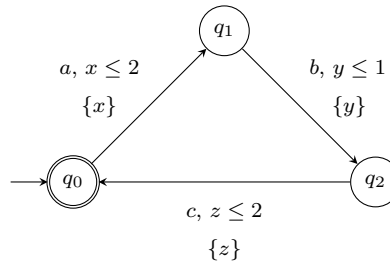


1. What is the timed word accepted by the following accepting run of some timed automaton with two clocks x and y ?

$$\begin{array}{ccccccc}
 q_0 & & q_0 & & q_1 & & q_1 & & q_F \\
 x : 0 & \xrightarrow{\delta_0} & x : 0.6 & \xrightarrow{a} & x : 0.6 & \xrightarrow{\delta_1} & x : 1.9 & \xrightarrow{a} & x : 0 \\
 y : 0 & & y : 0.6 & & y : 0 & & y : 1.3 & & y : 1.3
 \end{array}$$

2. Let \mathcal{B} be the following timed automaton:



Consider the timed word $s = (abcabc, 0.5, 1, 1.5, 1.8, 1.9, 3)$.

- a) Does \mathcal{B} accept s ? If so, write down the accepting run of \mathcal{B} on s .
 - b) For a timed word (w, τ) we define the *time span* of (w, τ) to be the time at which the last letter occurs, i.e., if $|w| = n$, then time span of (w, τ) is τ_n .
For every $k \in \mathbb{N}$, give a timed word in $\mathcal{L}(\mathcal{B})$ that has length greater than k and whose time span is lesser than 1.
3. Give a timed automaton for the timed language of all words in $(a + b)^*$ such that there exist two a -s which are at distance 1 apart *and* there exist two b -s which are at distance 2 apart
 4. Let $\Sigma = \{a, b\}$. Construct timed automata for the following languages:
 - (a) $\{(w, \tau) \mid \text{there is an } a \text{ at some time } t \text{ and no action occurs at time } t + 1\}$
 - (b) $\{(w, \tau) \mid \text{all } a\text{'s occur before time 1 and no two } a\text{'s happen at the same time}\}$
 - (c) $\{(a^k, \tau) \mid k \geq 1, \tau_1 = 1, \text{ and } \tau_{i+2} - \tau_i \leq 1 \text{ for all } 1 \leq i \leq k - 2\}$
 5. Group the following valuations over five clocks $\{x_1, x_2, \dots, x_5\}$ into regions. Assume that $M_{x_1} = 8, M_{x_2} = 3, M_{x_3} = 5, M_{x_4} = 2, M_{x_5} = 7$.

$$\begin{aligned}
 v_1 &:= (7.4, 2.1, 8.7, 5.4, 7.0) \\
 v_2 &:= (3.4, 2.0, 8.5, 10.0, 7.1) \\
 v_3 &:= (7.3, 2.2, 8.8, 5.2, 7.0) \\
 v_4 &:= (7.5, 2.1, 8.9, 5.5, 7.0) \\
 v_5 &:= (3.2, 2.0, 8.8, 10.0, 7.5) \\
 v_6 &:= (3.3, 2.0, 8.4, 10.0, 7.2)
 \end{aligned}$$

6. Consider an automaton with 2 clocks $\{x, y\}$. Let the maximum bounds function M for the automaton be given by: $M(x) = 3, M(y) = 4$. Draw the division of the xy -plane into regions.
7. Given 3 clocks $\{x, y, z\}$ and $M(x) = 2, M(y) = 1, M(z) = 2$, enumerate the set of regions.

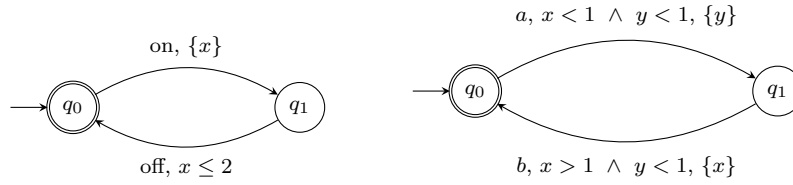


Figure 1:

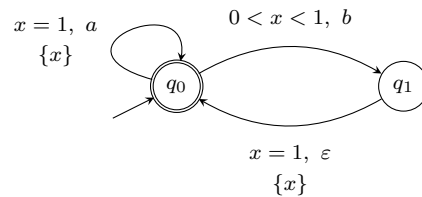
8. Let R be a region over clock set X and bound function M . Give an algorithm to compute the time-successors of a region R .
9. Draw the region automaton for the automata shown in Figure 1.
10. Suppose R is a region over clock set X and bound function M . Let x, y be two arbitrary clocks in X . Is the projection of R on to the xy -plane a region over $\{x, y\}$ with the bounds function M restricted to x and y ?
11. Construct a deterministic timed automaton for the language:

$$\{ (a, t_1)(a, t_2) \dots (a, t_n) \mid n \geq 2 \text{ and there exists some } i \in \{1, \dots, n - 2\} \text{ s.t. } t_{i+2} - t_i = 1 \}$$

12. Show that the following language is not timed regular:

$$\{ (a, t_1)(a, t_2) \dots (a, t_n) \mid n \geq 3 \text{ and for every } i \in \{2, \dots, n - 1, \text{ we have } t_i - t_{i-1} = t_{i+1} - t_i \}$$

13. What is the language accepted by the following automaton? **(8 marks)**



14. Draw the automaton (if needed with ϵ -transitions) for the language over $\Sigma = \{a, b\}$ given by:

$$\{ (w, \tau) \mid w \in \Sigma^*, \forall i \leq |w| : w_i = a \text{ implies } \tau_i \text{ is an integer and } w_i = b \text{ implies } \tau_i \text{ is not an integer} \}$$

where w_i denotes the i^{th} letter in the word w and τ_i denotes the corresponding time-stamp.