

TIMED AUTOMATA

LECTURE 8

GOALS OF TODAY'S LECTURE

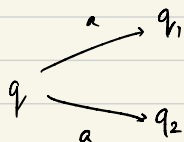
Event-clock Automata:

- a determinizable subclass of T.A.

- Alur, Fix, Henzinger

Problem with subset construction:

Untimed cases:



$$\{q\} \xrightarrow{a} \{q_1, q_2\}$$

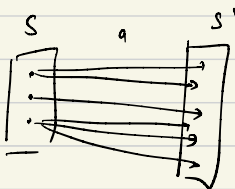
$$S \xrightarrow{a} S' \quad S \subseteq Q \text{ state.}$$

- 1. for every $q' \in S' \exists q \in S$ st.

$$q \xrightarrow{a} q'$$

- 2. Moreover, $\forall q \in S$. if $q \xrightarrow{a} q'$

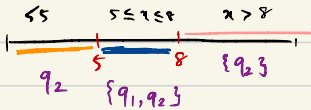
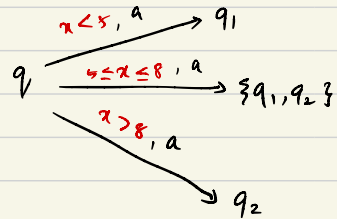
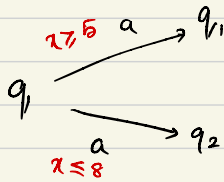
then $q' \in S'$.



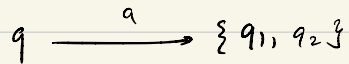
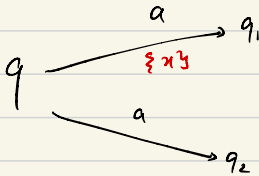
Timed case:

Guards a problem?

No.

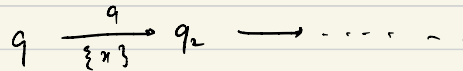


Resets a problem?

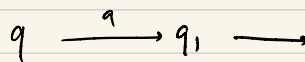


Should x be reset or not?

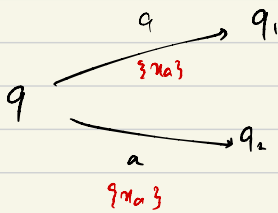
- If x is reset in $q \xrightarrow{a} \{q_1, q_2\}$ then there is potentially an incorrect run:



- If x is not reset, then there is an incorrect run:



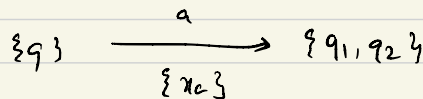
Main idea:



Whenever 'a' is on the transition
- ra has to be reset.

- ra cannot be reset in any other edge.

To do subset:



Summary:

- Problem with subset construction due to resets
- Can be circumvented by resetting a special clock 'ra' at a , ta .

Event-recording Automata (ERA):

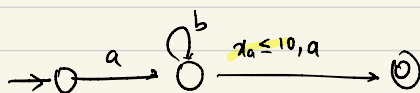
Σ : alphabet.

$$X_{\Sigma} : \{ x_a \mid a \in \Sigma \}$$

↳ Event-recording clocks.

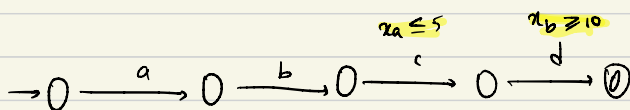
Example 1:

$\{ ab^*a \mid \text{distance between the two } a\text{'s is } \leq 10 \}$



Example 2:

$\{ (abcd, \tau_1, \tau_2, \tau_3, \tau_4) \mid \tau_3 - \tau_1 \leq 5 \wedge \tau_4 - \tau_2 \geq 10 \}$



Example 3:

$\{ (aaa, \tau_1, \tau_2, \tau_3) \mid \tau_3 - \tau_1 = 1 \}$

↳ no ERA



While reading third 'a' x_a maintains time since second 'a'.

Semantics of Event-recording clocks:

	a	b	a	a	b	b	a
	0.5	2.7	3.0	4.9	7.0	8.5	10.0
x_a	$\perp$	2.2	2.5	1.9	2.1	3.6	5.1
x_b	$\perp$	$\perp$	0.3	2.2	4.3	1.5	1.5

→ Values of x_a, x_b get determined by the input word!

