TIMED AUTOMATA

LECTURE 7

GOALS OF TODAY'S LECTURE

- 1. Deterministic Timed Automata
- 2. Closure properties
- 3. Determinizability of T.A:
- 4. Subclass of timed automata that is determinizable (Event recording automata)

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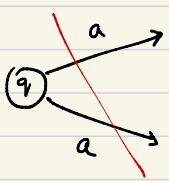
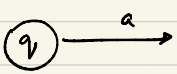
1. Deterministic timed automaton:

Finite automata: 1)

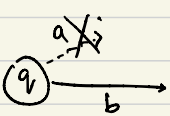




Unique transition on 'a' at 'q'
(for every 'a' and 'q')

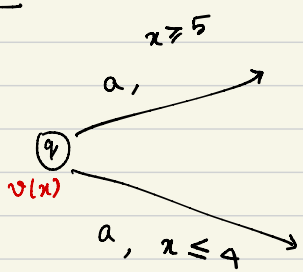
2) Unique start state.

1) and 2) ensure that given a word, there is a unique run of the automaton on the word

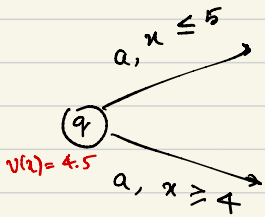


If in every state, there is some transition on every letter, then the automaton is said to be complete.

Timed automata:

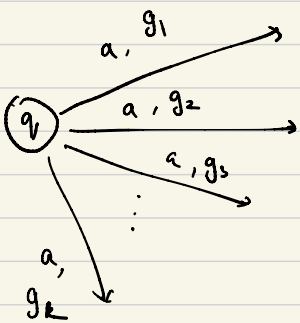


Yes, deterministic



no

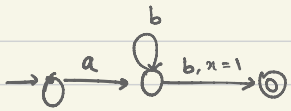
A T.A. is deterministic if for every state 'q', letter 'a':



$g_i \wedge g_j$ is unsatisfiable

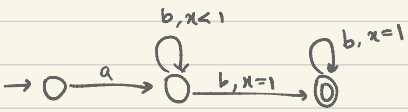
- complete if $g_1 \cup g_2 \cup g_3 \ldots \cup g_k$ is the set of all valuations

A_1:

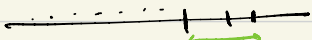


Not deterministic.

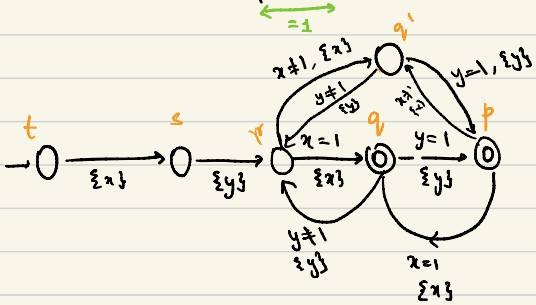
Question: Is there a deterministic T.A. accepting $\mathcal{L}(A_1)$?



A_2: at least 3 letters + 2nd letter from the last comes at distance 1 from the last letter.

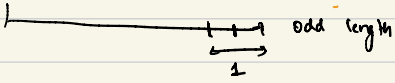


$\Sigma = \{a, b\}$



a a a a

1 2 2.5 3



Properties of deterministic timed automata:

1. Every timed word has a unique run (if the DTA is complete).

2. If DTA is not complete, then every timed word has atmost 1 run.

3. **Closure properties:** DTA are closed under union, intersection and complement

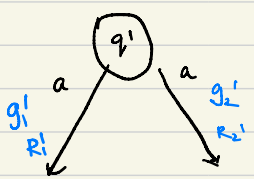
DTA $\mathcal{A}_1$ $\mathcal{A}_2$

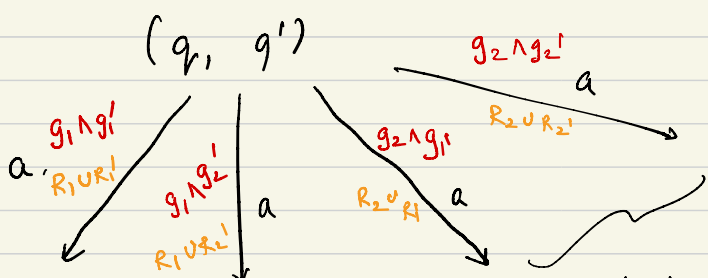
Acc. states:
$(*, f_2) \cup (f_1, *)$

DTA: $\mathcal{A}_1 \cup \mathcal{A}_2$ — Product construction.

$\mathcal{A}_1 \cap \mathcal{A}_2$

(f_1, f_2)





no two trans. have a non-empty guard intersection.

Since we start with DTA, $g_1 \wedge g_2$ is unsat.

$g_1' \wedge g_2'$ is unsat.

Complementation for DTA:

- complete the automaton and then: (there exists a unique run for every word)
- interchange accept and non-accept states. (the complement is accepted).

Theorem: Given DTA $\mathcal{A}$, and NTA $\mathcal{B}$
- Checking if $\mathcal{L}(\mathcal{A})$ is universal is decidable
- Checking $\mathcal{L}(\mathcal{B}) \subseteq \mathcal{L}(\mathcal{A})$ is decidable.

Proof: 1) $[\mathcal{L}(\mathcal{A})]^c = \overline{\mathcal{A}}$ DTA. check emptiness of $\overline{\mathcal{A}}$.

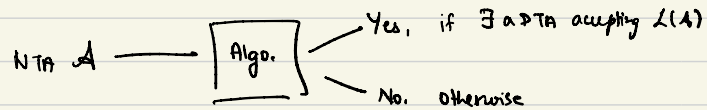
2) Check Emptiness of $\mathcal{L}(\mathcal{B}) \cap \mathcal{L}(\mathcal{A})^c$

Question: Is there a DTA for every NTA?

In a previous lecture, we have seen a timed regular language L whose complement is not timed regular.

- This language L cannot be accepted by a DTA.

Question: Given an NTA $\mathcal{A}$, decide whether there exists an equivalent DTA.



Undecidable [Finkel '06]

Summary:

- Deterministic TA., Examples
- Closure properties: union, intersection, complement
  ↓
  - decidable universality,
  - decidable inclusion $\mathcal{L}(B) \subseteq \mathcal{L}(A)$, when A is deterministic.
- Deciding whether a given NTA has an equivalent DTA is undecidable.