TIMED AUTOMATA

LECTURE 8

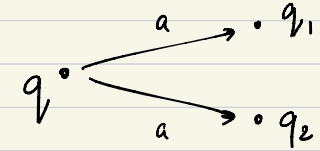
TODAY'S GOALS:

- 1. Problem with subset construction in T.A.
- 2. A restricted class where subset constrn. works.

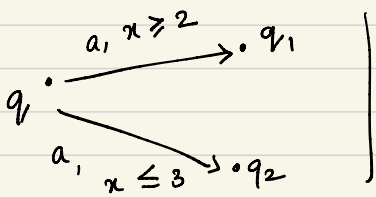
Event - clock automata

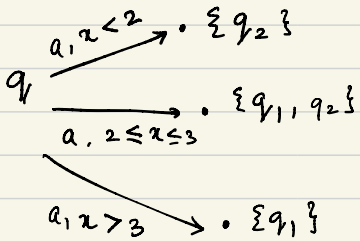
- Alur, Fix, Henzinger (1999)

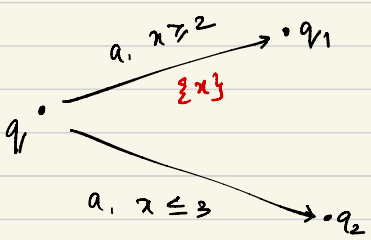
Problem with subset construction:

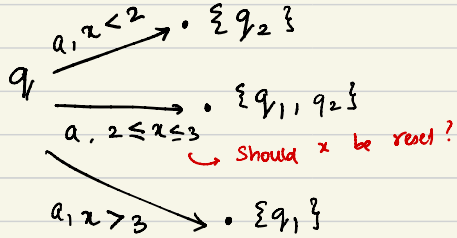


$\{q_1\} \xrightarrow{a} (\{q_1, q_2\})$





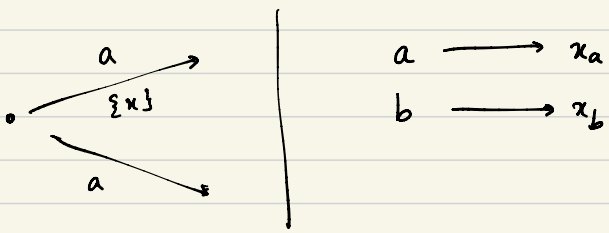




- Resets cause difficulty during subset construction.

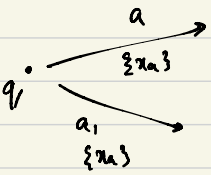
Event - recording clocks:

- For every $a \in \Sigma$, there is an x_a
- Value of x_a is determined by input word, and not by automaton.



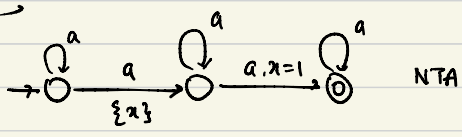
→ Whenever there is an 'a', x_a has to be reset

Moreover, x_a cannot be reset anywhere else.



→ x_a tracks the time since the last occurrence of 'a'.

"Non" - example:



Not an event recording clock

Example:

$$\{(abcd)^k, \tau \mid \tau_c - \tau_a \leq 2 \wedge \tau_d - \tau_b \geq 1\}$$

distance between $a \rightarrow c \leq 2$

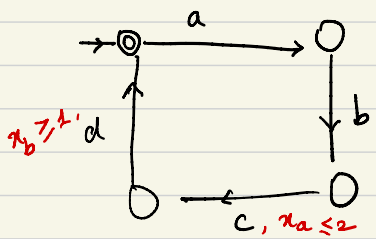
distance $b \rightarrow d \geq 1$

Example:

$$\{(abcd)^k, \tau \mid \tau_c - \tau_a \leq 2 \wedge \tau_d - \tau_b \geq 1\}$$

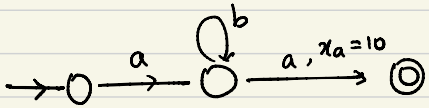
distance between $a \longrightarrow c \leq 2$

distance $b \longrightarrow d \geq 1$



Example:

$$\{ a b^* a \mid \text{time between the 2 a's is exactly 10} \}$$



Example: $\{ b b^* a \mid$ time between first and last letter is exactly 10$\}$

- Cannot be accepted by an Event-Recording automaton.

