UNIVERSALITY PROBLEM

in

1- CLOCK timed automata

Let $T\Sigma^*$ denote the set of **all timed words**

Universality: Given A, is $\mathcal{L}(A) = T\Sigma^*$?

Inclusion: Given A, B, is $\mathcal{L}(B) \subseteq \mathcal{L}(A)$?

Universality and inclusion are **undecidable** when A has **two clocks** or more

A theory of timed automata

Alur and Dill. *TCS'94*

A decidable case of the inclusion problem

Universality: Given A, is $\mathcal{L}(A) = T\Sigma^*$?

Inclusion: Given A, B, is $\mathcal{L}(B) \subseteq \mathcal{L}(A)$?

One-clock restriction

Universality and inclusion are **decidable** when A has at most **one clock**

On the language inclusion problem for timed automata: Closing a decidability gap

Ouaknine and Worrell. *LICS'05*

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One-clock restriction

Universality and inclusion are **decidable** when A has at most **one clock**

On the language inclusion problem for timed automata: Closing a decidability gap

Ouaknine and Worrell. *LICS'05*

In this lecture: universality for one clock TA

Step 0:

Well-quasi orders and Higman's Lemma

Quasi-order

Given a set Q, a quasi-order is a **reflexive** and **transitive** relation:

$$\sqsubseteq \quad \subseteq \quad Q \times Q$$

- $(\mathbb{N}, \leq)$
- $(\mathbb{Z}, \leq)$

$$\text{Let } \Lambda = \{A, B, \ldots, Z\}, \quad \Lambda^* = \{\text{set of words}\}$$

- $(\Lambda^*,$ lexicographic order $\sqsubseteq_L)$: $AAAB \sqsubseteq_L AAB \sqsubseteq_L AB$
- $(\Lambda^*,$ prefix order $\subseteq_P)$: $AB \subseteq_P ABA \subseteq_P ABAA$
- $(\Lambda^*,$ subword order $\preccurlyeq)$ $HIGMAN \preccurlyeq HIGHMOUNTAIN$ [OW'05]

Well-quasi-order

An infinite sequence $\langle q_1, q_2, \ldots \rangle$ in $(Q, \sqsubseteq)$ is saturating if $\exists\, i < j : q_i \sqsubseteq q_j$

A quasi-order $\sqsubseteq$ is a well-quasi-order (wqo) if **every** infinite sequence is saturating

- $(\mathbb{N}, \leq)$: wqo
- $(\mathbb{Z}, \leq)$ $-1 \geq -2 \geq -3 \geq \ldots$
- $(\Lambda^*,$ lexicographic order $\sqsubseteq_L)$:
- $(\Lambda^*,$ prefix order $\subseteq_P)$:
- $(\Lambda^*,$ subword order $\preccurlyeq)$

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- $(\mathbb{N}, \leq)$ $\surd$
- $(\mathbb{Z}, \leq)$ $\times$ $-1 \geq -2 \geq -3, \ldots$
- $(\Lambda^*,$ lexicographic order $\sqsubseteq_L)$: $\times$ $B \sqsupseteq_L AB \sqsupseteq_L AAB \ldots$
- $(\Lambda^*,$ prefix order $\subseteq_P)$: $\times$ $B, AB, AAB, \ldots$
- $(\Lambda^*,$ subword order $\preccurlyeq)$

Well-quasi-order

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- $(\mathbb{N}, \leq)$ ✓
- $(\mathbb{Z}, \leq)$ × $-1 \geq -2 \geq -3, \ldots$
- (Λ^*, lexicographic order $\sqsubseteq_L$): × $B \sqsupseteq_L AB \sqsupseteq_L AAB \ldots$
- (Λ^*, prefix order $\subseteq_P$): × $B, AB, AAB, \ldots$
- (Λ^*, subword order $\preccurlyeq$) ?

Higman's lemma

Let $\sqsubseteq$ be a quasi-order on Λ

Define the induced **monotone domination order** $\preccurlyeq$ on Λ^* as follows:

$a_1 \ldots a_m \preccurlyeq b_1 \ldots b_n$ if there exists a **strictly increasing** function

$f : \{1, \ldots, m\} \mapsto \{1, \ldots, n\}$ s.t

$\forall\, 1 \leq i \leq m : a_i \sqsubseteq b_{f(i)}$

Higman's lemma

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$$f : \{1, \ldots, m\} \mapsto \{1, \ldots, n\} \text{ s.t}$$

$$\forall\, 1 \leq i \leq m : \; a_i \sqsubseteq b_{f(i)}$$

Higman'52

If $\sqsubseteq$ is a wqo on Λ, then the induced monotone domination order $\preccurlyeq$ is a wqo on Λ^*

Subword order

$$\Lambda \; := \; \{A, B, \ldots, Z\}$$
$$\sqsubseteq \; := \; x \sqsubseteq y \text{ if } x = y$$

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$\sqsubseteq$ is a **wqo** as Λ is **finite**

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Induced monotone domination order $\preccurlyeq$ is the subword order

$$HIGMAN \preccurlyeq HIGHMOUNTAIN$$

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Induced monotone domination order $\preccurlyeq$ is the subword order

$$HIGMAN \preccurlyeq HIGHMOUNTAIN$$

By Higman's lemma, $\preccurlyeq$ is a wqo too

If we start writing an **infinite sequence** of words, we will **eventually** write down a **superword** of an earlier word in the sequence

