TIMED AUTOMATA

LECTURE 20

Theorem

Deterministic timed automata are **closed under complement**

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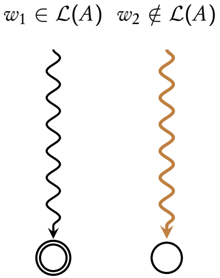
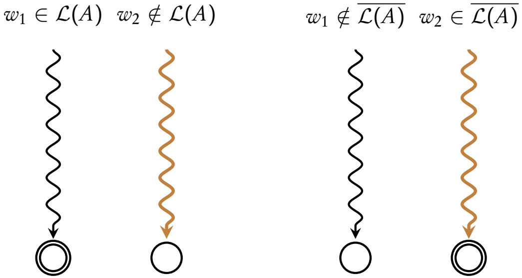
1. **Unique** run for every timed word



Theorem

Deterministic timed automata are **closed under complement**

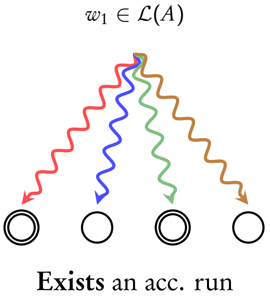
1. **Unique** run for every timed word
2. **Complementation:** **Interchange** acc. and non-acc. states

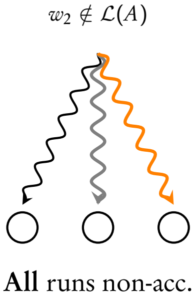


Theorem (Lecture 1)

Non-deterministic timed automata are **not closed under complement**

Many runs for a timed word

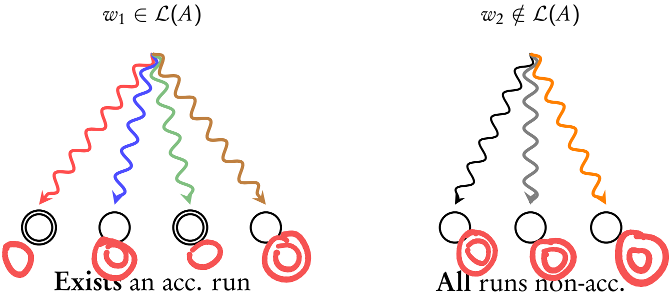




Theorem (Lecture 1)

Non-deterministic timed automata are **not closed under complement**

Many runs for a timed word



Complementation: **interchange** acc/non-acc + ask are **all runs acc.** ?

A timed automaton model with **existential** and **universal** semantics for acceptance

Alternating timed automata

Lasota and Walukiewicz. *FoSSaCS'05*, *ACM TOCL'2008*

Section 1:

Introduction to ATA

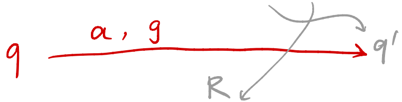
- X : set of **clocks**
- $\Phi(X)$: set of clock constraints σ (**guards**)

$$\sigma : \; x < c \mid x \leq c \mid \sigma_1 \wedge \sigma_2 \mid \neg\sigma$$

c is a non-negative **integer**

- Timed automaton A: $(Q, Q_0, \Sigma, X, T, F)$

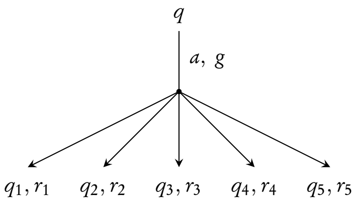
$$T \;\subseteq\; Q \times \Sigma \times \Phi(X) \times Q \times \mathcal{P}(X)$$



$$T \subseteq Q \times \Sigma \times \Phi(X) \times Q \times \mathcal{P}(X)$$

$$\downarrow$$

$$T : Q \times \Sigma \times \Phi(X) \mapsto \mathcal{P}(Q \times \mathcal{P}(X))$$



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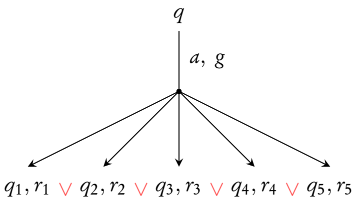

