

# TIMED AUTOMATA

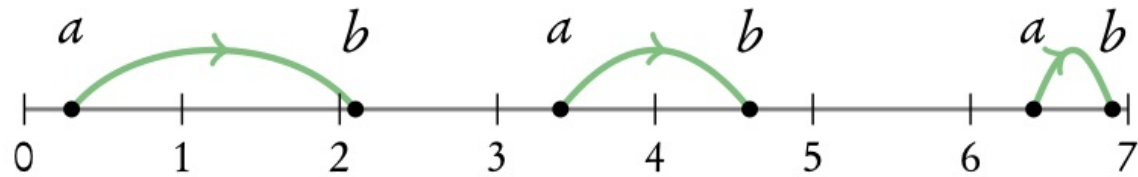
## LECTURE 2

## GOALS OF TODAY'S LECTURE

- 1. Languages not accepted by Timed Automata
- 2. Timed regular languages
- 3. Closure properties
- 4. Course plan

$$L_7 = \{ ( (ab)^k, \tau ) \mid \tau_{2i+2} - \tau_{2i+1} < \tau_{2i} - \tau_{2i-1} \text{ for each } i \geq 1 \}$$

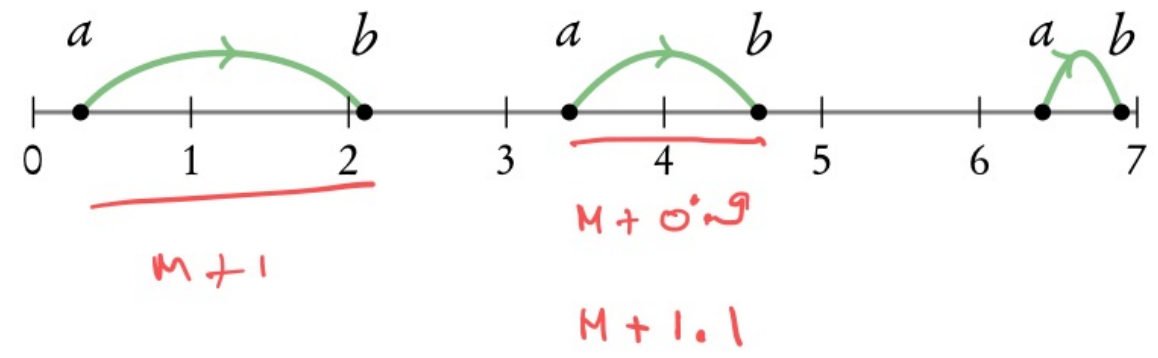
Converging  $ab$  distances



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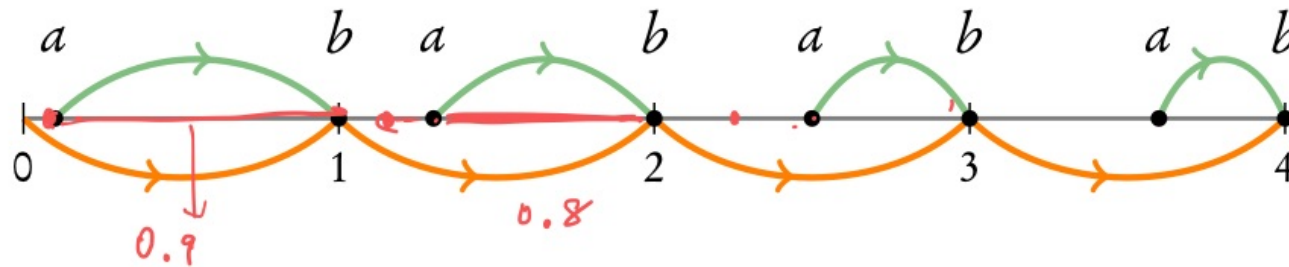
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Exercise: Prove that no timed automaton can accept  $L_7$