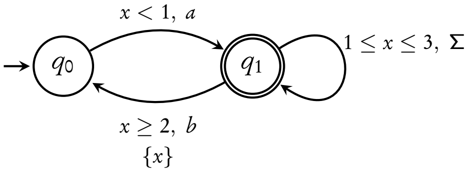
Step 1:

A naive procedure for universality of one-clock TA

Terminology

Let $A = (Q, \Sigma, Q_0, \{x\}, T, F)$ be a timed automaton with one clock

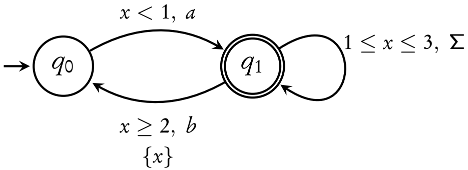
- **Location:** $q_0, q_1, \dots \in Q$
- **State:** (q, u) where $u \in \mathbb{R}_{\geq 0}$ gives value of the clock
- **Configuration:** **finite** set of states



Terminology

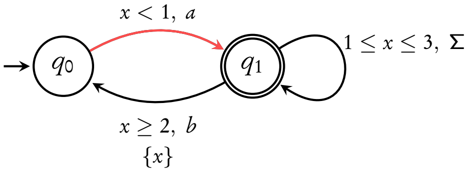
Let $A = (Q, \Sigma, Q_0, \{x\}, T, F)$ be a timed automaton with one clock

- **Location:** $q_0, q_1, \dots \in Q$
- **State:** (q, u) where $u \in \mathbb{R}_{\geq 0}$ gives value of the clock
- **Configuration:** **finite** set of states $\{(q_1, 2.3), (q_0, 0)\}$



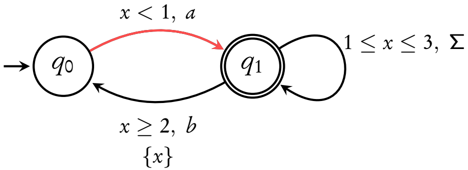
Transition between configurations:

$\{(q_0, 0)\} \xrightarrow{0.2,\ a}$



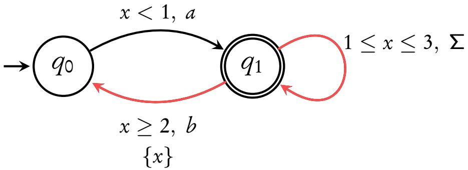
Transition between configurations:

$$\{(q_0, 0)\} \xrightarrow{0.2,\ a} \{(q_1, 0.2)\}$$



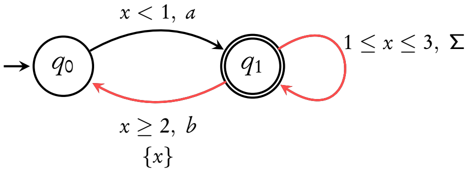
Transition between configurations:

$\{(q_0, 0)\} \xrightarrow{0.2,\ a} \{(q_1, 0.2)\} \xrightarrow{2.1,\ b}$



Transition between configurations:

$$\{(q_0, 0)\} \xrightarrow{0.2,\ a} \{(q_1, 0.2)\} \xrightarrow{2.1,\ b} \{(q_1, 2.3), (q_0, 0)\} \ldots$$



Transition between configurations:

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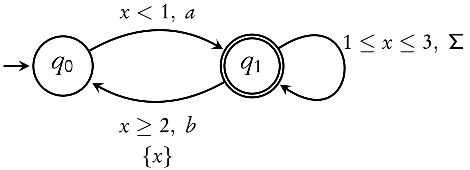
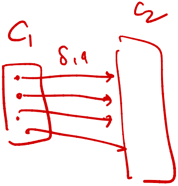
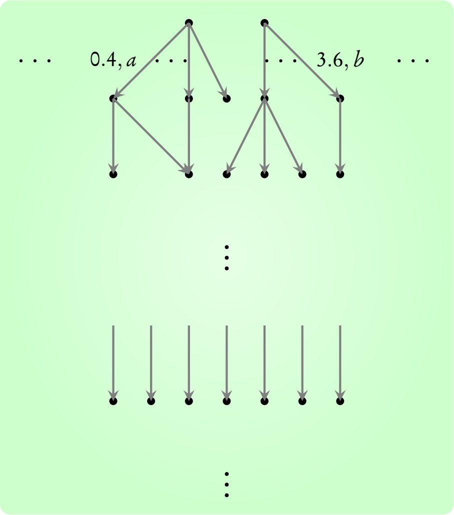




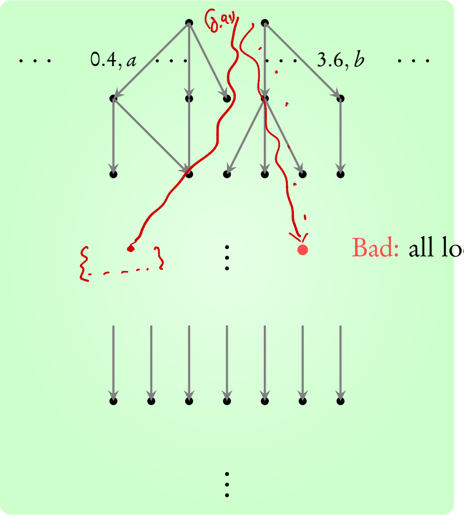
$$C_1 \xrightarrow{\delta,\ a} C_2 \text{ if}$$

$$C_2 = \{\ (q_2, u_2) \mid \exists (q_1, u_1) \in C_1 \text{ s. t. } (q_1, u_1) \xrightarrow{\delta,\ a} (q_2, u_2)\}$$

