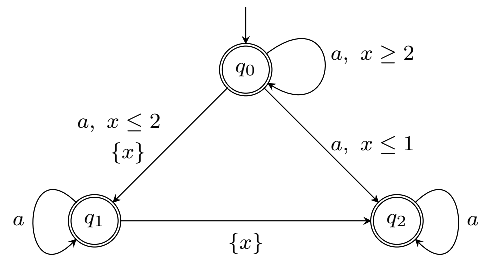
1. The purpose of this question is to understand the algorithm by Ouaknine and Worrell for checking the universality of one-clock timed automata. Let us call it the OW-algorithm. The definitions and notations used in this question are identical to those in the lecture.

   Consider the following automaton over the singleton alphabet $\{a\}$:



   The set of regions $REG = \{r_0, r_{01}, r_1, r_{12}, r_2, r_{2\infty}\}$

   a) What is the set Λ? Recall that nodes in the graph built by the OW-algorithm would be labeled by words in Λ^*.

   b) Consider the word $W = \{(q_1, r_0)\}\{(q_2, r_{01})\}$. List two different configurations C_1 and C_2 of the automaton that map to this word by the encoding H defined in the lecture, i.e., $H(C_1) = H(C_2) = W$.

   c) For the C_1 and C_2 that you have chosen, prove that for $\delta_1 = 1.3$, there exists a $\delta_2 \in \mathbb{R}_{\geq 0}$ such that

   $$\text{if } C_1 \xrightarrow{\delta_1, a} C_1' \qquad \text{then } C_2 \xrightarrow{\delta_2, a} C_2'$$

   such that $H(C_1') = H(C_2')$.

   d) Run the OW-algorithm on the above automaton and check if it accepts all timed words.

2. Given a general timed automaton B and a one-clock timed automaton A design an algorithm for $L(B) \subseteq L(A)$.

3. We know that the universality problem for one-clock timed automata is decidable. Suppose we consider timed automata with multiple clocks, but restrict guards to contain only the constant 0; that is, for a set of clocks X, the guards come from the set $\Phi_0(X)$ defined inductively as

   $$\Phi_0(X) \ := \ x = 0 \mid x > 0 \mid \Phi_0(X) \ \wedge \ \Phi_0(X)$$

   where $x \in X$. For instance, if $y, z \in X$, then $y = 0 \wedge z > 0$ is a guard in $\Phi_0(X)$.

   Show that the universality problem is decidable for timed automata with multiple clocks, but whose guards come only from the set $\Phi_0(X)$.