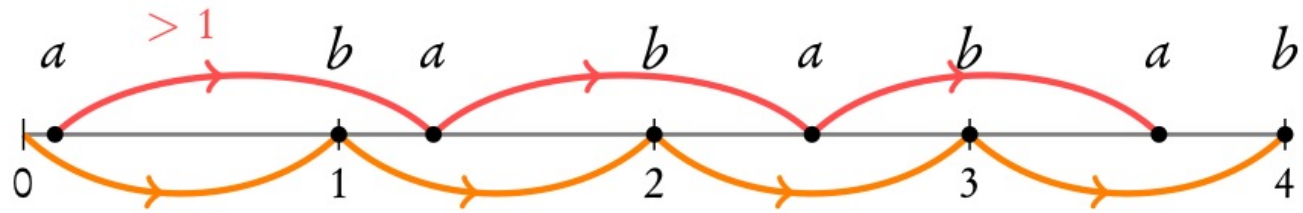
$$L_7 = \{ ( (ab)^k, \tau ) \mid \tau_{2i} = i \text{ and } \tau_{2i+2} - \tau_{2i+1} < \tau_{2i} - \tau_{2i-1} \}$$

Pivoted converging  $ab$  distances



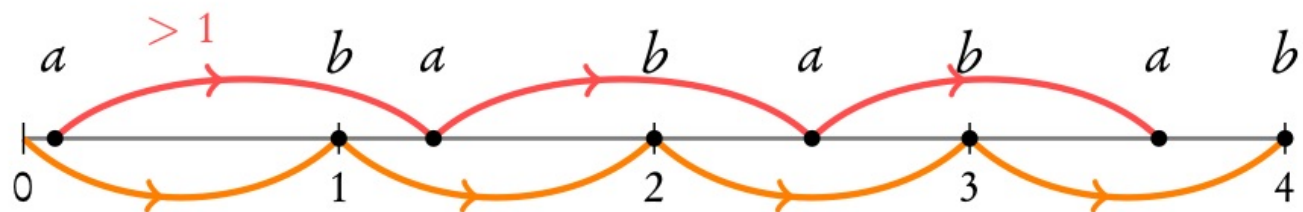
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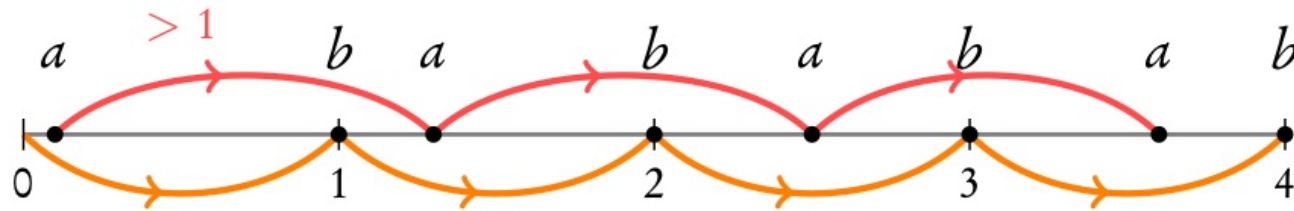
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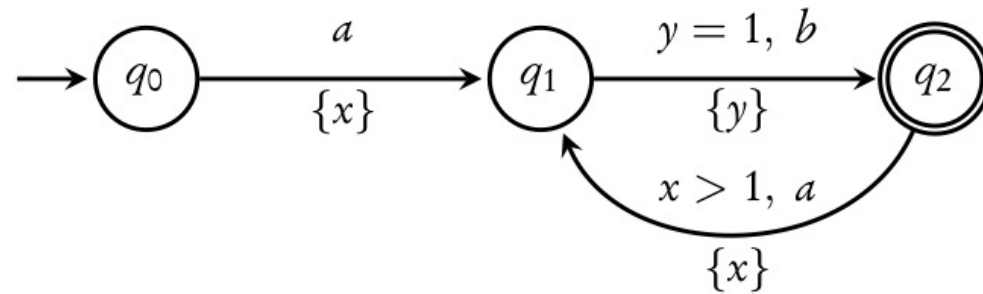
$$\begin{aligned} \tau_{2i+2} - \tau_{2i+1} < \tau_{2i} - \tau_{2i-1} &\Leftrightarrow \tau_{2i+2} - \tau_{2i} < \tau_{2i+1} - \tau_{2i-1} \\ &\Leftrightarrow 1 < \tau_{2i+1} - \tau_{2i-1} \end{aligned}$$