Event - recording automata: (ERA)

- Σ : a finite alphabet.

  X_Σ : $\{ x_a \mid a \in \Sigma \}$

	a	a	b	b	a
	2.5	3.2	4.0	5.1	6.0
x_a	$\perp$	0.7	0.8	1.9	2.8
x_b	$\perp$	$\perp$	$\perp$	1.1	0.9

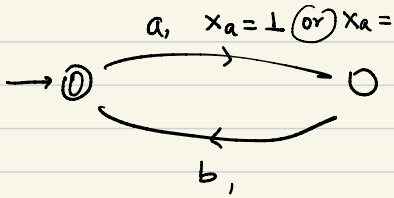
Values of x_a, x_b are determined by the input word, and not by the automaton.

Guards:

$$\varphi := x \leq c \mid x \geq c \mid \neg\varphi \mid \varphi \wedge \varphi$$

$\Phi(\Sigma)$: set of all guards.

where $c \in \mathbb{N} \cup \{\perp\}$

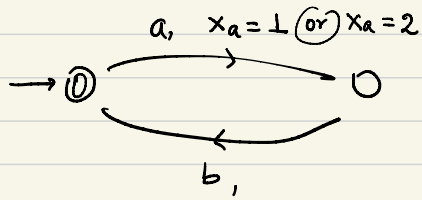


a b a b a b . . .

$=2$ $=2$ $=2$

$\perp \leq \perp$ is true.

- every other combination: $\perp \leq 3$, $5 \geq \perp$ ... are false.



a b a b a b ...

	a	b	a	b	a	b
	5	6	7	8	9	10
x_a	$\perp$	1	2	1	2	1

ERA:

Q : states.

Q_0 : initial states.

Σ : alph.

E : $q \xrightarrow{a,\ \Phi(\Sigma)} q'$

F : acc. states.

- Given a timed word, how do we mark the values of clock.

a_0	a_1	a_2	.....	a_n
t_0	t_1	t_2		t_n
γ_0	γ_1	γ_2	.....	γ_n

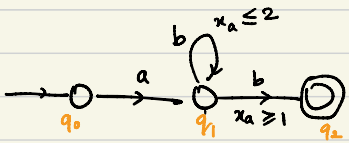
γ_i : clock valuation at i^{th} letter.

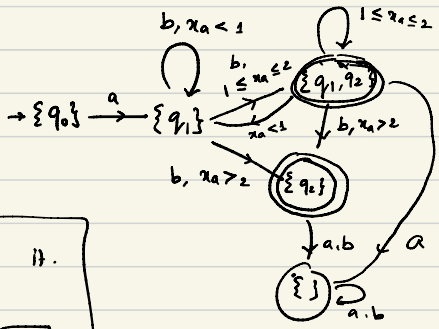
$$\gamma_i(x_a) = \begin{cases} t_i - t_j & \text{if } \exists j < i \text{ s.t. } a_j = a \text{ and } \forall j < k < i \; a_k \neq a \\ \bot & \text{otherwise} \end{cases}$$

- A word $(a_0, t_0)\ (a_1, t_1)\ \dots\ (a_n, t_n)$ is accepted by an ERA. if there exists a sequence of transitions:

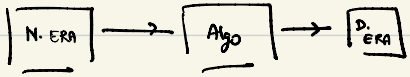
$$q_0 \xrightarrow{a_0, g_0} q_1 \xrightarrow{a_1, g_1} q_1 \longrightarrow \dots \xrightarrow{a_n, g_n} q_n$$

s.t. γ_i satisfies guard g_i.





