ a b^* a \mid$ distance between the two a's is $\leq 10 \}$



Example 2:

$\{ (abcd, \tau_1 \tau_2 \tau_3 \tau_4) \mid \tau_3 - \tau_1 \leq 5 \wedge \tau_4 - \tau_2 \geq 10 \}$



Example 3:

$\{ (aaa, \tau_1 \tau_2 \tau_3) \mid \tau_3 - \tau_1 = 1 \}$

↳ no ERA

While reading third 'a' x_a maintains time since second 'a'.

Semantics of Event-recording clocks:

	a	b	a	a	b	b	a
	0.5	2.7	3.0	4.9	7.0	8.5	10.0
x_a	$\perp$	2.2	2.5	1.9	2.1	3.6	5.1
x_b	$\perp$	$\perp$	0.3	2.2	4.3	1.5	1.5

→ Values of x_a, x_b get determined by the input word!

– and not by the automaton

Given an input word $w = (a_1 a_2 \ldots a_k, \tau_1 \tau_2, \ldots, \tau_k)$

we define a function $\gamma_i : X_\Sigma \longrightarrow \{\perp\} \cup \mathbb{R}_{\geq 0}$

for all $i \in \{1, 2, \ldots k\}$

$$\gamma_i(x_a) = \begin{cases} t_i - t_j & \text{if there exists } j.\ j < i,\ a_j = a,\ \forall j < m < i,\ a_m \neq a \\ \perp & \text{otherwise} \end{cases}$$

Guards: $\Phi ::= x_a \sim c \mid \phi \wedge \phi \mid \phi \vee \phi$

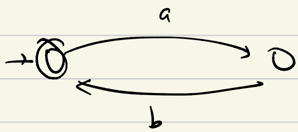
$\Phi(X_\Sigma)$ where $c \in \mathbb{N}$ or $c = \bot$

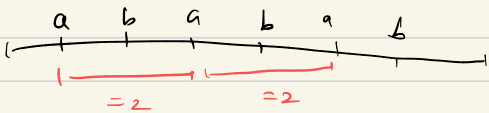
- $\bot \geq \bot$, $\bot \leq \bot$ is true
- Any other comparison is false.

$\bot \leq 5$, $10 > \bot$ are false.

Example 4:

$x_a = 1$ or $x_a = 2$





ERA defn: $\mathcal{A}: (Q, \Sigma, X_\Sigma, \Delta, F)$

$\Delta \subseteq Q \times \Sigma \times \Phi(X_\Sigma) \times Q$

Timed word w: $(a_1 a_2 \ldots a_k, \tau_1 \tau_2 \ldots \tau_k)$

When does $\mathcal{A}$ accept w?

$\mathcal{A}$ accepts w if there exists a run:

$$q_0 \xrightarrow{g_1, a_1} q_1 \xrightarrow{g_2, a_2} q_2 \longrightarrow \ldots \xrightarrow{g_k\ a_k} q_k$$

- s.t. $\gamma_i \models g_i$
- q_k is accepting.

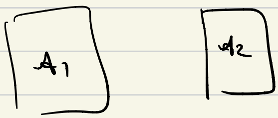
$\mathcal{L}(\mathcal{A}) = \{ w \mid \mathcal{A} \text{ accepts } w \}$

Closure properties:

$\mathcal{A}_1, \mathcal{A}_2$: ERA.

Union: ERA for $\mathcal{L}(\mathcal{A}_1) \cup \mathcal{L}(\mathcal{A}_2)$?

↳ disjoint union of $\mathcal{A}_1, \mathcal{A}_2$



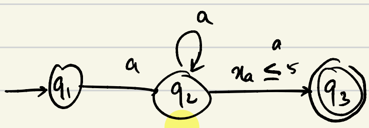
Intersection: ERA for $\mathcal{L}(\mathcal{A}_1) \cap \mathcal{L}(\mathcal{A}_2)$?

↳ product construction as done for DTA.

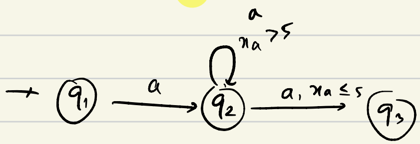
Complementation: ?

→ First consider determinization problem.