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# Timed automata

Runs

$1 \text{ clock} < 2 \text{ clocks} < \dots$

Role of max constant

## Timed automata

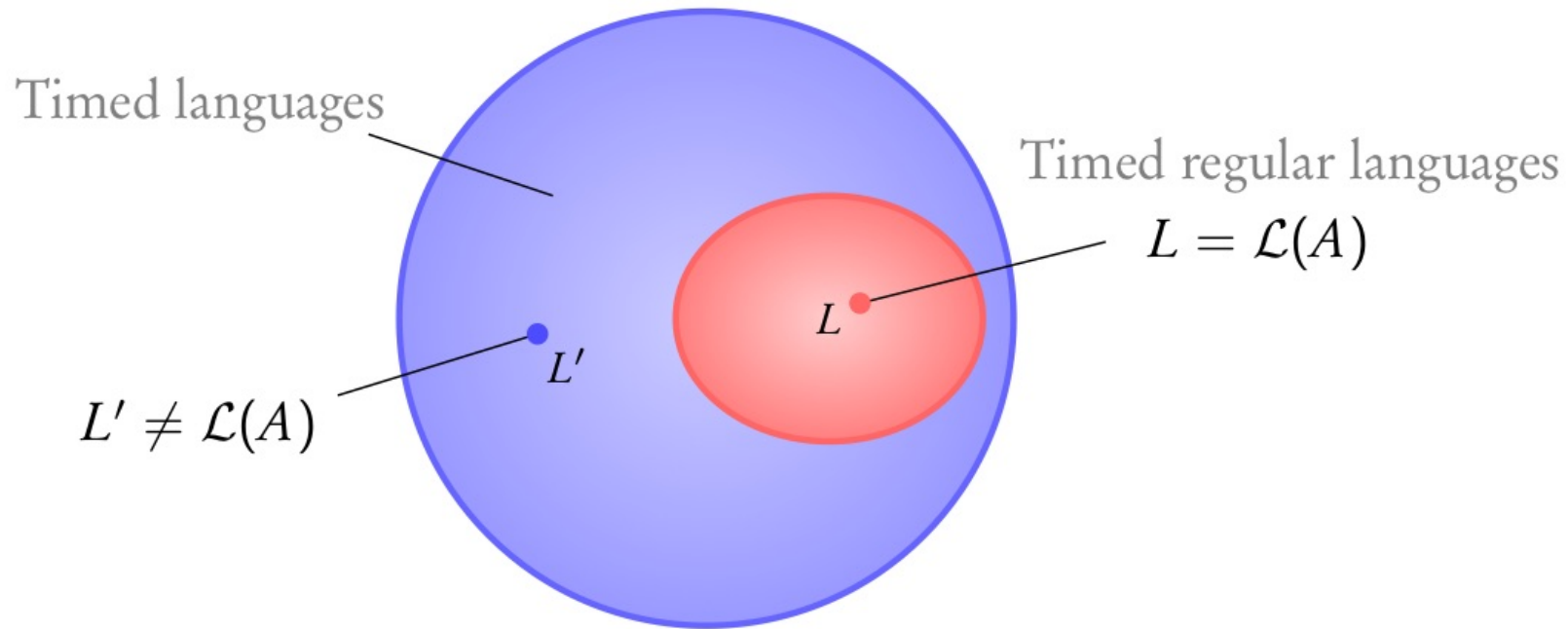
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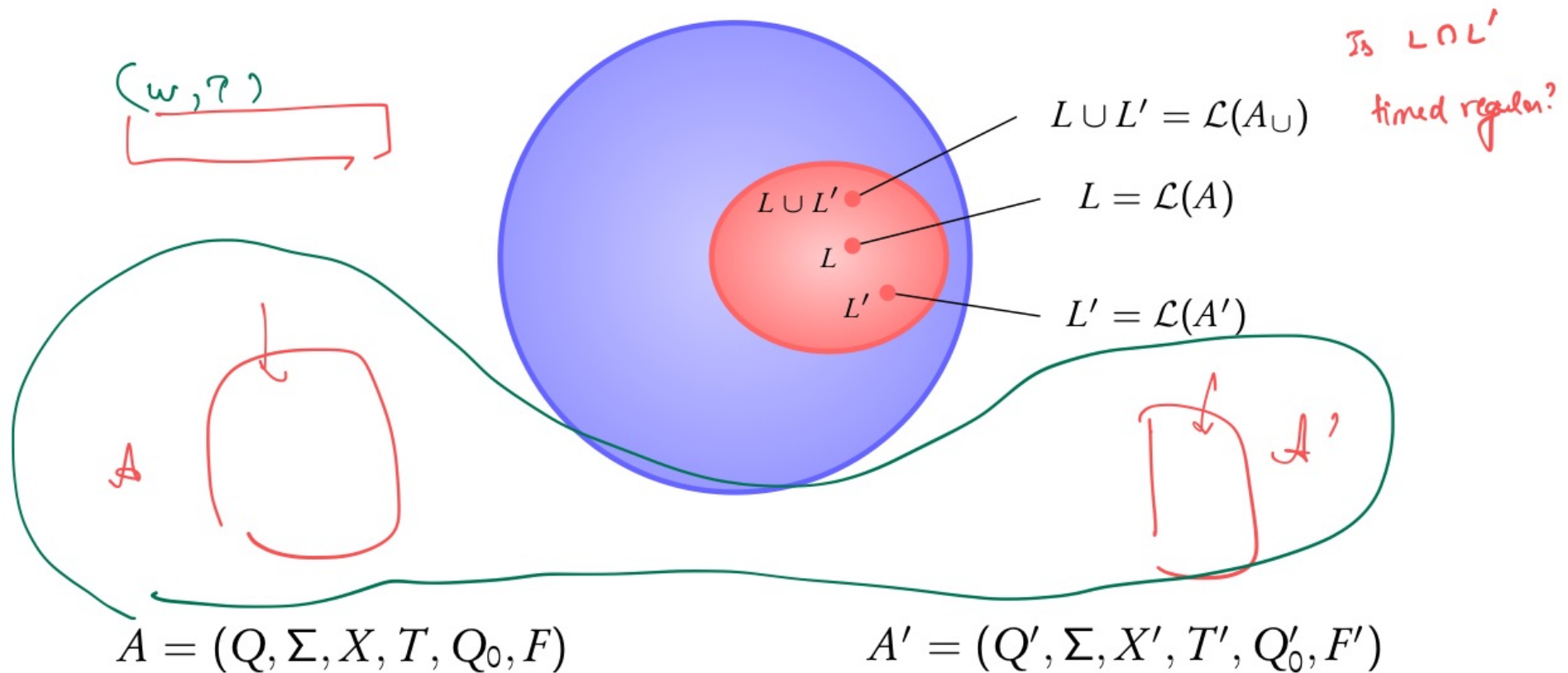
## Timed regular lngs.