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$\downarrow$ $\mathcal{B}^+(S)$ is all $\phi ::= S \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2$

$$T : Q \times \Sigma \times \Phi(X) \;\mapsto\; \mathcal{B}^+(Q \times \mathcal{P}(X))$$

$$Q \times P(X) = \{ (q_1, r_1), (q_2, r_2), \ldots, (q_n, r_n) \}$$

$$[(q_1, r_1) \vee (q_2, r_2)] \wedge (q_3, r_3)$$

$$T : Q \times \Sigma \times \Phi(X) \mapsto \mathcal{P}(Q \times \mathcal{P}(X))$$

$\mathcal{B}^+(S)$ is all $\phi ::= S \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2$

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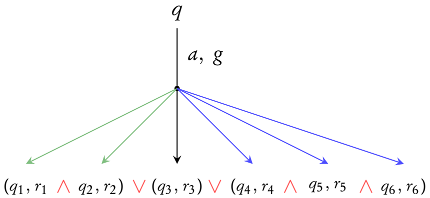

Alternating Timed Automata

An **ATA** is a tuple $A = (Q, q_0, \Sigma, X, T, F)$ where:

$$T : Q \times \Sigma \times \Phi(X) \mapsto \mathcal{B}^+(Q \times \mathcal{P}(X))$$

is a **finite partial function.**

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is a **finite partial function.**

$\Phi(x) = \{ g_1, g_2, \ldots \}$

Partition: For every q, a the set

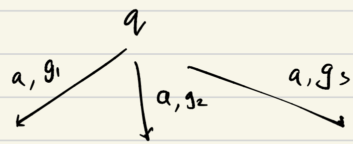
$$\{ [\sigma] \mid T(q, a, \sigma) \text{ is defined } \}$$

gives a finite partition of $\mathbb{R}_{\geq 0}^X$

(q, a, g_1)

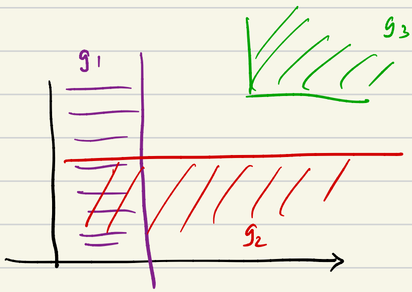
(q, a, g_2)

~~(q, a, g_3)~~

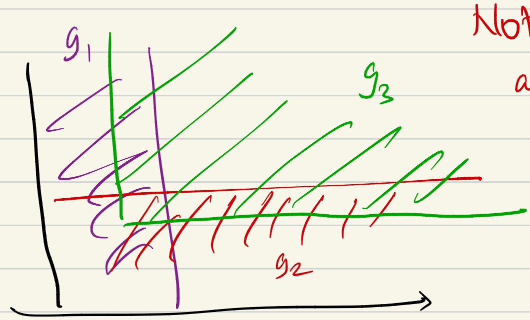


$g_1 \cup g_2 \cup g_3$ gives all valuations

$X = \{x, y\}$

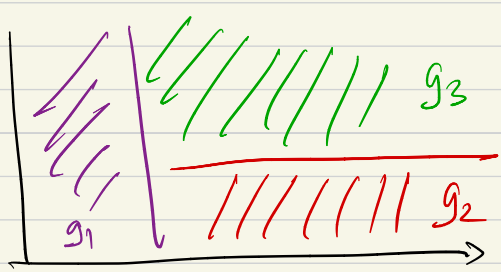


Not allowed.