Labeled transition system of **configurations**



Labeled transition system of **configurations**

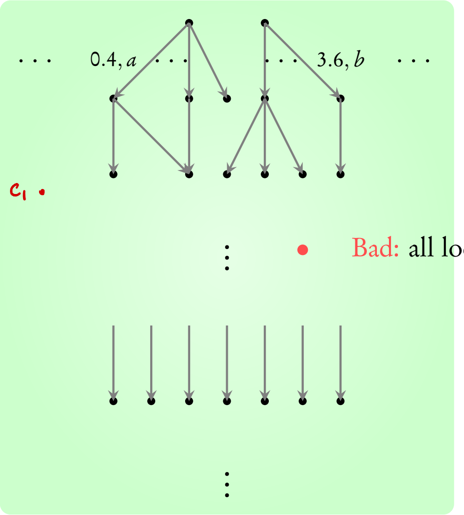


Bad: all locations **non-accepting**

$(\delta_1, a_1)(\delta_2, a_2) \ldots (\delta_n, a_n)$

- unique path for each word in this transition system.

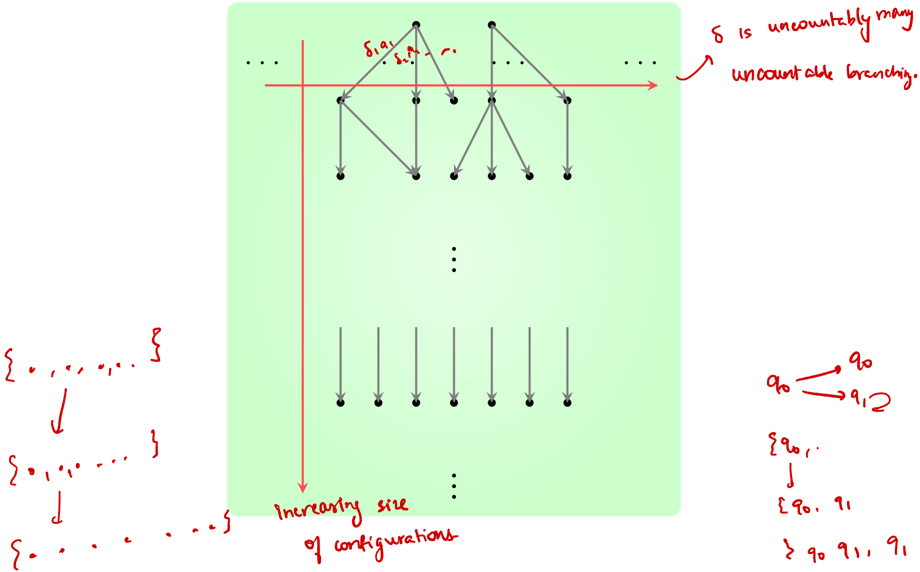
Labeled transition system of **configurations**



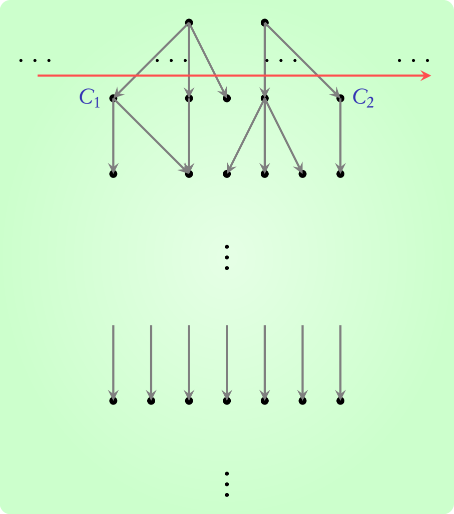
Bad: all locations **non-accepting**

Checking universality of A reduces to this question:

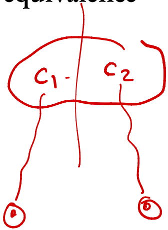
Is a bad configuration **reachable** from some **initial** configuration?



Need to handle **two dimensions** of infinity!



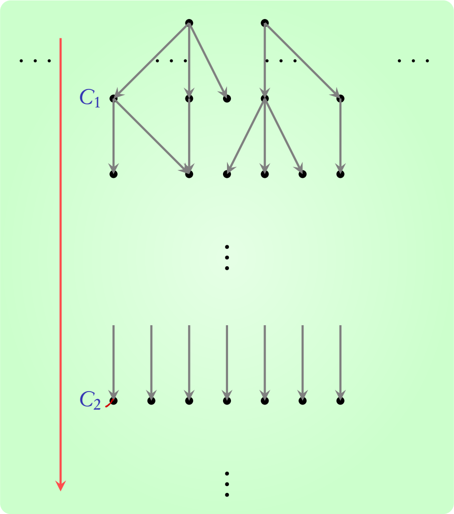
abstraction by **equivalence** $\sim$



$C_1 \sim C_2$ should imply:

C_1 goes to a **bad** config. $\Leftrightarrow$ C_2 goes to a **bad** config.

