1. Draw the automaton (if needed with ε-transitions) for the language over $\Sigma = \{a, b\}$ given by:

$$\begin{aligned}\{\ (w,\tau) \mid w \in \Sigma^*,\ \forall i \leq |w| :\ & w_i = a \text{ implies } \tau_i \text{ is an integer and} \\ & w_i = b \text{ implies } \tau_i \text{ is not an integer}\}\end{aligned}$$

   where w_i denotes the i^{th} letter in the word w and τ_i denotes the corresponding time-stamp.

2. Which of the following are well-quasi-orders? Provide a short justification.

   Assume that Σ is a finite alphabet.

   (a) $(\Sigma^*, \sqsubseteq_1)$ where $w_1 \sqsubseteq_1 w_2$ if w_1 is a suffix of w_2

   (b) $(\Sigma^*, \sqsubseteq_2)$ where $w_1 \sqsubseteq_2 w_2$ if w_1 is a prefix of w_2 *or* w_1 is a subword of w_2.

   (c) $(\mathcal{Z}(X), \subseteq)$ where $\mathcal{Z}(X)$ is the set of all zones over clocks X and $\subseteq$ is the usual subset inclusion.

3. Let A be a timed automaton. Is the following decision problem decidable? If you think the problem is decidable, give an algorithm. Else, give a proof of undecidability.

   Given A and a constraint g, denote A_g to be the automaton obtained by removing all occurrences of g in A: in other words, all transitions $q \xrightarrow{g \wedge \phi, R} q'$ in A are changed to $q \xrightarrow{\phi, R} q'$.

   Consider the decision problem: is $\mathcal{L}(A) = \mathcal{L}(A_g)$?

   Essentially, this decision problem asks if a particular guard occuring in an automaton is useful at all.