# Timed regular languages



## Definition

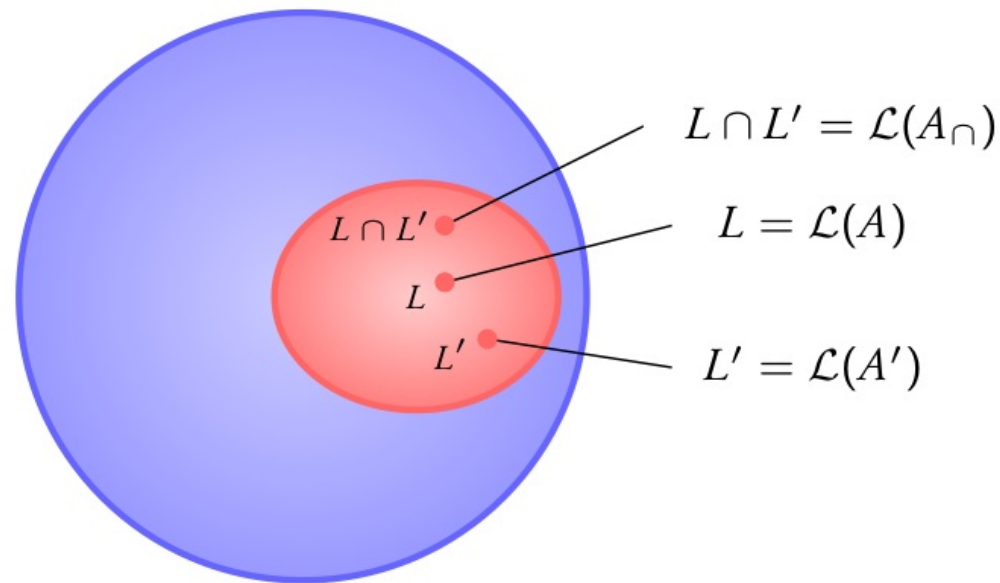
A timed language is called **timed regular** if it can be **accepted** by a timed automaton