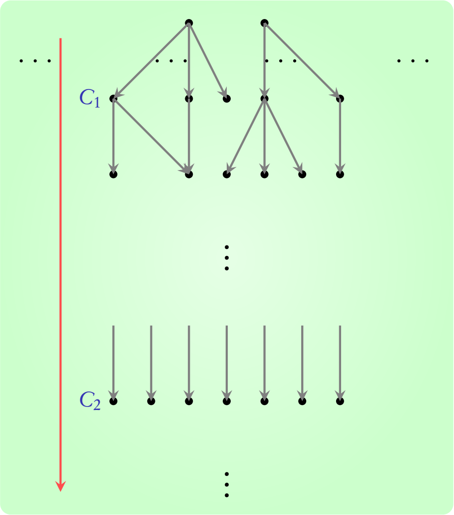
finite **domination** order $\preccurlyeq$



$C_1 \preccurlyeq C_2$ should imply:

C_2 goes to a **bad** config $\Rightarrow$ C_1 goes to a **bad** config. too

finite **domination** order $\preccurlyeq$



$C_1 \preccurlyeq C_2$ iff:

C_2 goes to a **bad** config $\Rightarrow$ C_1 goes to a **bad** config. too

No need to explore C_2!

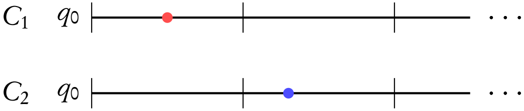
Step 2:

The equivalence

Credits: Examples in this part taken from one of **Ouaknine's talks**

Equivalent configurations: Examples

$$C_1 = \{(q_0, 0.5)\} \not\sim C_2 = \{(q_0, 1.3)\}$$



Equivalent configurations: Examples

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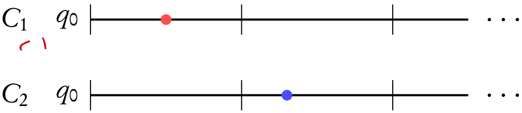
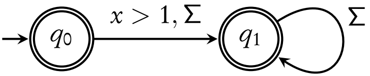
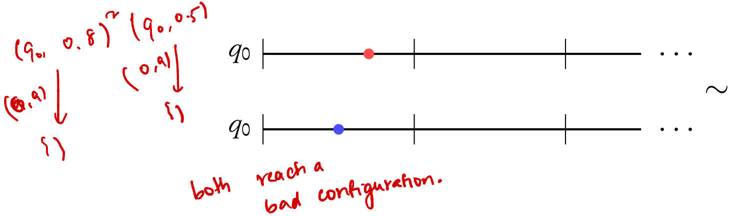
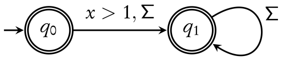




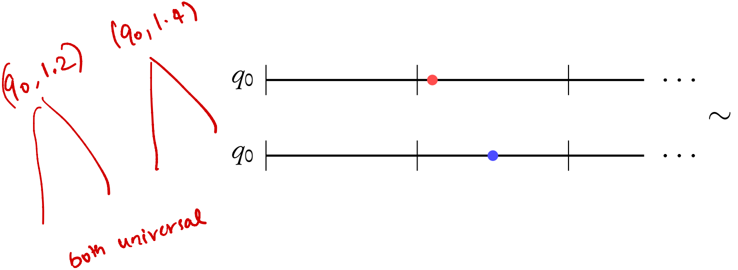
C_2 is universal, but C_1 **rejects** $(a, 0)$



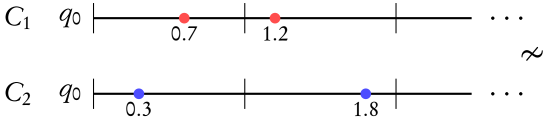
(q0, 0.8)
(q0, 0.5)
(0.9)
(0.9)
{}
{}
q0
q0
~
both reach a bad configuration.



q0
q1
x > 1, Σ
Σ

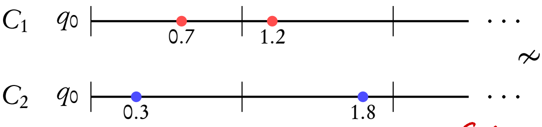


(q0, 1.2)
(q0, 1.4)
q0
q0
~
both universal



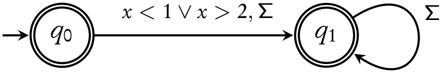
$C_1 = \{ (q_0, 0.7), (q_0, 1.2) \}$

$\{(q_0, 0.7), (q_0, 1.2)\}$



$\{(q_0, 0.3), (q_0, 1.8)\}$

$C_3 = \{(q_0, 0.5), (q_0, 1.4)\}$



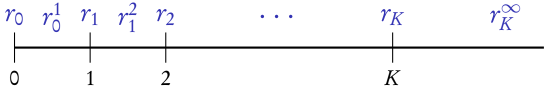
$C_1 \sim C_3$? ✓

$C_2 \sim C_3$? ✗

C_2 is universal, but C_1 **rejects** $(a, 0.5)$

Let K be the largest constant appearing in A

Define $REG = \{r_0, r_0^1, r_1, \ldots, r_K, r_K^\infty\}$



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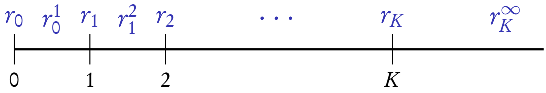

$$C = \{(q_1, 0.0), (q_1, 0.3), (q_1, 1.2), (q_2, 1.0), (q_3, 0.8), (q_3, 1.3)\}$$