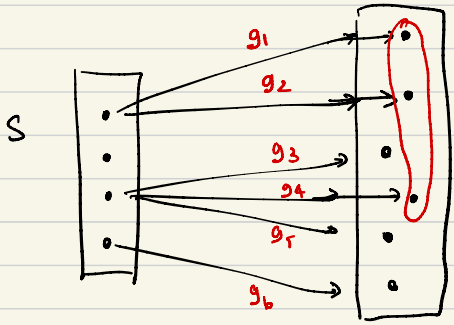
→ Given an ERA, how to determinize it.



- Subset construction taking care of guards alongside.

ERA A: Subset construction gives a DERA.

States: $S \subseteq Q$ (subset)



$\varphi_{s''}$

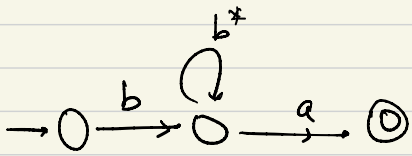
$g_1 \wedge g_2 \wedge g_4 \wedge \neg g_3 \wedge \neg g_5 \wedge \neg g_6$

For $s'' \subseteq s'$:

$S \xrightarrow{a,\ \varphi_{s''}} s''$

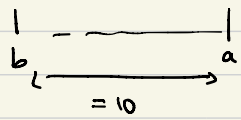
s' = set of all states that can be reached on an 'a' from s.

Event-predicting clocks and Event-predicting automata (EPA)



$y_a = 10$

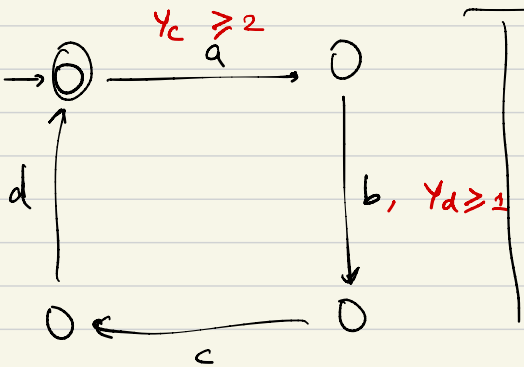
↳ the "next" 'a' occurs at time 10 from now.



b b b a

1 1.5 3.5 11

$y_a = 10$



	a	b	a	a	b	b
	2.5	3.3	4.5	7	10.2	11
y_a	2	1.2	2.5	1	1	1
y_b	0.8	0.9	5.7	3.2	0.8	1

Event predicting automaton:

- Analogous to the definition of ERA.

$a_0 \quad a_1 \quad a_2 \quad \ldots \quad a_n$

$t_0 \quad t_1 \quad \ldots \quad t_n$

$\gamma_0 \quad \gamma_1 \quad \ldots \quad \gamma_n$

$$\gamma_i(y_a) = \begin{cases} t_j - t_i & \exists\, j > i \text{ s.t. } a_j = a \\ & \forall\, i < k < j \quad a_k \neq a \\ \bot & \text{otherwise.} \end{cases}$$

- Rest same (syntax, run.).

Determinizing EPA:

→ Same subset construction as before with guards taken care of.

Non-deterministic EPA $\xrightarrow{\text{subset}}$ Det. E.P.A.

Event - clock automata (ECA)

$bb^* a$ (EPA, but not ERA)

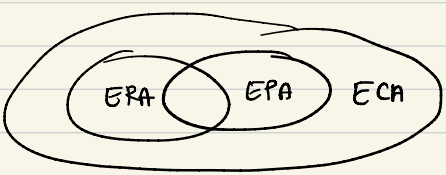
$=10$

$a b^* b$ (ERA, but not EPA)

$=10$

ECA: automata where we can have both event recording clocks and event predicting clocks.

→ can be determinized by the subset construction.



Next: Relating ECA with timed automata.