TIMED AUTOMATA

LECTURE 16

Recall:

Problem: Given a one-clock T.A. $\mathcal{A}$, is $\mathcal{L}(\mathcal{A})$ universal?

Strategy: Building a transition system over configurations.

$$\{(q_0, 0)\} \qquad\qquad T_{\mathcal{A}}$$

$$\xrightarrow{\delta,\, a}$$

$$\{(q_1, \delta), (q_2, 0), (q_3, \delta)\}$$

$\vdots$

- For every timed word, there is a unique path in this system corresponding to the word, leading to a unique node.

Bad: A node of $T_{\mathcal{A}}$ is bad if every state contains a non-accepting location.

Property: $\mathcal{A}$ is not universal $\iff$ there exist a path leading to a bad node in $T_{\mathcal{A}}$.

How to make $T_{\mathcal{A}}$ finite?

Step 3: Equivalence between configurations

$C_1 = \{ \_, \_, \_, \_, \_ \}$ $\qquad$ $C_2 = \{ \_, \_, \_, \_ \}$

$$\begin{tikzcd}
C_1 \arrow[d, "\delta_1, a_1"'] & \sim & C_2 \arrow[d, "\delta_1', a_1"] \\
C_1' \arrow[d, "\delta_2, a_2"'] & \sim & C_2' \arrow[d, "\delta_2', a_2"] \\
C_1'' & \sim & C_2''
\end{tikzcd}$$

The locations present in C_1 are same as those of C_2.

- Goal is to come up with such an equivalence.

Suppose we have such an equivalence:

$$\begin{tikzcd}
C_1 \arrow[d, rightsquigarrow] & \sim & C_2 \arrow[d, rightsquigarrow] \\
C_{bad} & \sim & C'_{bad}
\end{tikzcd}$$

C'_{bad} will also be bad if C_{bad} is bad.

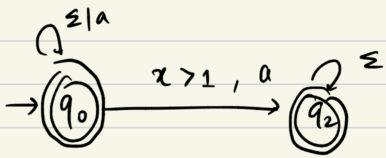
Once we have this, we can build a graph over equivalence classes.

Formulating the equivalence:

- We will make use of the max constant M, and parameterize the equivalence wrt. M.

Example:

$$c_1 : \{ (q_0, 0.5) \} \not\sim c_2 : \{ (q_0, 1.7) \} \; ?$$



From c_2, all words are accepted.

$$c_2 = \{ (q_0, 1.7) \}$$

$$\{ (q_0, 1.7) \} \xrightarrow{(0,a)} \{ (q_2, 1.7) \} \qquad \{ (q_0, 1.7) \} \xrightarrow{(0.1,a)} \{ (q_2, 1.8) \} \quad \ldots$$

From c_1, the word $(0,a)$ is rejected.

$$c_1 = \{ (q_0, 0.5) \}$$

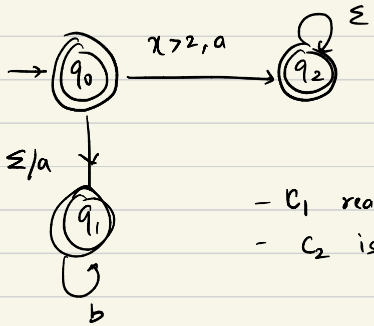
$$\{ (q_0, 0.5) \} \xrightarrow{(0,a)} \{ \} \leftarrow \text{Bad}$$

We cannot relate c_1 and c_2.

Example 2:

$C_1 = \{ (q_0, 0.5), (q_1, 1.7) \}$ $C_2 = \{ (q_0, 2.4) (q_1, 1.7) \}$

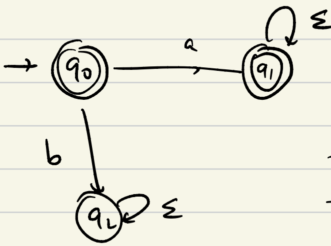
A similar automaton as in the previous case will show that C_1 cannot be made equivalent to C_2.