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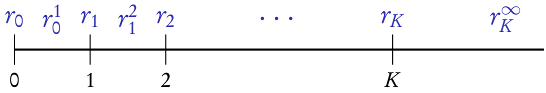

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$$\{(q_1, r_0, 0), (q_1, r_0^1, 0.3), (q_1, r_1^2, 0.2), (q_2, r_1, 0), (q_3, r_0^1, 0.8), (q_3, r_1^2, 0.3)\}$$

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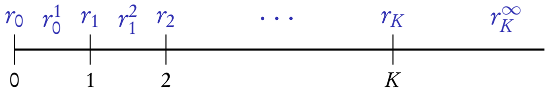

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$$\{(q_1, r_0, 0), (q_2, r_1, 0)\}\{(q_1, r_1^2, 0.2)\}\{(q_1, r_0^1, 0.3)(q_3, r_1^2, 0.3)\}\{(q_3, r_0^1, 0.8)\}$$

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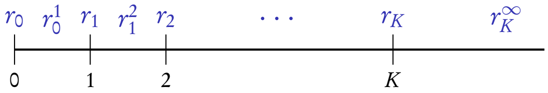

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$$\{(q_1, r_0, 0), (q_1, r_0^1, 0.3), (q_1, r_1^2, 0.2), (q_2, r_1, 0), (q_3, r_0^1, 0.8), (q_3, r_1^2, 0.3)\}$$

$$\{(q_1, r_0, 0), (q_2, r_1, 0)\}\ \{(q_1, r_1^2, 0.2)\}\ \{(q_1, r_0^1, 0.3)(q_3, r_1^2, 0.3)\}\ \{(q_3, r_0^1, 0.8)\}$$

$$H(C) = \{(q_1, r_0), (q_2, r_1)\}\ \{(q_1, r_1^2)\}\ \{(q_1, r_0^1)(q_3, r_1^2)\}\ \{(q_3, r_0^1)\}$$

Let K be the largest constant appearing in A

$$REG := \{r_0, r_0^1, r_1, \ldots, r_K, r_K^\infty\}$$

$$\Lambda := \mathcal{P}(\ Q \times REG\ )$$

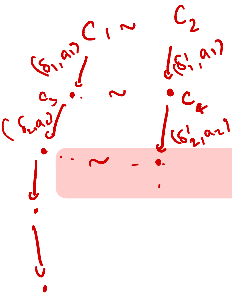
We can give $H : C \mapsto \Lambda^*$ that remembers:

- **integral** part of the clock value (modulo K) in each state of C,
- **order** of **fractional** parts of the clock among different states in C

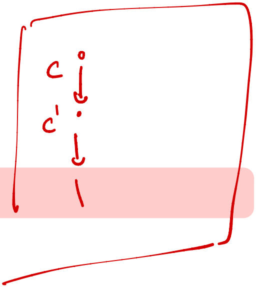
Equivalence

$$C_1 \sim C_2 \quad \text{if} \quad H(C_1) = H(C_2)$$

Equivalence

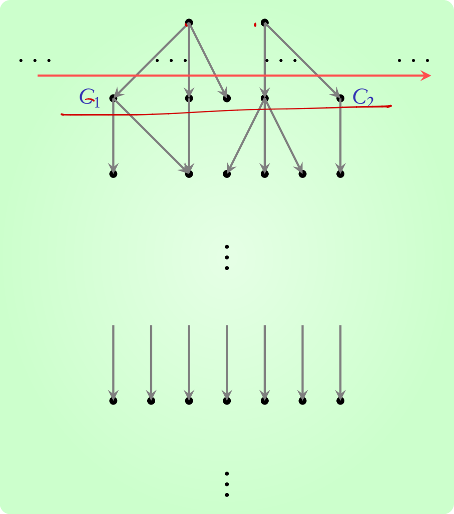


$C_1 \sim C_2$ if $H(C_1) = H(C_2)$

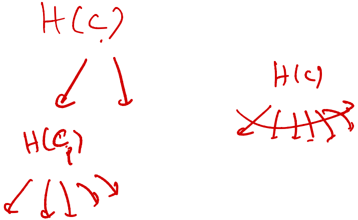


It can be shown that $\sim$ is a **bisimulation**

C_1 goes to a **bad** config. $\Leftrightarrow$ C_2 goes to a **bad** config.



abstraction by **equivalence** $\sim$



$C_1 \sim C_2$ iff:

C_1 goes to a **bad** config. $\Leftrightarrow$ C_2 goes to a **bad** config.