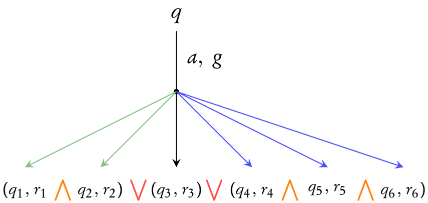


Not allowed as not a partition

✓

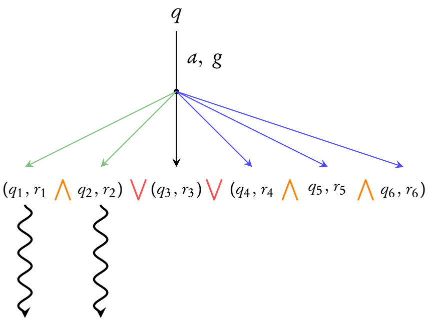


Acceptance



Accepting run from q iff:

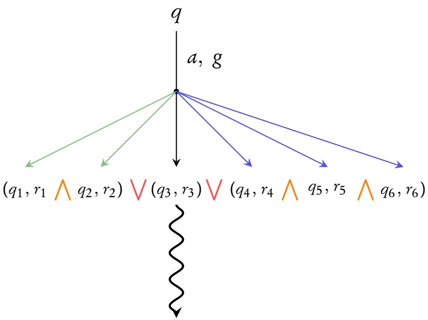
Acceptance



Accepting run from q iff:

- accepting run from q_1 **and** q_2,

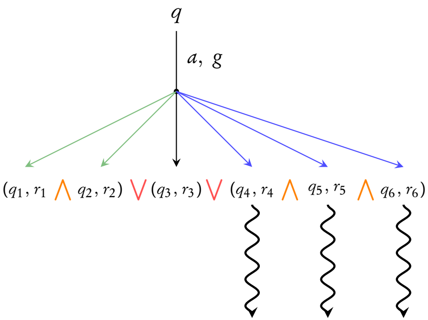
Acceptance



Accepting run from q iff:

- accepting run from q_1 **and** q_2,
- **or** accepting run from q_3,

Acceptance

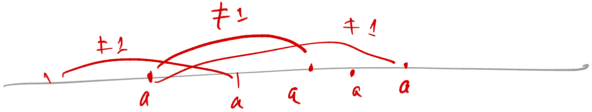


Accepting run from q iff:

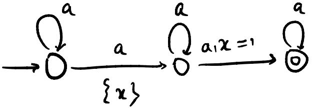
- accepting run from q_1 **and** q_2,
- **or** accepting run from q_3,
- **or** accepting run from q_4 **and** q_5 **and** q_6

L : timed words over $\{a\}$ containing **no two** $a's$ at distance 1

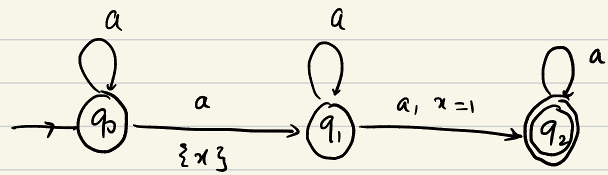
(Not expressible by non-deterministic TA)



Complement of L: ∃ 2 a's at distance 1 apart.



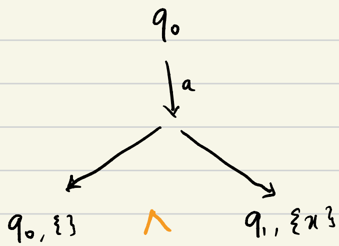
I



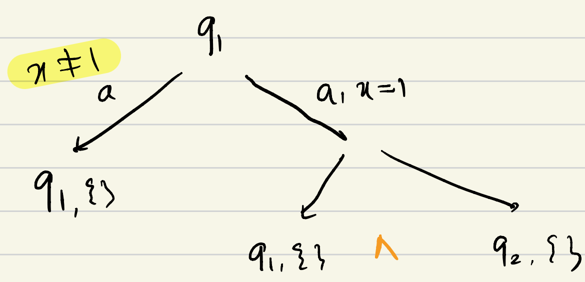
Non-deterministic TA.

ATA for C

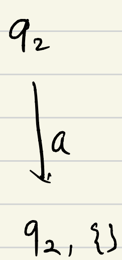
T1.



T2.

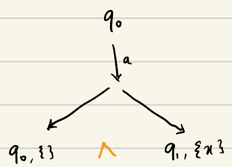


T3.