- C_1 reaches a bad configuration on $(0, a)$
- C_2 is universal

Example 3:

$C_1 = \{ (q_0, 0) \}$ $C_2 = \{ q_1, 0) \}$



- C_1 reaches a bad config.
- C_2 is universal.

From Examples 1, 2, 3, we learn that the equivalence should satisfy foll. properties:

$C_1 \sim C_2$ should imply:

There exist bijection $f : C_1 \longmapsto C_2$ mapping each state of C_1 to a state of C_2 s.t.

- 1. $f[(q,u)]$ has same location q

- 2. Suppose $f(q,u) = (q, u')$. Then

$$\lfloor u \rfloor = \lfloor u' \rfloor$$

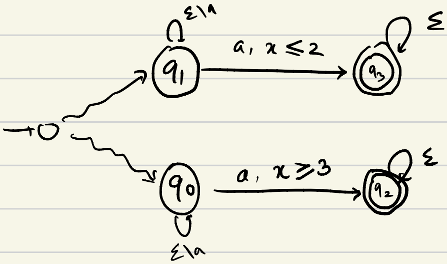
Is this sufficient?

Example 4: $M = 10$

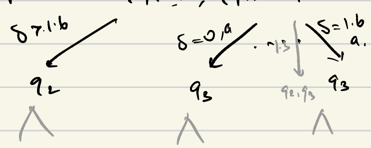
$C_2 = \{(q_0, 1.7), (q_1, 0.4)\}$

$C_1 = \{(q_0, 1.7), (q_1, 0.9)\}$

Want an example of an aut. s.t. C_1 is bad, C_2 is not bad



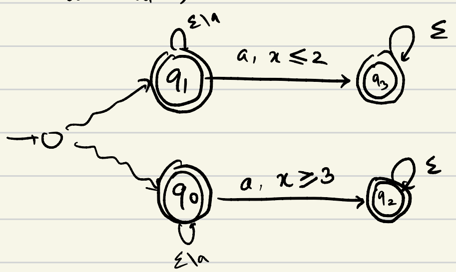
$C_1 : \{(q_0, 1.7), (q_1, 0.4)\}$



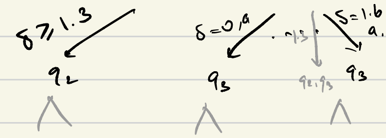
Example 4: $M = 10$

$C_2 = \{(q_0, 1.7), (q_1, 0.4)\}$
$C_1 = \{(q_0, 1.7), (q_1, 0.9)\}$

Want an example of an aut. s.t. C_1 is bad, C_2 is not bad

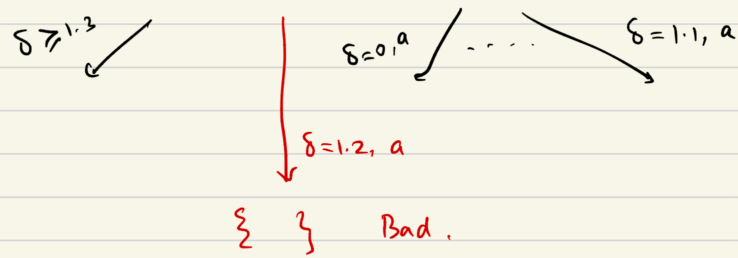


C_2: $\{(q_0, 1.7), (q_1, 0.4)\}$



C_2 is universal
– No bad location is reachable

C_1: $\{(q_0, 1.7), (q_1, 0.9)\}$



∴ It is also important to consider the order between fractional values while equating the configuration.