- and not by the automaton

Given an input word $w = (a_1 a_2 \dots a_k, \tau_1 \tau_2, \dots, \tau_n)$

We define a function $\gamma_i : X_{\Sigma} \mapsto \{\perp\} \cup \mathbb{R}_{\geq 0}$

for all $i \in \{1, 2, \dots, k\}$

$$\gamma_i(x_a) = \begin{cases} t_i - t_j & \text{if there exists } j. \\ & j < i, a_j = a, \\ & \forall j < m < i, a_m \neq a \\ \perp & \text{otherwise} \end{cases}$$

Guards: $\Phi := x_a \sim c \mid \Phi \wedge \Phi \mid \Phi \vee \Phi$

$\Phi(x_\Sigma)$

where $c \in \mathbb{N}$ or $c = \perp$

$\perp \geq \perp$

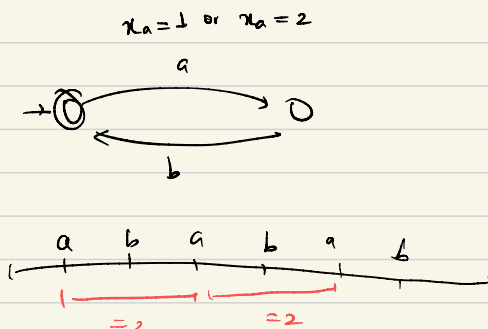
- $\perp \leq \perp$ is true

- Any other comparison is false.

$\perp \leq 5, 10 > \perp$

are false.

Example 4:



ERA defn: $\mathcal{A}: (\mathcal{Q}, \Sigma, x_\Sigma, \Delta, F)$

$$\Delta \subseteq \mathcal{Q} \times \Sigma \times \Phi(x_\Sigma) \times \mathcal{Q}$$

Timed word $w: (a_1 a_2 \dots a_k, \tau_1 \tau_2 \dots \tau_k)$

When does $\mathcal{A}$ accept w ?

$\mathcal{A}$ accepts w if there exists a run:

$$q_0 \xrightarrow{g_1, a_1} q_1 \xrightarrow{g_2, a_2} q_2 \dots \xrightarrow{g_k, a_k} q_k$$

- s.t. $\gamma_i \models g_i$

- q_k is accepting.

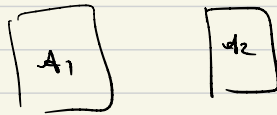
$$\mathcal{L}(\mathcal{A}) = \{ w \mid \mathcal{A} \text{ accepts } w \}$$

Closure properties:

A_1, A_2 : ERA.

Union: ERA for $\mathcal{L}(A_1) \cup \mathcal{L}(A_2)$?

↳ disjoint union of A_1, A_2



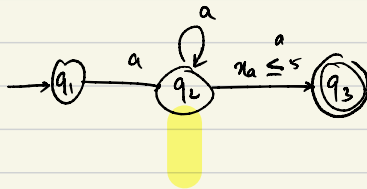
Intersection: ERA for $\mathcal{L}(A_1) \cap \mathcal{L}(A_2)$?

↳ product construction as done for DFA.

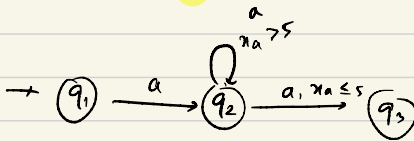
Complementation: ?

→ First consider determinization problem.

Determinization of ERA:



This is non-deterministic (NERA)

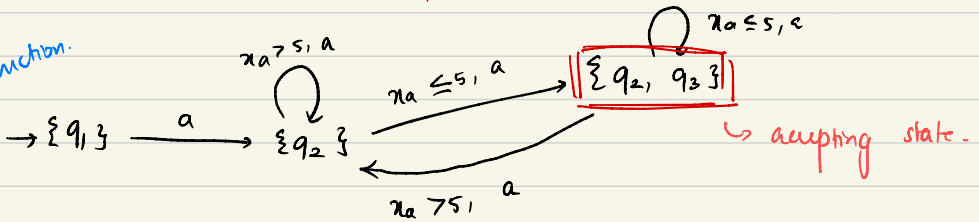


don not accept.

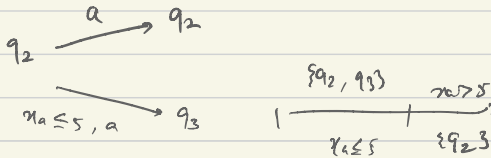
a	a	a
0	5	10

Wrong automaton

Example of subset construction.

