In the lecture we have seen neighbourhood equivalence when there are only two clocks $\{x, y\}$. This can be extended to the general case with multiple clocks by considering clocks pairwise. Here is the definition.

Definition 1 (Neighbourhood equivalence) Two valuations v and v' are said to be neighbourhood equivalent, written as $v \simeq_{\mathsf{nbd}} v'$ if:

1. $\lfloor v(x) \rfloor = \lfloor v'(x) \rfloor$ for all clocks x,
2. $\{v(x)\} = 0$ iff $\{v'(x)\} = 0$ for all clocks x
3. for every pair of clocks x, y:
   (a) $\{v(x)\} < \{v(y)\} \Leftrightarrow \{v'(x)\} < \{v'(y)\}$
   (b) $\{v(x)\} = \{v(y)\} \Leftrightarrow \{v'(x)\} = \{v'(y)\}$

Each equivalence class of $\simeq_{\mathsf{nbd}}$ will be called a *neighbourhood*.

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1. Group the following valuations over two clocks into neighbourhoods.

$$\begin{aligned} v_1 &:= (3.4,\ 7.2) \\ v_2 &:= (2.1,\ 10.0) \\ v_3 &:= (3.3,\ 7.4) \\ v_4 &:= (3.7,\ 7.2) \\ v_5 &:= (2.1,\ 10.1) \\ v_6 &:= (2.7,\ 10.0) \\ v_7 &:= (3.9,\ 7.4) \end{aligned}$$

2. Below are combinations of v, δ and v' with $v \simeq_{\mathsf{nbd}} v'$. For each of them, find a $\delta' \geq 0$ such that $v + \delta \simeq_{\mathsf{nbd}} v' + \delta'$.

$v = (2.2, 3.7)$	$\delta = 0.3$	$v' = (2.1, 3.9)$
$v = (5.0, 8.2)$	$\delta = 3.8$	$v' = (5.0, 8.9)$
$v = (3.2, 2.1)$	$\delta = 0.9$	$v' = (3.7, 2.2)$
$v = (3.2, 2.1)$	$\delta = 2.9$	$v' = (3.7, 2.2)$

   Recall that $v + \delta$ is the valuation obtained by adding δ to each coordinate of v.

3. Group the following valuations over five clocks into neighbourhoods.

$$\begin{aligned} v_1 &:= (7.4,\ 2.1,\ 8.7,\ 5.4,\ 7.0) \\ v_2 &:= (3.4,\ 2.0,\ 8.5,\ 10.0,\ 7.1) \\ v_3 &:= (7.3,\ 2.2,\ 8.8,\ 5.2,\ 7.0) \\ v_4 &:= (7.5,\ 2.1,\ 8.9,\ 5.5,\ 7.0) \\ v_5 &:= (3.2,\ 2.0,\ 8.8, 10.0,\ 7.5) \\ v_6 &:= (3.3,\ 2.0,\ 8.4,\ 10.0,\ 7.2) \end{aligned}$$

4. Below are combinations of v, δ and v' with $v \simeq_{\mathsf{nbd}} v'$. For each of them, find a $\delta' \geq 0$ such that $v + \delta \simeq_{\mathsf{nbd}} v' + \delta'$.

$v = (1.1,\ 2.0,\ 3.0,\ 4.0,\ 5.9)$	$\delta = 3.4$	$v' = (1.5,\ 2.0,\ 3.0,\ 4.0,\ 5.6)$
$v = (10.2,\ 4.8,\ 19.1,\ 2.0,\ 8.5)$	$\delta = 5.4$	$v' = (10.1,\ 4.9,\ 19.05,\ 2.0,\ 8.7)$

5. Now we look at the general case. Let v, v' be valuations such that $v \simeq_{\mathsf{nbd}} v'$. Show that for every $\delta \geq 0$, there exists a $\delta' \geq 0$ such that $v + \delta \simeq_{\mathsf{nbd}} v' + \delta'$