$$A \cup A' = (Q \cup Q', \Sigma, X \cup X', T \cup T', Q_0 \cup Q'_0, F \cup F')$$

$$\mathcal{L}(A) \cup \mathcal{L}(A') = \mathcal{L}(A \cup A')$$

Timed regular languages are **closed** under **union**



$$A = (Q, \Sigma, X, T, Q_0, F)$$

$$A' = (Q', \Sigma, X', T', Q'_0, F')$$

$$A_{\cap} = (Q \times Q', \Sigma, X \cup X', T_{\cap}, Q_0 \times Q'_0, F \times F')$$

$$T_{\cap} : (q_1, q'_1) \xrightarrow[R \cup R']{a, g \wedge g'} (q_2, q'_2) \text{ if}$$

$$q_1 \xrightarrow[R]{a, g} q_2 \in T \text{ and } q'_1 \xrightarrow[R']{a, g'} q'_2 \in T'$$

Timed regular languages are **closed** under **intersection**



$L$  : a timed language over  $\Sigma$

$aa$

$$\text{Untime}(L) \equiv \{w \in \Sigma^* \mid \exists \tau. (w, \tau) \in L\}$$

## Untiming construction

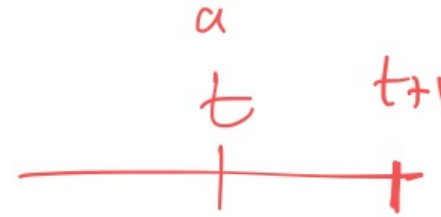
For every **timed** automaton  $A$  there is a **finite automaton**  $A_u$  s.t.

$$\text{Untime}( \mathcal{L}(A) ) = \mathcal{L}(A_u)$$

more about this later ...

# Closure under Complementation

$$\Sigma : \{a, b\}$$

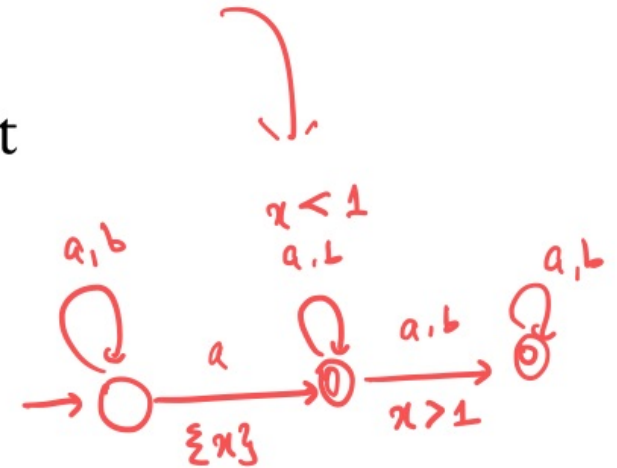
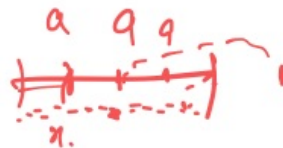


a b x  
1 1

a b (a) b ✓  
1 1 2 2.5

$L = \{ (\omega, \tau) \mid \text{there is an } a \text{ at some time } t \text{ and no action occurs at time } t + 1 \}$

$\bar{L} = \{ (\omega, \tau) \mid \text{every } a \text{ has an action at a distance 1 from it} \}$



# Complementation

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**Claim:** No timed automaton can accept  $\bar{L}$

