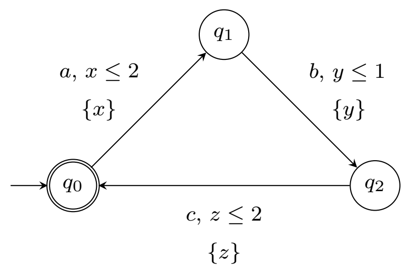
1. Give a timed automaton over $\Sigma = \{a, b\}$ that accepts all timed words.

2. Let $\mathcal{A} = (\{q\}, \{a, b\}, \{x\}, T, \{q\}, \{q\})$ be a timed automaton with a single state q and a single clock x. Note that q is also an accepting state. Let T be the set of transitions. Give an instance of T that makes $\mathcal{A}$ reject at least one timed word.

3. What is the timed word accepted by the following accepting run of some timed automaton with two clocks x and y?

$$\begin{array}{c} q_0 \\ x:0 \\ y:0 \end{array} \xrightarrow{\delta_0} \begin{array}{c} q_0 \\ x:0.6 \\ y:0.6 \end{array} \xrightarrow{a} \begin{array}{c} q_1 \\ x:0.6 \\ y:0 \end{array} \xrightarrow{\delta_1} \begin{array}{c} q_1 \\ x:1.9 \\ y:1.3 \end{array} \xrightarrow{a} \begin{array}{c} q_F \\ x:0 \\ y:1.3 \end{array}$$

4. Let $\mathcal{B}$ be the following timed automaton:

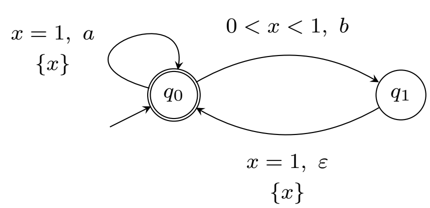


Consider the timed word $s = (abcabc, 0.5, 1, 1.5, 1.8, 1.9, 3)$.

a) Does $\mathcal{B}$ accept s? If so, write down the accepting run of $\mathcal{B}$ on s.

b) For a timed word (w, τ) we define the *time span* of (w, τ) to be the time at which the last letter occurs, i.e., if $|w| = n$, then time span of (w, τ) is τ_n.
For every $k \in \mathbb{N}$, give a timed word in $\mathcal{L}(\mathcal{B})$ that has length greater than k and whose time span is lesser than 1.

5. (a) What is the language accepted by the following automaton?



(b) Draw the automaton (if needed with ε-transitions) for the language over $\Sigma = \{a, b\}$ given by:

$$\{\ (w, \tau) \mid w \in \Sigma^*,\ \forall i \le |w| :\ \begin{array}{l} w_i = a \text{ implies } \tau_i \text{ is an integer and} \\ w_i = b \text{ implies } \tau_i \text{ is not an integer}\} \end{array}$$

where w_i denotes the i^{th} letter in the word w and τ_i denotes the corresponding time-stamp.

6. Give a timed automaton for the timed language of all words in $(a + b)^*$ such that there exist two a-s which are at distance 1 apart *and* there exist two b-s which are at distance 2 apart.

7. Let us add an extra feature to the timed automaton model. Suppose in addition to resets that set a clock to 0, we also allow resetting a clock to 1: that is, each transition is of the form (q, a, g, R_0, R_1, q') where g is the guard, R_0 is the set of clocks that have to be reset to 0 and R_1 is the set of clocks that need to be reset to 1 (assume that $R_0 \cap R_1 = \emptyset$ in every transition).

   Let TA_{+1} denote the set of timed automata that have these special resets to either 0 or 1.

   Show that this extra feature does not add expressive power to the model. In other words, prove that for every automaton $\mathcal{A}$ in TA_{+1} there exists a normal timed automaton $\mathcal{B}$ that has resets only to 0, such that $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{B})$.