Associating a word to each configuration:

$K = 10$

C_1 : $\{ (q_1, 5.7), (q_1, 2.3), (q_2, 7.3), (q_2, 10.4), (q_3, 2.9) \}$

$\downarrow$

Encoding:

$H(C_1)$: $\{(q_1, r_2^3), (q_2, r_7^8)\} \; \{(q_1, r_5^6)\} \; \{(q_3, r_2^3)\} \; \{(q_2, r_{10}^{\infty})\}$

Time-Successors of H(C):

(iii) (i) (ii)

$H(C_1) \xrightarrow{\tau} \{(q_3, r_3)\} \; \{(q_1, r_2^3), (q_2, r_7^8)\} \; \{(q_1, r_5^6)\} \; \{(q_2, r_{10}^{\infty})\}$

$\downarrow \tau$

$\{(q_3, r_3^4)\} \; \{(q_1, r_2^3), (q_2, r_7^8)\} \; \{(q_1, r_5^6)\} \; \{(q_2, r_{10}^{\infty})\}$

$\downarrow \tau$

$\{(q_1, r_6)\} \; \{(q_3, r_3^4)\} \; \{(q_1, r_2^3), (q_2, r_7^8)\} \; \{(q_2, r_{10}^{\infty})\}$

$\downarrow \tau$

$\{(q_1, r_6^7)\} \; \{(q_3, r_3^4)\} \; \{(q_1, r_2^3), (q_2, r_7^8)\} \; \{(q_2, r_{10}^{\infty})\}$

$\downarrow \tau$

$\vdots$

$\{ (q_1, r_{10}^{\infty}), (q_2, r_{10}^{\infty}), (q_3, r_{10}^{\infty}) \}$

Claim: Let H_1, H_2 be encodings s.t.

$$H_1 \xrightarrow{\tau} H_2$$

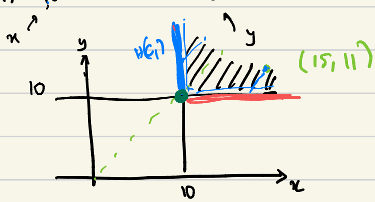
Then: for every $C_2 \in H_2$, there exists $C_1 \in H_1$ s.t.

$$C_1 \xrightarrow{\delta} C_2 \quad \text{for some } \delta > 0$$

This claim is false!

- Consider $H(C_1)$: $\{(q_1, r_{10})\}$ $\{(q_2, r_{10}^{\infty})\}$
  H_1

$$H_1 \xrightarrow{\tau} \{(q_1, r_{10}^{\infty}), (q_2, r_{10}^{\infty})\}$$



Lemma: Let H_1, H_2 be encodings s.t.

$$H_1 \xrightarrow{T} H_2$$

Then: for every $C_1 \in H_1$ there exists a $C_2 \in H_2$ s.t.

$$C_1 \xrightarrow{\delta} C_2 . \quad \text{for some} \quad \delta > 0$$

Proof: (Sketch). Given $H_1 = w_1 \; w_2 \; \ldots \; w_n \; w_\infty \xrightarrow{T} H_2$

H_2 : - 1. If w_1 "is an integer" :

$$H_2 = w_1' \; w_2 \ldots \; w_n \; w_\infty'$$

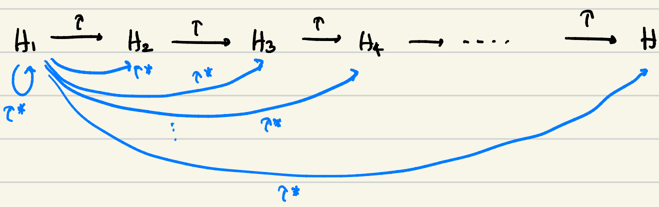
- 2. If w_1 "is not an integer"

$$H_2 = w_n' \; w_1 \; w_2 \ldots \; w_n \; w_\infty$$

- In each case, find a specific δ based on the given configuration $C_1 \in H_1$.

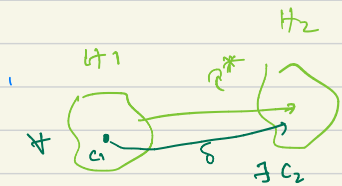
Reflexive-transitive closure of τ:

τ : immediate successor



τ^* : reflexive transitive closure of τ:

The previous lemma carries over to τ^*:



Lemma: Let H_1, H_2 be encodings s.t.

$$H_1 \xrightarrow{\tau^*} H_2$$

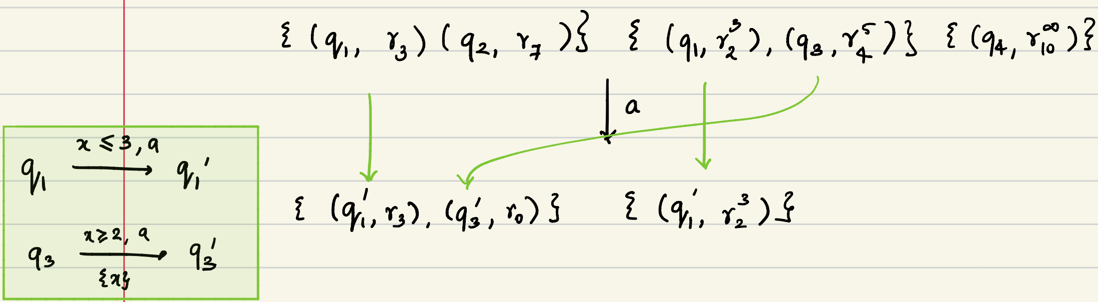
Then: for every $c_1 \in H_1$ there exist a $c_2 \in H_2$ s.t.

$$c_1 \xrightarrow{\delta} c_2 \,. \quad \text{for some } \delta \geq 0$$

Moral:

- Time successors of an encoding can be effectively computed
- Above lemma.