Definition of equivalence:

Difference from the region equivalence:

$$(q, \langle x_1, x_2, \dots, x_n \rangle) \sim_M (q, \langle x_1', x_2', \dots, x_n' \rangle)$$

- $\lfloor x_i \rfloor = \lfloor x_i' \rfloor$ or $x_i, x_i' > M$
- For all $x_i, x_j \leq M$,
  $\{x_i\} \leq \{x_j\} \iff \{x_i'\} \leq \{x_j'\}$

Now:

$$\{(q, x_1), (q, x_2), \dots (q, x_n)\} \sim \{(q, x_1') (q, x_2'), \dots (q, x_n')\}$$

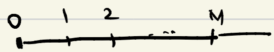
$$\{(q, 1.7), (q, 2.3), (q, 5.4)\} \quad \{(q, 1.8), (q, 2.2), (q, 5.3)\}$$

The bijection is apriori not evident.

To encode the order of fractionals, the integrals and the correspondence between locations, the paper OW'05 makes use of an intermediate representation.

Associating a word over a finite alphabet to each configuration:

Suppose the max constant is M.



$$REG = \{ r_0,\ r_0^1,\ r_1,\ r_1^2,\ \ldots,\ r_M,\ r_M^\infty \}$$

Suppose $M = 10$

$$C = \{ (q_1, 2.3)\ (q_2, 5.7),\ (q_3, 7.1),\ (q_4, 5),\ (q_5, 7.3)\ (q_6, 10.7) \}$$

$\downarrow$ → order based on fr. values modulo M + group together same fr. values

$$\{(q_4, 5)\}\ \{(q_3, 7.1)\}\ \{(q_1, 2.3), (q_5, 7.3)\}\ \{(q_2, 5.7)\}\ \{(q_6, 10.7)\}$$

unbdd. group tog.

replace exact value with the region ← $\downarrow$

$$\{(q_4, r_5)\}\ \{(q_3, r_7^8)\}\ \{(q_1, r_2^3), (q_5, r_7^8)\}\ \{(q_2, r_5^6)\}\ \{(q_6, r_{10}^\infty)\}$$

$\downarrow$ $H(C)$ $\quad$ $\downarrow$ words over Power set of $(Q \times REG)$ → Σ

$$\boxed{C_1 \sim C_2 \quad \text{iff} \quad H(C_1) = H(C_2)}$$

Example: $M = 10$

1) $C_1 = \{(q_0, 1.7)\} \not\sim C_2 = \{(q_0, 0.5)\}$

$H(C_1) = \{(q_0, r_1^2)\} \neq C_2 = \{(q_0, r_0^1)\}$

2) $C_1 = \{(q_0, 0.5)\} \not\sim C_2 = \{(q_1, 0.5)\}$

$H(C_1) = \{(q_0, r_0^1)\} \neq H(C_2) = \{(q_1, r_0^1)\}$

3) $C_1 = \{(q_1, 1.7), (q_2, 0.4)\} \not\sim C_2 = \{(q_1, 1.7)\ (q_2, 0.9)\}$

$H(C_1) = \{(q_2, r_0^1)\}\ \{(q_1, r_1^2)\} \neq H(C_2) = \{(q_1, r_1^2)\}\ \{(q_2, r_0^1)\}$

4) $C_1 = \{(q_1, 1.7), (q_2, 0.4)\} \sim C_2 = \{(q_1, 1.5), (q_2, 0.1)\}$

$H(C_1) = \{(q_2, r_0^1)\}\ \{(q_1, r_1^2)\} = H(C_2) = \{(q_2, r_0^1)\}$
$\{(q_1, r_1^2)\}$

5) $C_1 = \{(q, 1.1), (q, 1.2)\} \not\sim C_2 = \{(q, 1.5)\}$

$H(C_1) = \{(q, r_0^1)\}\ \{(q, r_0^1)\} \neq H(C_2) = \{(q, r_0^1)\}$

$aa \neq a$

Summary:

Equivalence $c_1 \sim c_2$ iff $H(c_1) = H(c_2)$

Next class:

- $\sim$ has nice properties
- a graph can be constructed where node is an $H(c)$
- termination?