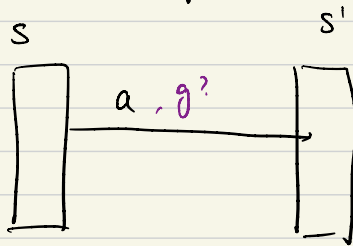


accepting state.

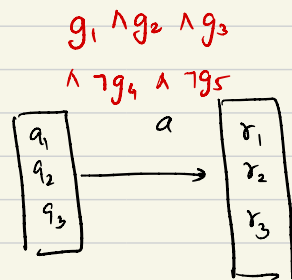
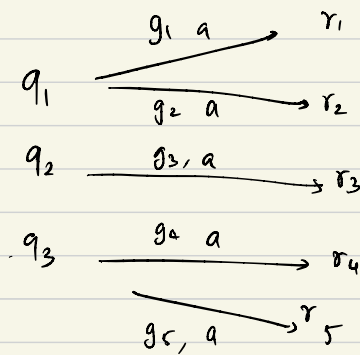


Subset construction for NFA:

States: subset of Q .



$$S = \{q_1, q_2, q_3\}$$



$S: \{ q_1, q_2, \dots, q_k \}$

for every subset T of transition on 'a' from S :

$$T = \{ (q_{i_1}, a, g_{i_1}, r_{i_1}), \dots, (q_{i_m}, a, g_{i_m}, r_{i_m}) \}$$

We have

$$S \xrightarrow[\phi]{a} \{ r_{i_1}, r_{i_2}, \dots, r_{i_m} \}$$

$$\phi: g_{i_1} \wedge g_{i_2} \wedge \dots \wedge g_{i_m} \wedge \bigwedge_{\substack{q \xrightarrow{a, s} q' \\ q \in S, (q, a, g, q') \notin T}} \neg g$$

Subset construction:

States: subsets

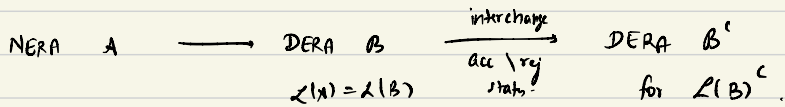
Transition relation: as above

Accepting states: Subsets that intersect with F

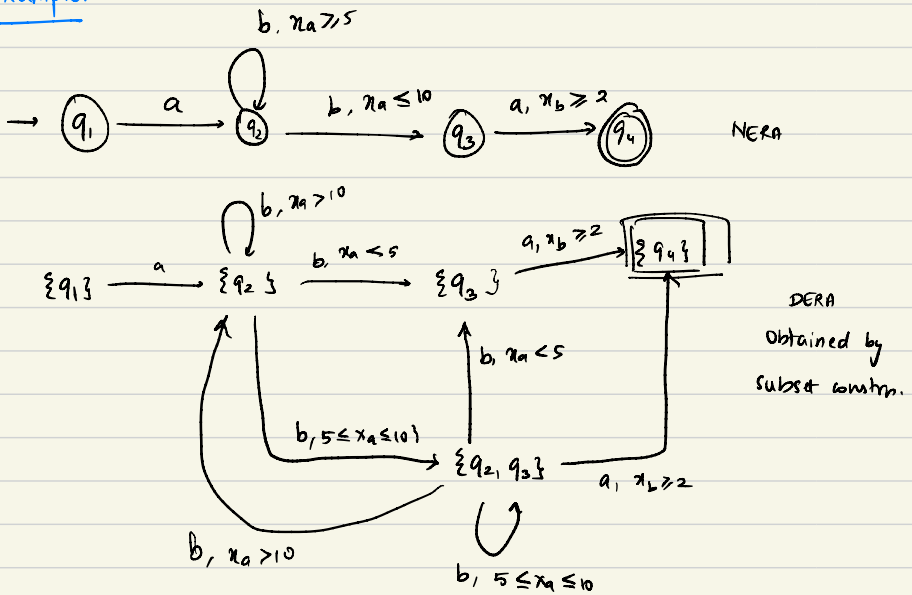
Thm:

Deterministic ERAs are as expressive as NERAs.

Complementation:



Example:



Summary:

- 1. Event Recording Automata: determinizable
 - closed under boolean operations.
- 2. Some examples, some non-examples.