Determinization of ERA:



This is non-deterministic (NERA)

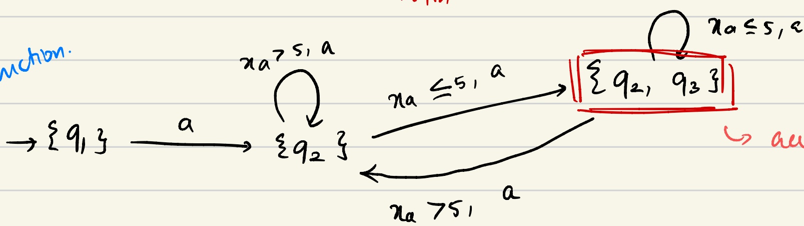


Wrong automaton

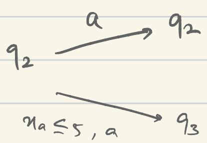
does not accept.

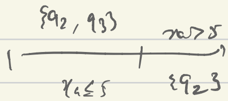
a	a	a
0	5	10

Example of subset construction.



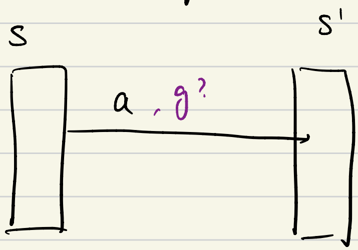
↳ accepting state.



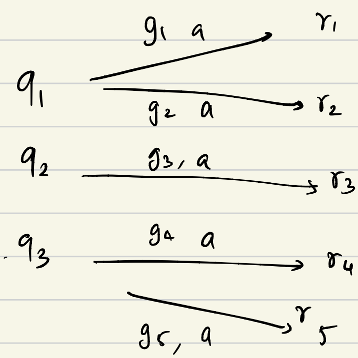


Subset construction for NERAs:

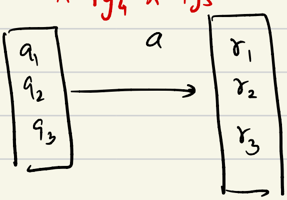
States: subsets of Q.



$S = \{ q_1, q_2, q_3, \}$



$g_1 \wedge g_2 \wedge g_3 \wedge \neg g_4 \wedge \neg g_5$



$S: \{q_1, q_2, \ldots\ q_k\}$

For every subset T of transitions on 'a' from S:

$T = \{(q_{i_1}, a, g_{i_1}, r_{i_1}) \ldots\ (q_{i_m}, a, g_{i_m}, r_{i_m})\}$

We have

$$S \xrightarrow[\phi]{a} \{r_{i_1}, r_{i_2}, \ldots, r_{i_m}\}$$

$$\phi: g_{i_1} \wedge g_{i_2} \wedge \ldots\ g_{i_m} \wedge \bigwedge_{\substack{q \xrightarrow{a,g} q' \\ q \in S,\ (q,a,g,q') \notin T}} \neg g$$

Subset construction:

States: subsets

Transition relation: as above

Accepting states: Subsets that intersect with F

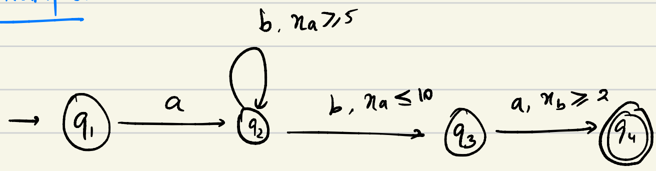
Thm: Deterministic ERAs are as expressive as NERAs.

Complementation:

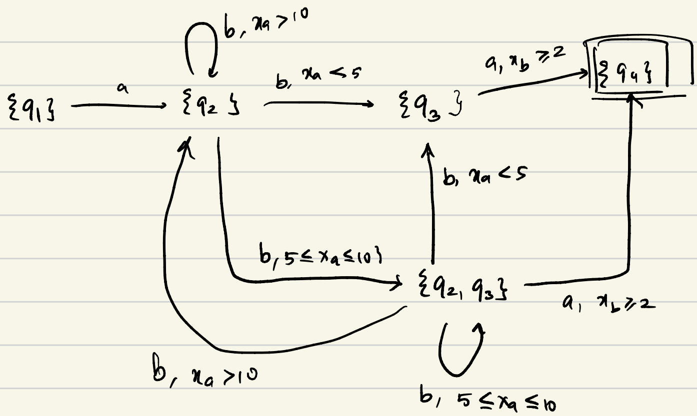
NERA $\mathcal{A}$ ⟶ DERA $\mathcal{B}$ $\xrightarrow[\text{acc} \setminus \text{rej states}]{\text{interchange}}$ DERA $\mathcal{B}^c$

$\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{B})$ for $\mathcal{L}(\mathcal{B})^c$.

Example:



NERA



DERA
Obtained by subset constrn.

Summary:

– 1. Event Recording Automata: determinizable
   - closed under boolean operations.

– 2. Some examples, some non-examples.