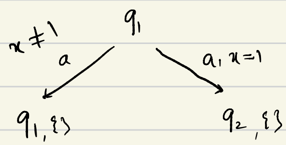


Acc. state $\{q_0, q_1\}$

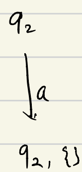
T1.

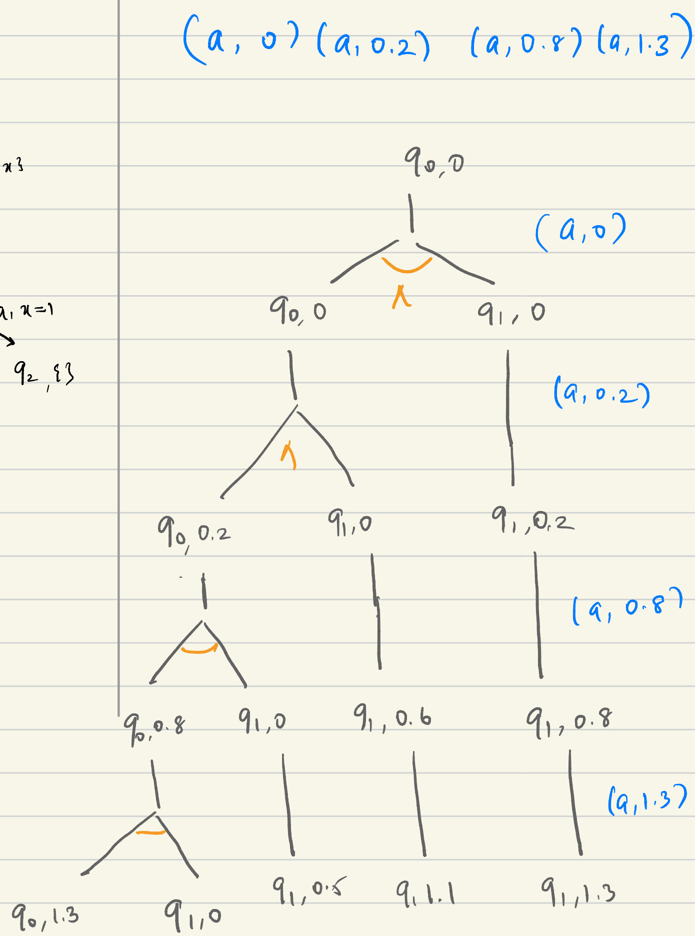


T2.



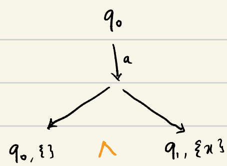
T3.



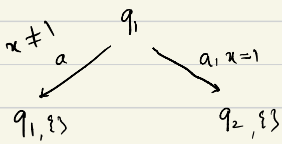


Accepting

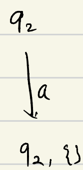
T1.

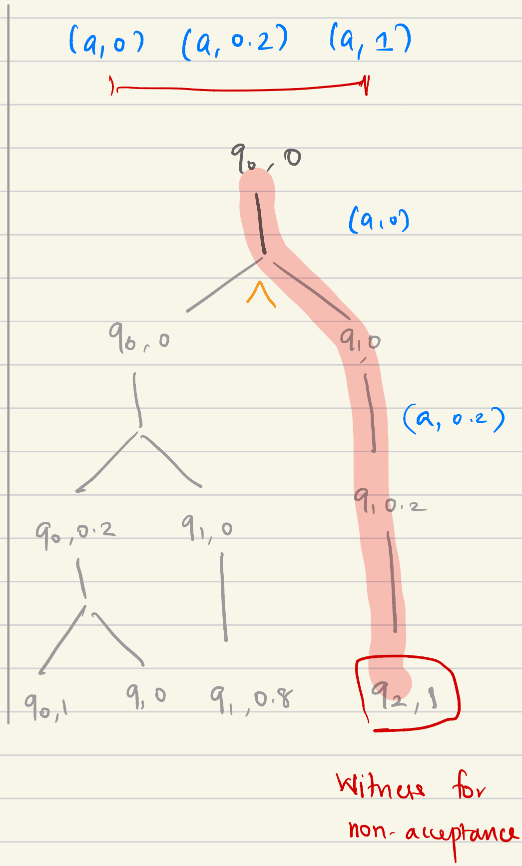

q0
a
q0, {}
∧
q1, {x}

T2.


