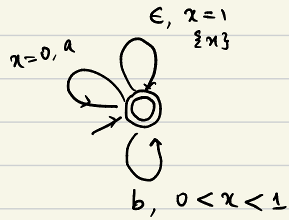
TIMED AUTOMATA

LECTURE 24

19/11/2020

Solutions to Tutorial 6:

1. $\{ (w, \tau) \mid w \in \Sigma^*, \forall i \leq |w| : \; w_i = a \Rightarrow \tau_i$ is integer, $w_i = b \Rightarrow \tau_i$ is not integer $\}$



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2. (a) Not a well quasi-order $ba^i \quad ba^j$

$b \quad ba \quad baa \quad baaa \quad \ldots$

(b) Yes, it is a well quasi-order since subword is a wqo.

$w_1 \quad w_2 \quad w_3 \quad \ldots \quad w_i \quad w_j$, $\sqsupseteq$

(c) No.

$x - y = 1 \quad x - y = 2 \quad x - y = 3 \quad \ldots$

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3. Problem is undecidable.

Firstly, note that $\mathcal{L}(A) \subseteq \mathcal{L}(A_g)$ since every run of A is also a run of A_g.

Hence: $\mathcal{L}(A) = \mathcal{L}(A_g)$ iff $\mathcal{L}(A_g) \subseteq \mathcal{L}(A)$.

We will reduce universality of T.A. to this problem.

- Suppose there is an algorithm to check if $\mathcal{L}(A_g) \subseteq \mathcal{L}(A)$.

Given automaton $\mathcal{B}$, we want to check if $T\Sigma^* \subseteq \mathcal{L}(\mathcal{B})$.

Repeat:
- Pick some guard g of $\mathcal{B}$
- If $\mathcal{L}(\mathcal{B}_g) \nsubseteq \mathcal{L}(\mathcal{B})$ then return "Not universal"
- Else // $\mathcal{L}(\mathcal{B}_g) = \mathcal{L}(\mathcal{B})$

  replace $\mathcal{B} := \mathcal{B}_g$

Until $\mathcal{B}$ has no guards

If $\mathcal{B}$ (the untimed automaton) is universal, report "Universal
Else report "Not universal".

Every timed extension of a $w \in \text{Untime}(\mathcal{L}(\mathcal{B}))$ is present in $\mathcal{L}(\mathcal{B})$