

TIMED AUTOMATA

LECTURE 24

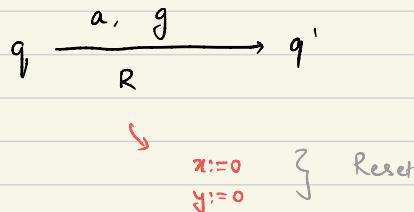
Updatable Timed Automata

- Bouyer, Sutour, Fleury, Petit
Theoretical Computer Science (2004)

Our goals:

- Syntax, semantics, examples (today)
- Emptiness problem
- Expressive power

Update:



$$\begin{aligned}x &= 5 \\x &= y \\x &= y + 5 \\x &= y - 2\end{aligned}$$

Let X be a set of clocks. An "update" is an expression generated by the following grammar:

$$x := c \quad | \quad x := y + d \quad \rightarrow \text{an update to } x.$$

$$c \in \mathbb{N} \quad d \in \mathbb{Z}$$

An update function maps each clock ' x ' to an "update to i ".

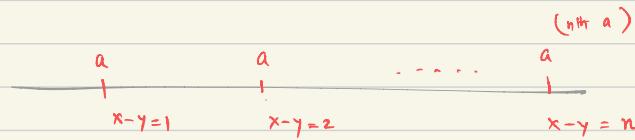
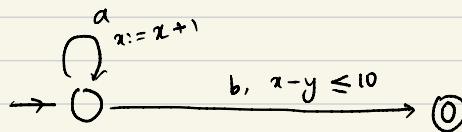
Eg: $X = \{x, y, z\}$

update functions

$$\begin{array}{ccc}x := 2 & | & z := x + 2 \\y := y & | & y := y + 1 \\z := x + 2 & | & z := z\end{array}$$

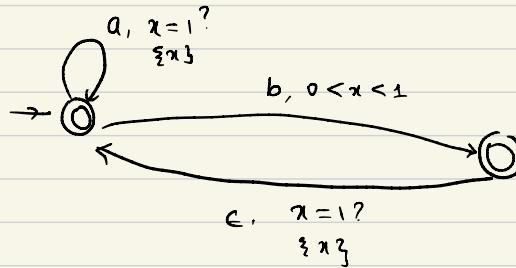
same ↪

Example:



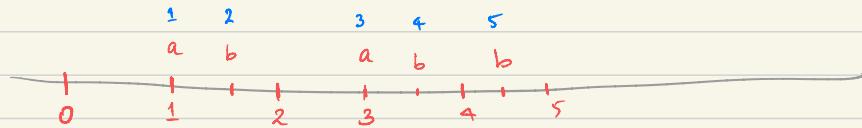
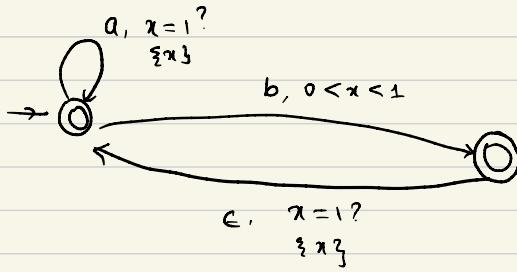
$$= \{ (a^k b, \tau) \mid k \leq 10, \tau_1 \leq \tau_2 \dots \tau_k \leq \tau_{k+1} \}$$

Example:



classical
timed automaton.

Consider the above automaton with ϵ -transitions. What is its language?



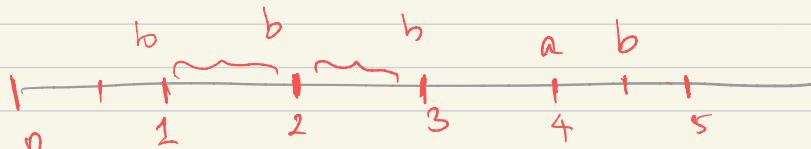
$(w_1, w_2, \dots, w_n, \tau_1, \tau_2, \dots, \tau_n)$ is accepted if:

if $w_1 = a$, then $\tau_1 = 1$
 if $w_1 = b$, then $\tau_1 \in (0, 1)$

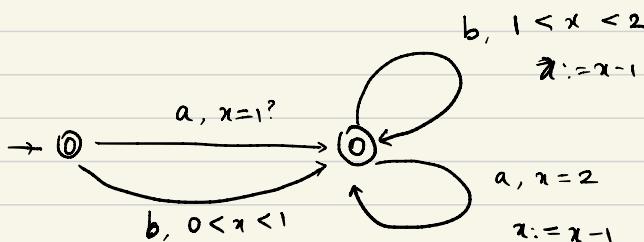
if $w_2 = a$, then $\tau_2 = 2$
 if $w_2 = b$, then $\tau_2 \in (1, 2)$

⋮

if $1 \leq i \leq n$, if $w_i = a$, then $\tau_i = i$
 if $w_i = b$, then $\tau_i \in (i-1, i)$

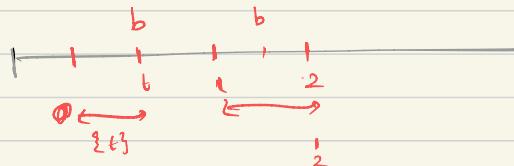


- Construct an updatable TA for the previous language that has no ϵ -transitions.