Decision problems for timed automata: A survey

Alur, Madhusudhan. *SFM'04: RT*



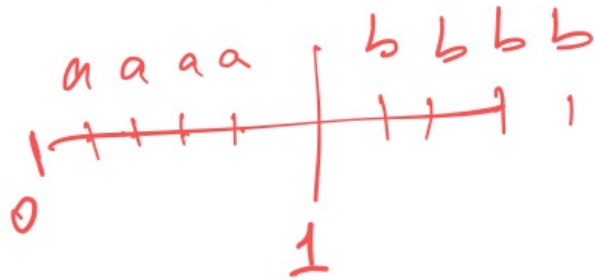
Step 1:  $\bar{L} = \{ (w, \tau) \mid \text{every } a \text{ has an action at a distance 1 from it } \}$

*Suppose*  $\bar{L}$  is timed regular

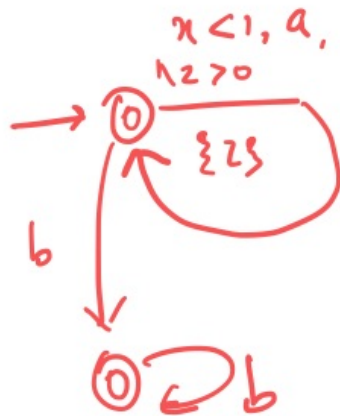
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Clearly  $L'$  is timed regular



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$L' \cap \bar{L}$  should be timed regular

Step 3:  $\text{Untime}( \bar{L} \cap L' )$  should be a regular language

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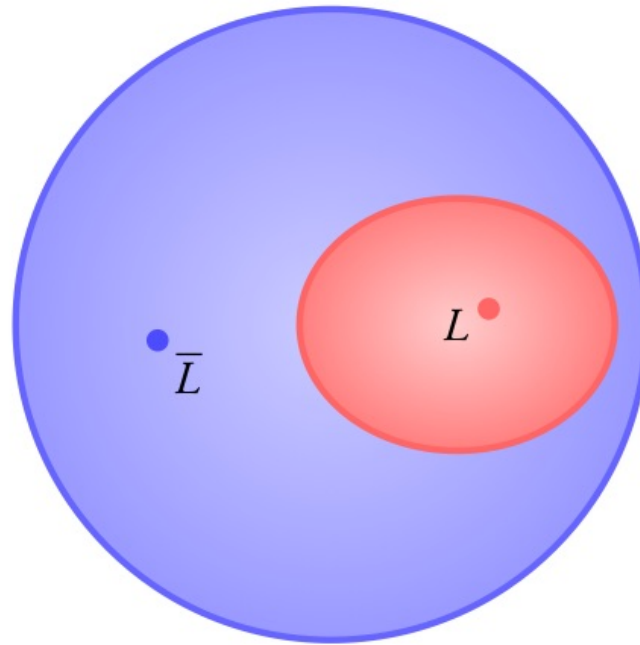
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Step 4: But,  $\text{Untime}(\bar{L} \cap L') = \{a^n b^m \mid m \geq n\}$ , *not regular!*

Therefore  $\bar{L}$  cannot be timed regular  $\square$



Timed regular languages are **not closed** under **complementation**

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1 clock < 2 clocks < ...

Role of max constant

## Timed regular lngs.

Closure under  $\cup, \cap$

Non-closure under complement



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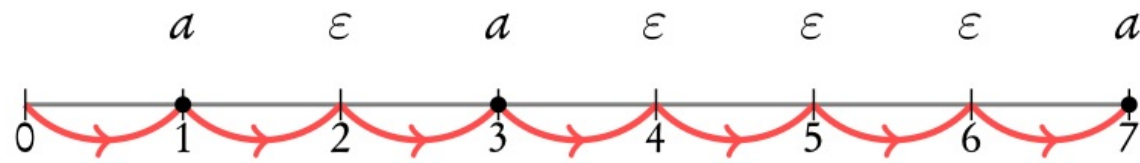
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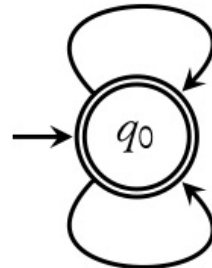
## $\epsilon$ -transitions



$$L_6 := \{ ( a^k, \tau ) \mid \tau_i \text{ is some integer for each } i \}$$



$$x = 1, \varepsilon, \{x\}$$



$$x = 1, a, \{x\}$$

## $\varepsilon$ -transitions

$\varepsilon$ -transitions **add expressive power** to timed automata.