q1
x ≠ 1
a
a, x=1
q1, {}
q2, {}

T3.


q2
a
q2, {}


(a, 0) (a, 0.2) (a, 1)
q0, 0
(a, 0)
∧
q0, 0
q1, 0
(a, 0.2)
q0, 0.2
q1, 0
q1, 0.2
q0, 1
q1, 0
q1, 0.8
q2, 1
Witness for
non-acceptance

L : timed words over $\{a\}$ containing **no two** $a's$ at distance 1

(Not expressible by non-deterministic TA)

ATA:

$$q_0, a, tt \mapsto (q_0, \emptyset) \wedge (q_1, \{x\})$$

$$q_1, a, x = 1 \mapsto (q_2, \emptyset)$$

$$q_1, a, x \neq 1 \mapsto (q_1, \emptyset)$$

$$q_2, a, tt \mapsto (q_2, \emptyset)$$

q_0, q_1 are acc., q_2 is non-acc.

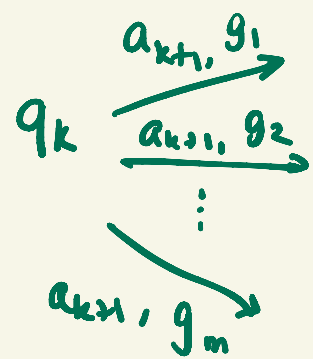
Acceptance Game: $\mathcal{G}_{A,w}$

$$w := (a_0, t_0)\,(a_1, t_1)\,(a_2, t_2) \,\dots\, (a_{k+1}, t_{k+1}) \,\dots\, (a_n, t_n)$$

Phase 0   Phase 1   Phase $k+1$

(q_k, v_k)

$$\bar{v} = v_k + t_{k+1} - t_k$$



- Let σ be unique constraint s.t. $\bar{v}$ satisfies σ

$$b = \delta(q_{k+1}, a_{k+1}, \sigma)$$

- $b = b_1 \wedge b_2$: Adam chooses a subformula and game continues with the subformula.

- $b = b_1 \vee b_2$: Eve

- $b = (q, r) \in Q \times \mathcal{P}(C)$

$((q_1, r_1) \vee (q_2, r_2)) \wedge (q_3, r_3)$

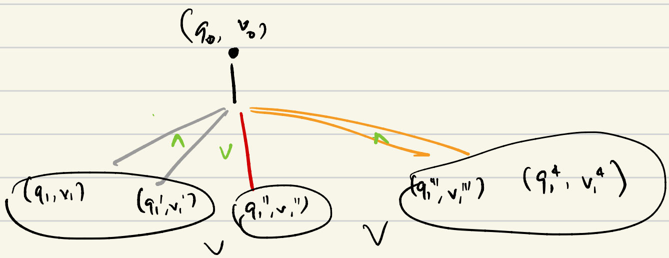
- Phase ends with

$$(q_{k+1}, v_{k+1}) := (q, \bar{v}[r := 0])$$

→ Play ends with (q_{n+1}, v_{n+1})

- Eve wins the play if q_{n+1} is accepting; otherwise Adam wins.

- $w \in \mathcal{L}(\mathcal{A})$ if Eve has a strategy to win $\mathcal{G}_{\mathcal{A}, w}$. Else $w \notin \mathcal{L}(\mathcal{A})$.



Acceptance game $\mathcal{G}_{A,w}$:

$$\mathcal{L}(A) = \{ w \mid \text{Eve wins } \mathcal{G}_{A,w} \}$$

Summary:

- A model involving existential and universal transitions.
  - ↳ Alternating T.A.
- 1 Example
- Acceptance game $\mathcal{G}_{A,w}$.