Discrete - successors of H(C)



There can also be multiple edges on 'a' from a state.

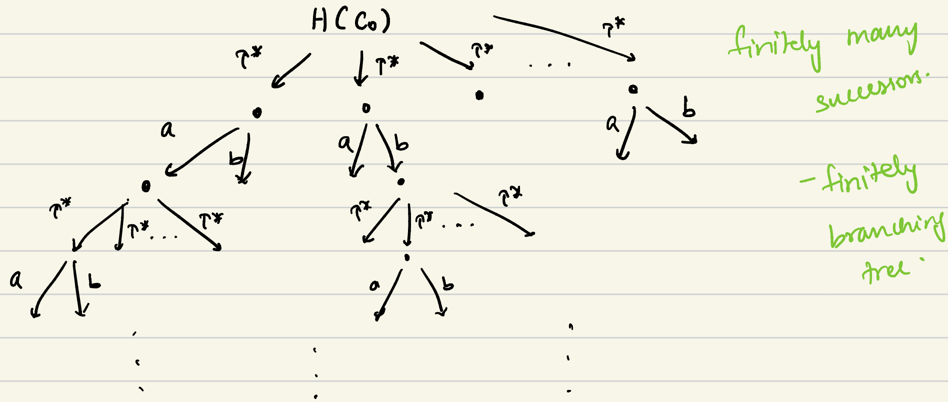
Same lemma as before applies to discrete successors as well.

> **Lemma:** Let H_1, H_2 be encodings s.t.
>
> $$H_1 \xrightarrow{a} H_2$$
>
> Then: for every $C_1 \in H_1$ there exist a $C_2 \in H_2$ s.t.
>
> $$C_1 \xrightarrow{a} C_2 .$$

Moral:

- Discrete successor computation is also effective
- Above lemma

Construction of a tree :– "Encoding tree"



Soundness:

Proposition: For every path $H_0 \xrightarrow{\tau^*} \cdot \xrightarrow{a_1} H_1 \xrightarrow{\tau^*} \xrightarrow{a_2} H_2 \rightarrow \cdots \xrightarrow{a_n} H_n$

there exists a run: $C_0 \xrightarrow{\delta_1, a_1} C_1 \xrightarrow{\delta_2, a_2} C_2 \rightarrow \cdots\cdots \xrightarrow{\delta_n, a_n} C_n$

s.t. $C_i \in H_i$

Completeness:

Proposition: For every run $C_0 \xrightarrow{\delta_1, a_1} C_1 \xrightarrow{\delta_2, a_2} C_2 \rightarrow \cdots\cdots \xrightarrow{\delta_n, a_n} C_n$

there exists a path: $H(C_0) \xrightarrow{\tau^*} \xrightarrow{a_1} H(C_1) \xrightarrow{\tau^*} \xrightarrow{a_2} H(C_2) \rightarrow \cdots \xrightarrow{a_n} H(C_n)$

Theorem: Automaton is not universal

iff

a bad node is reached in the encoding tree.

Encoding tree can be constructed on-the-fly.

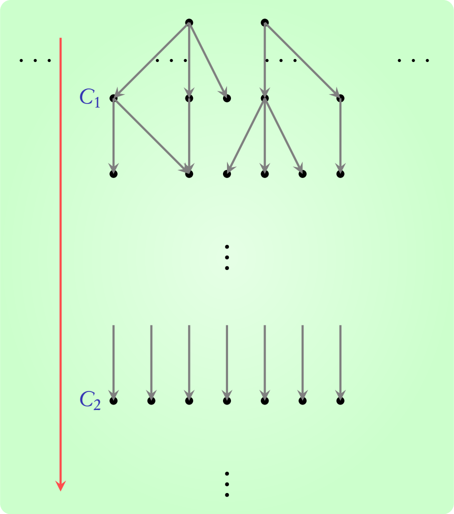
Termination ??

A bad node in the tree is an encoding where all the locations are non-accepting.

Step 3:

The domination order

finite **domination** order $\preccurlyeq$



$C_1 \preccurlyeq C_2$ iff:

C_2 goes to a **bad** config $\Rightarrow$ C_1 goes to a **bad** config. too

Look at $H(C_1)$ and $H(C_2)$, the words over Λ^*

$$\Lambda = \mathcal{P}(\ Q \times REG\ )$$

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Let $\subseteq$ be the **inclusion** (quasi-)order on Λ

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Let $\subseteq$ be the **inclusion** (quasi-)order on Λ

Consider the induced monotone domination order $\preccurlyeq$ over Λ^*

H(C1): $\{(q_0, r_0)\}\ \ \{(q_1, r_0^1), (q_0, r_2^3)\}$

≼ $\qquad \preccurlyeq$

H(C2): $\{(q_0, r_0), (q_1, r_1)\}\ \ \{(q_2, r_2^3)\}\ \ \{(q_1, r_0^1), (q_0, r_2^3), (q_2, r_1^2)\}$

