TIMED AUTOMATA

LECTURE 18

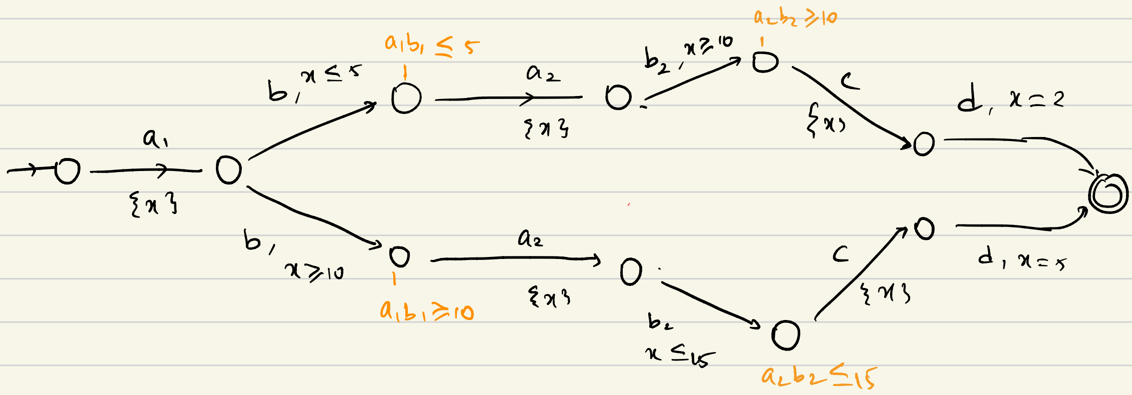
Timed automata with diagonal constraints:

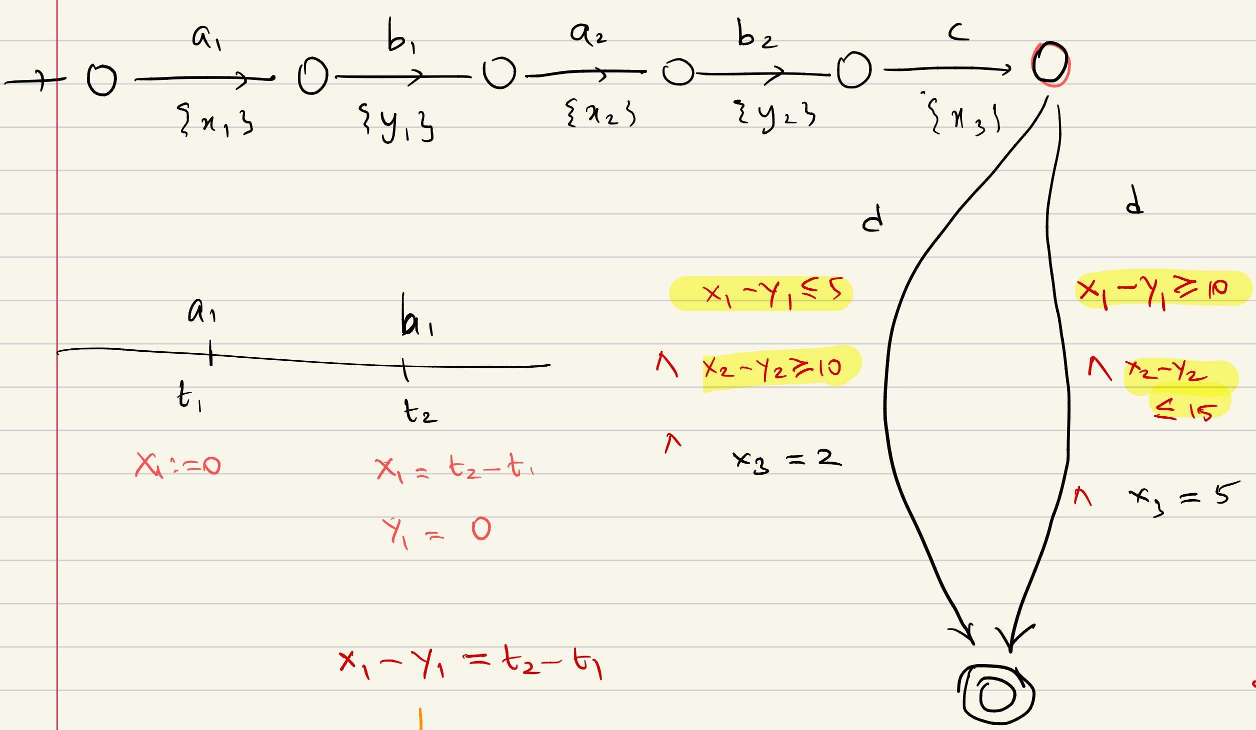
Example:

$a_1 \quad b_1 \quad a_2 \quad b_2 \quad c \quad d$

either time $(a_1 b_1) \leq 5$ and time $(a_2 b_2) \geq 10$ and time$(c\, d) = 2$

or time $(a_1 b_1) \geq 10$ and time $(a_2\, b_2) \leq 15$ and time$(c, d) = 5$





$x_1 := 0$

$x_1 = t_2 - t_1$

$y_1 = 0$

$x_1 - y_1 = t_2 - t_1$

↳ this difference does not change by time elapse

diagonal constraints.

Questions:

-1) Do diagonal constraints add expressive power? No

-2) Do diagonal constraints give succinctness? Yes

Motivation: Even though diagonal constraints do not add expressive power, they offer modeling convenience & at times they can produce exponentially succinct models.

Diagonal constraints:

X: set of clocks

$$x - y \sim c \qquad c \in \mathbb{N} \qquad \sim \in \{<, \leq, =, \geq, >\}$$

Guard:

$$\phi := x \sim c \mid x - y \sim c \mid \phi \wedge \phi$$

$$x, y \in X$$

Eg: $y - x \leq 2 \quad \wedge \; z = 5$

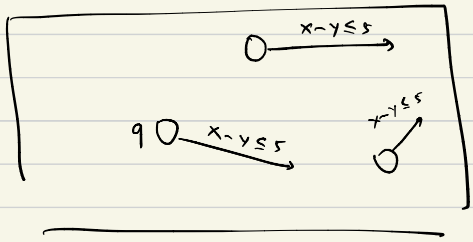
d- Timed automata: Timed automata where guards can contain diagonal constraints.

Theorem: For every d-timed automaton $\mathcal{A}$, there exists a timed automaton $\mathcal{B}$ s.t. $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{B})$

Eliminating diagonal constraints:

Assume that $\mathcal{A}$ has a single diagonal constraint: $x - y \leq c$

$\mathcal{A}$



Idea: At state q the automaton needs to know whether $x-y \leq 5$ or not.

Split every state p into two copies:

$(p, 1)$ $\qquad$ $(p, 0)$

When this copy is reached, we should have $x-y \leq 5$ $\qquad$ When this copy is reached, $x-y > 5$

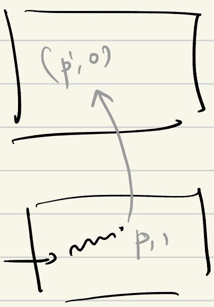
- Suppose q_0 is the initial state.

  - Initially $x-y = 0$, $\therefore$ $x-y \leq 5$ is true

    Hence start with $(q_0, 1)$

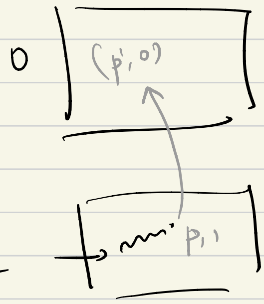
$(q_0, 1) \longrightarrow (q_1, 1) \longrightarrow (q_2, 1)$

0



- When do we jump from 0-copy to 1-copy? 1

  $(p, 1) \longrightarrow (p', 0)$



- When do we jump from 0-copy to 1-copy?

$$(p, 1) \longrightarrow (p', 0)$$

i) Suppose x is reset.

Value of $x-y$ becomes $0-y$

which is still ≤ 5

- So automaton does not go to the 0-copy.

ii) Suppose y is reset.

Value of $x-y$ becomes $x-0$

↳ if $x \leq 5$, then stay in 1-copy

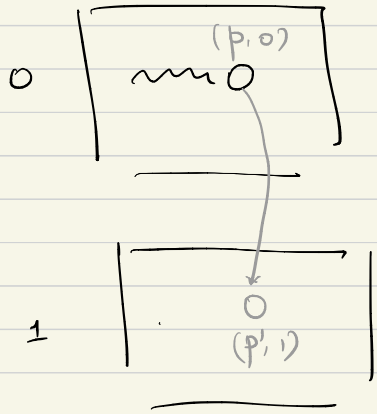
→ else $x > 5$, so go to 0-copy.

$$p \xrightarrow[\{y\}]{} p'$$

$$(p, 1) \xrightarrow[\{y\}]{x \leq 5} (p', 1)$$

$$(p, 1) \xrightarrow[\{y\}]{x > 5} (p', 0)$$

When do we jump from 0-copy to 1-copy?