Characterization of the expressive power of silent transitions in timed automata

Bérard, Diekert, Gastin, Petit. *Fundamenta Informaticae*'98

## $\varepsilon$ -transitions

$\varepsilon$ -transitions **add expressive power** to timed automata. However, they add power **only** when a clock is **reset** in an  $\varepsilon$ -transition.

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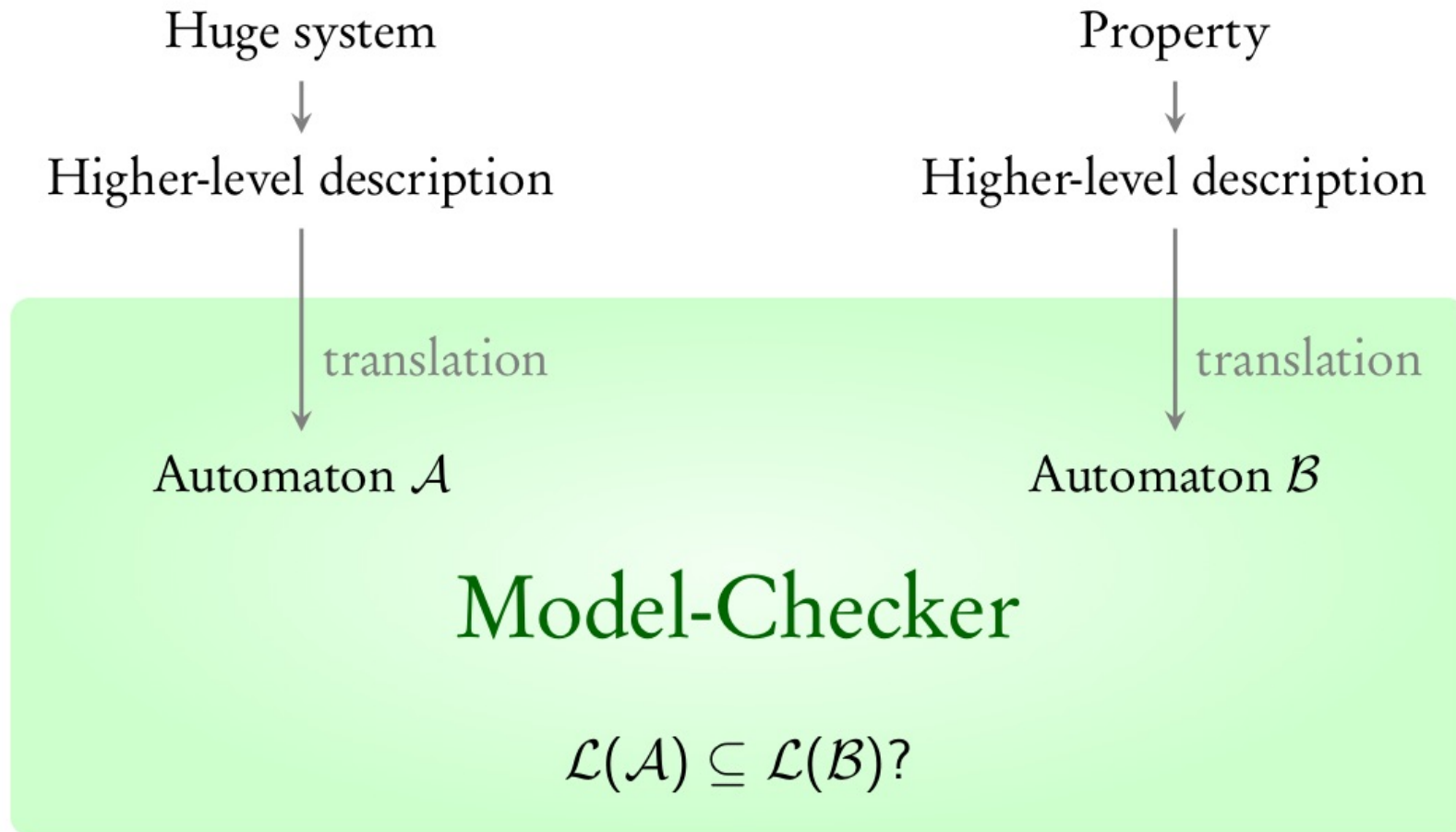
Non-closure under complement

