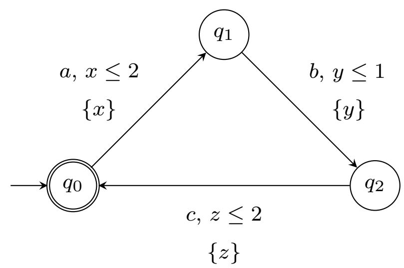
1. Give a timed automaton over $\Sigma = \{a, b\}$ that accepts all timed words.

2. Let $\mathcal{A} = (\{q\}, \{a, b\}, \{x\}, T, \{q\}, \{q\})$ be a timed automaton with a single state q and a single clock x. Note that q is also an accepting state. Let T be the set of transitions. Give an instance of T that makes $\mathcal{A}$ reject at least one timed word.

3. What is the timed word accepted by the following accepting run of some timed automaton with two clocks x and y?

$$\begin{array}{ccccccccc} q_0 & & q_0 & & q_1 & & q_1 & & q_F \\ x:0 & \xrightarrow{\delta_0} & x:0.6 & \xrightarrow{a} & x:0.6 & \xrightarrow{\delta_1} & x:1.9 & \xrightarrow{a} & x:0 \\ y:0 & & y:0.6 & & y:0 & & y:1.3 & & y:1.3 \end{array}$$

4. Let $\mathcal{B}$ be the following timed automaton:



Consider the timed word $s = (abcabc, 0.5, 1, 1.5, 1.8, 1.9, 3)$.

a) Does $\mathcal{B}$ accept s? If so, write down the accepting run of $\mathcal{B}$ on s.

b) For a timed word (w, τ) we define the *time span* of (w, τ) to be the time at which the last letter occurs, i.e., if $|w| = n$, then time span of (w, τ) is τ_n.
For every $k \in \mathbb{N}$, give a timed word in $\mathcal{L}(\mathcal{B})$ that has length greater than k and whose time span is lesser than 1.

5. Give a timed automaton for the timed language of all words in $(a + b)^*$ such that there exist two a-s which are at distance 1 apart *and* there exist two b-s which are at distance 2 apart.

6. Let $\Sigma = \{a, b\}$. Construct timed automata for the following languages:

(a) $\{(w, \tau) \mid$ there is an a at some time t and no action occurs at time $t + 1$ $\}$

(b) $\{(w, \tau) \mid$ all a's occur before time 1 and no two a's happen at the same time $\}$

(c) $\{(a^k, \tau) \mid k \geq 1, \tau_1 = 1,$ and $\tau_{i+2} - \tau_i \leq 1$ for all $1 \leq i \leq k - 2\}$