Suppose $p \xrightarrow[\{x\}]{} p'$ or $p \xrightarrow[\{x,y\}]{} p'$

$\to \quad (p,0) \xrightarrow[\{x\}]{} (p',1) \qquad (p,0) \xrightarrow[\{x,y\}]{} (p',1)$

Suppose $p \xrightarrow[\{y\}]{} p'$

$$\begin{array}{c} x-y \\ \downarrow \\ x-0 \end{array}$$

$\to \quad (p,0) \xrightarrow[\{y\}]{x \leq 5} (p',1)$ $\longrightarrow$ But this transition is not possible

(Nabin's observation)

$\qquad (p,0) \xrightarrow[\{y\}]{x > 5} (p',0)$

$(p,0) \xrightarrow[\{y\}]{} (p',0)$

Given $\mathcal{A}$: d-timed automaton: $\mathcal{A} = (Q^A, q_0^A, X, \Sigma, \Delta^A, F^A)$

- Assume that $\mathcal{A}$ has a single diagonal constraint $x - y \leq c$

Equivalent timed automaton $\mathcal{B}$:

$Q^B = Q^A \times \{0, 1\}$

$q_0^B = (q_0^A, 1)$ since $0 \leq c$ $c \in \mathbb{N}$

Δ^B: For every $p \xrightarrow[R]{a,\, g} p'$,

- Suppose g contains $x - y \leq c$.

g' is g without $x - y \leq c$

$(p, 1) \xrightarrow[R]{a,\, g'} (p', 1)$ if R contains x

$(p, 1) \xrightarrow[R]{a,\, g' \wedge x \leq c} (p', 1)$
$\quad\ \ \xrightarrow[R]{a,\, g' \wedge x > c} (p', 0)$ } else, if R contains y

$(p, 1) \xrightarrow[R]{a,\, g'} (p', 1)$ else (when R contains neither x nor y)

- Suppose g does not contain $x - y \leq c$.

In addition to above, we have:

$(p, 0) \xrightarrow[R]{a,\, g} (p', 1)$ if R contains x.

$(p, 0) \xrightarrow[R]{a,\, g} (p', 0)$ else.

- $F^B = F^A \times \{0, 1\}$

Exercise: When the diagonal constraint is $x - y \geq c$, $c > 0$

For every $p \xrightarrow[R]{a,\, g} p'$

Suppose g contains $x - y \geq c$

$(p, 1) \xrightarrow[R]{a,\, g'} (p', 0)$ if R contains x

$(p, 1) \xrightarrow[R]{a,\, g' \wedge x \geq c} (p', 1)$ else, if R contains y

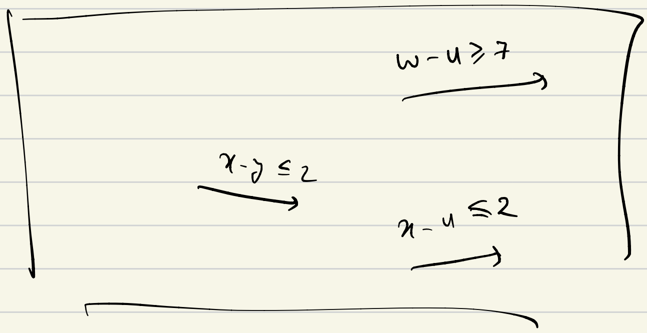
$\quad\quad \xrightarrow[R]{a,\, g' \wedge x < c} (p', 0)$

$(p, 1) \xrightarrow[R]{a,\, g'} (p', 1)$ else (R contains neither)

Suppose g does not contain $x - y \geq c$, there are additional transitions:

Exercise!

When $\mathcal{A}$ contains more than one diagonal constraint?



Eliminate one diagonal constraint at a time.

In the end, there is a diagonal free automaton whose states are of the form:

$$\langle q, 01100 \rangle$$

a bit vector with length equal to no. of diagonals in $\mathcal{A}$.

→ The transitions update the truth of the diagonals.

Complexity: The resulting timed automaton $\mathcal{B}$ is exponentially bigger.

Summary:

- Timed automata with diagonals are as expressive as diagonal-free timed automata.
- Construction of eliminating diagonals is in this paper:

Bérard, Gastin, Dieckert, Petit '98