## $\varepsilon$ -transitions

More expressive

$\xrightarrow{\varepsilon}$  without reset  $\equiv$  TA

# Recall...



$$\mathcal{L}(A) \subseteq \mathcal{L}(B)$$

iff

$$\mathcal{L}(A) \cap \overline{\mathcal{L}(B)} = \emptyset$$

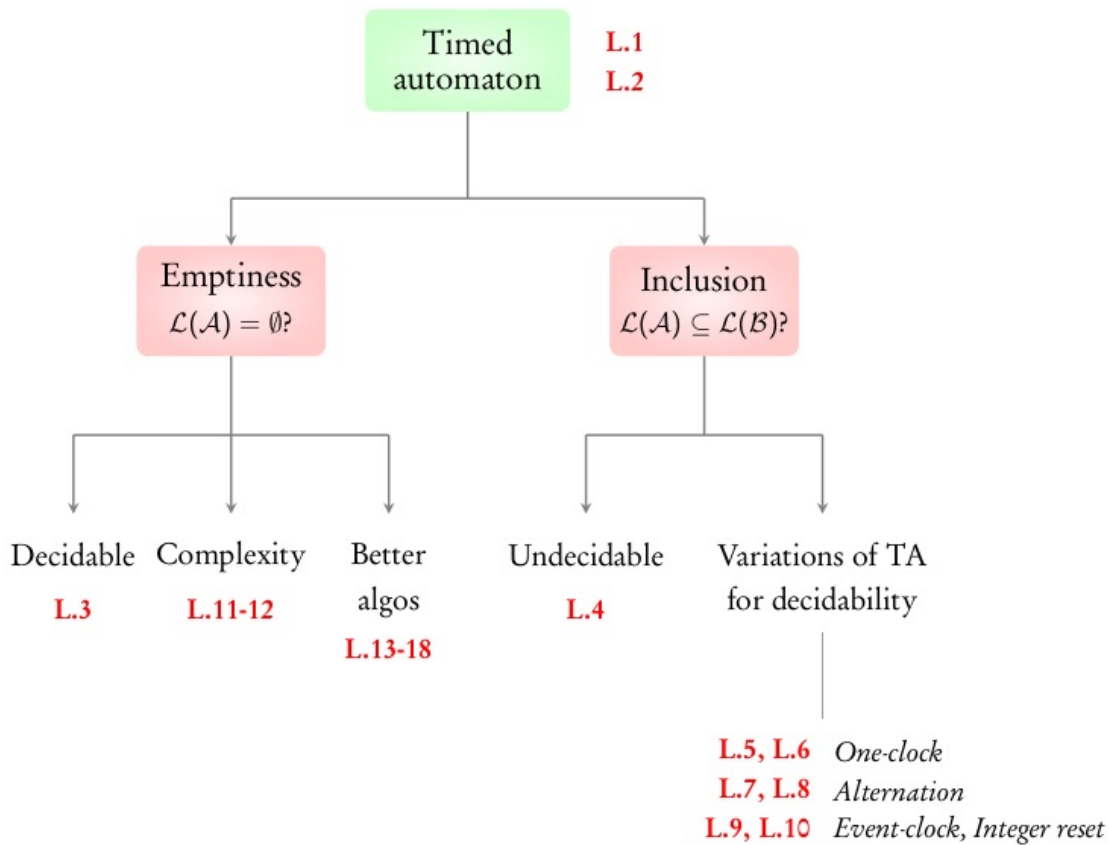
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non-closure under complement  $\Rightarrow$  the above **cannot be done** for TA!

# Course plan





# Course plan

Project plan:

- October 7 + 15 : choose your project.

- First phase evaluation: October 30

- Second phase evaluation: November 30

- Final report / software: December 15