Look at $H(C_1)$ and $H(C_2)$, the words over Λ^*

$$\Lambda = \mathcal{P}(\ Q \times REG\ )$$

Let $\subseteq$ be the **inclusion** (quasi-)order on Λ

Consider the induced monotone domination order $\preccurlyeq$ over Λ^*

$$\{(q_0, r_0)\}\ \{(q_1, r_0^1), (q_0, r_2^3)\}$$

$$\preccurlyeq$$

$$\{(q_0, r_0), (q_1, r_1)\}\ \{(q_2, r_2^3)\}\ \{(q_1, r_0^1), (q_0, r_2^3), (q_2, r_1^2)\}$$

Theorem: If $H(C_1) \preccurlyeq H(C_2)$, then $\exists C_2' \subseteq C_2$ s.t. $C_1 \sim C_2'$

Look at $H(C_1)$ and $H(C_2)$, the words over Λ^*

$$\Lambda = \mathcal{P}(\ Q \times REG\ )$$

Let $\subseteq$ be the **inclusion** (quasi-)order on Λ

Consider the induced monotone domination order $\preccurlyeq$ over Λ^*

$$\{(q_0, r_0)\}\ \ \{(q_1, r_0^1), (q_0, r_2^3)\}$$

$$\preccurlyeq$$

$$\{(q_0, r_0), (q_1, r_1)\}\ \ \{(q_2, r_2^3)\}\ \ \{(q_1, r_0^1), (q_0, r_2^3), (q_2, r_1^2)\}$$

Theorem: If $H(C_1) \preccurlyeq H(C_2)$, then $\exists C_2' \subseteq C_2$ s.t. $C_1 \sim C_2'$

$\subseteq$ is a wqo as Λ is **finite**. Therefore, $\preccurlyeq$ is a **wqo** due to **Higman's lemma**

Final algorithm

- Start from $H(C_0)$, where C_0 is the **initial configuration**
- Successor computation is **effective**
- Termination guaranteed as **domination order is wqo**

— Before each successor computation, check if there is a smaller word. If yes, do not explore this node.

A is **universal** iff the algorithm does **not reach a bad node**

One-clock

Universality is **decidable** for one-clock timed automata

One-clock

Universality is **decidable** for one-clock timed automata

For **two clocks**, we know universality is undecidable

One-clock

Universality is **decidable** for one-clock timed automata

For **two clocks**, we know universality is undecidable

Where does this algorithm go wrong when A has two clocks?

Two clocks

State: (q, u, v)

Configuration: $\{(q_1, u_1, v_1), (q_2, u_2, v_2), \ldots, (q_n, u_n, v_n)\}$

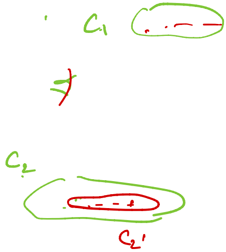
At the **least**, the following should be remembered while abstracting:

- relative ordering between fractional parts of x
- relative ordering between fractional parts of y

Current encoding can remember **only one** of them

Other encodings possible?

Consider some domination order $\preccurlyeq$

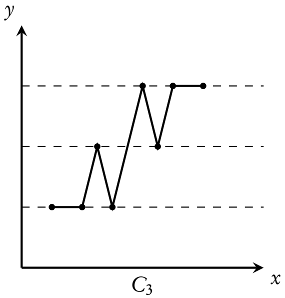


$C_1 \not\preccurlyeq C_2$ if for all $C_2' \subseteq C_2$:

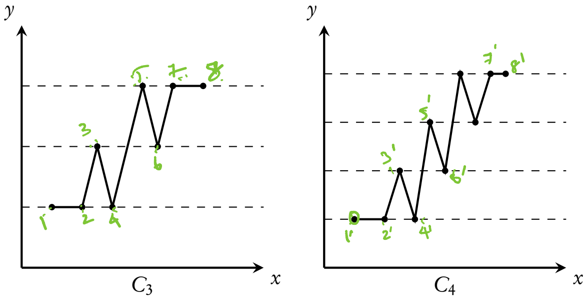
- either relative order of clock x does not match
- or relative order of clock y does not match

In the next slide: **No wqo** possible!

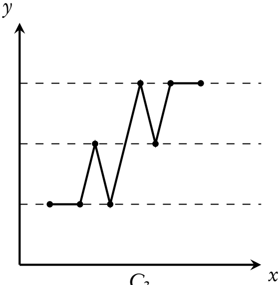
An infinite **non-saturating** sequence $C_1, C_2, C_3, \ldots$



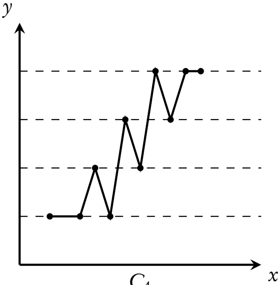
An infinite **non-saturating** sequence $C_1, C_2, C_3, \ldots$



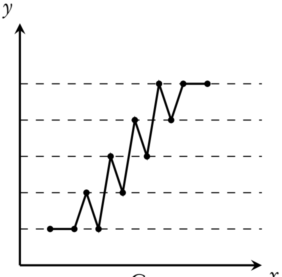
An infinite **non-saturating** sequence $C_1, C_2, C_3, \ldots$



C_3



C_4



C_5

Conclusion

- An algorithm for **universality** when A has one clock
- Can be **extended** for $\mathcal{L}(B) \subseteq \mathcal{L}(A)$ when A has one-clock