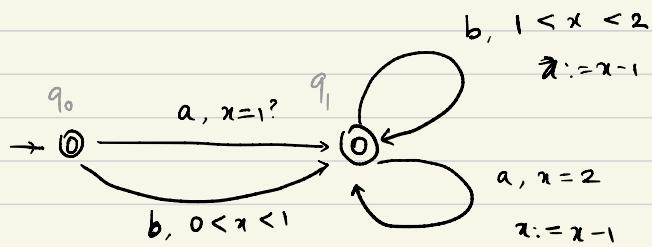


Invariant maintained by above automaton:

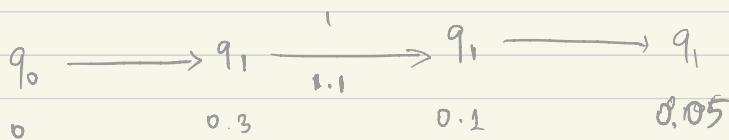
- After reading 'a' at 't', value of $x = 1$
- After reading 'b' at 'b', value of $x = \{2\}$



Because of above invariant: the next 'a' is read at $x=2$
the next 'b' is read at $1 < x < 2$



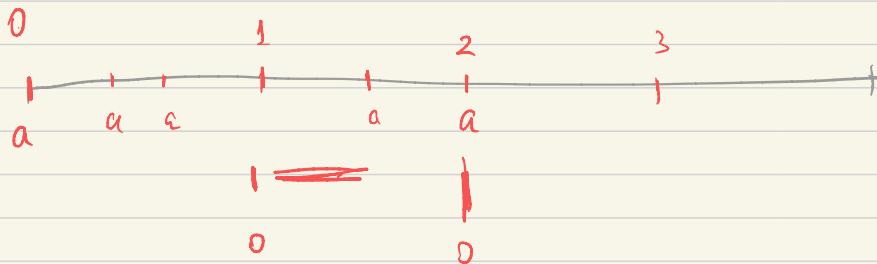
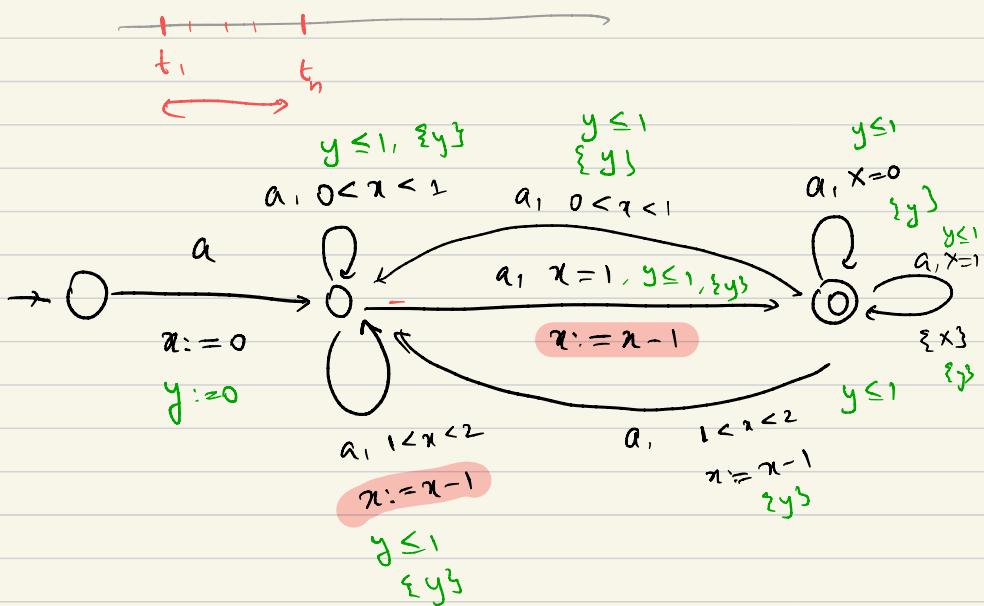
b
 0.3
 b
 1.1
 b
 2.05



Example:

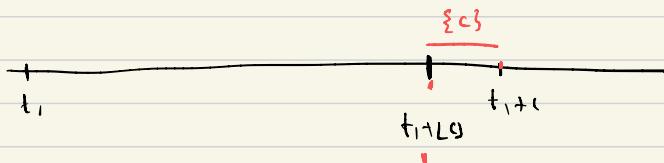
$$\{ (a^n, t_1, t_2, \dots, t_n) \mid t_{i+1} - t_i \leq 1$$

$t_n - t_1$ is an integer }



If the last 'a' seen is at $t_1 + c$, then

$t_1 + \lfloor c \rfloor$ becomes the new reference point.



Summary:

- introduction to updatable TA.
- Some examples