TIMED AUTOMATA

LECTURE 23

TODAY'S GOALS

- Expressiveness of Updatable T.A.

(Recall): **Updatable Timed automaton (UTA):**

Resets generalized to **updates**

Updates:

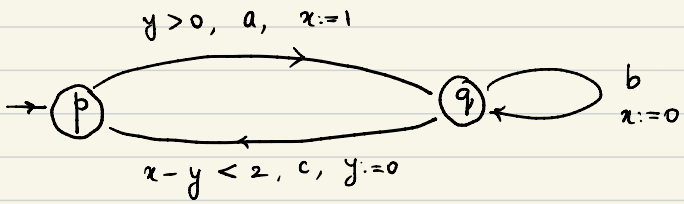
X: a set of clocks

For each clock $x \in X$, an **update on x** takes the following form:

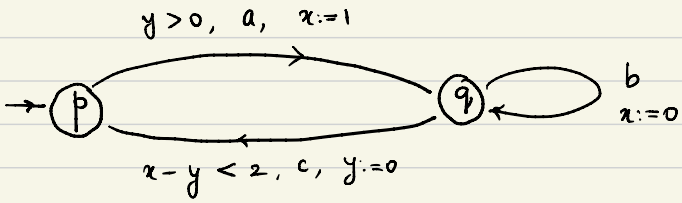
$$x := c \quad | \quad x := y + d \qquad c \in \mathbb{N},\ d \in \mathbb{Z},\ y \in X$$

Examples: $x := 5$, $x := x - 1$, $x := y + 2$, $x := y$, $x := 2 - 1$

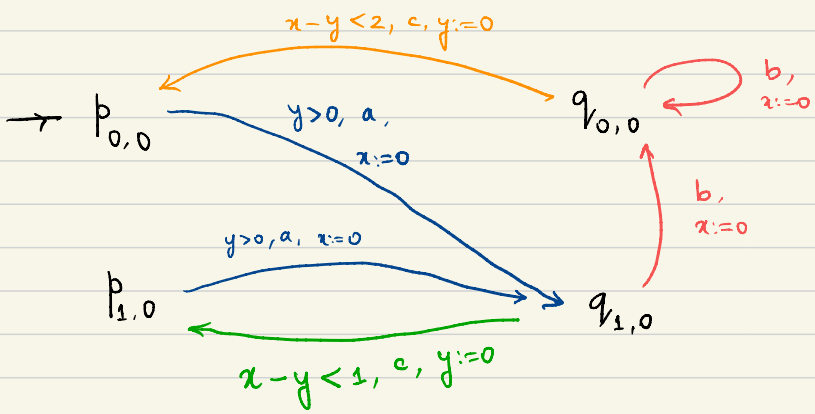
Question 1: Convert the following UTA into a TA:



Solution:



Idea: Remember the last update constant of each clock in the state.
Modify the guards depending on the appropriate constant in the state.



Question 2: Consider UTA restricted to updates of the form $x := c$ $(c \in \mathbb{N})$. Construct an equivalent timed automaton with only resets for this class of UTA.

Solution: Generalizing the previous construction:

Convert a UTA with $x := c$ updates to a TA.

UTA $A := (Q, Q_0, X, \Sigma, \Delta, F)$

We will construct a TA B.

States: Let $S_x := \{ c \mid x := c \text{ appears in some transition} \}$

$$S = \prod_{x \in X} S_x$$

$x = \{3, 5\}$

$y = \{4, 7, 8\}$

$S = (3,4)\ (3,7) \ldots$

States of B are of the form: $Q \times S$

Transitions:

For every $q \xrightarrow[up]{a,\ g} q'$ in A

we have a transition from every (q, σ)

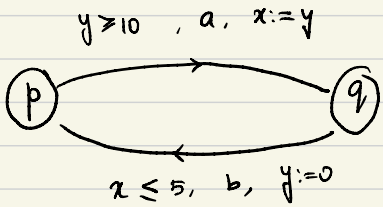
$$(q, \sigma) \xrightarrow[R_{up}]{a,\ g'} (q', \sigma')$$

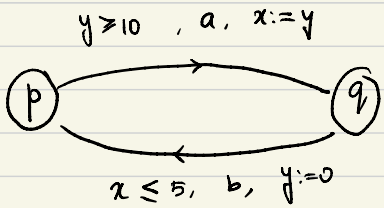
where $g' := g[x \to x + \sigma(x)]$ (replace x with $x + \sigma(x)$)

$R_{up} :=$ Set of clocks updated in up

$\sigma'(x) = c$ if $x := c \in up$, else $\sigma'(x) = \sigma(x)$.

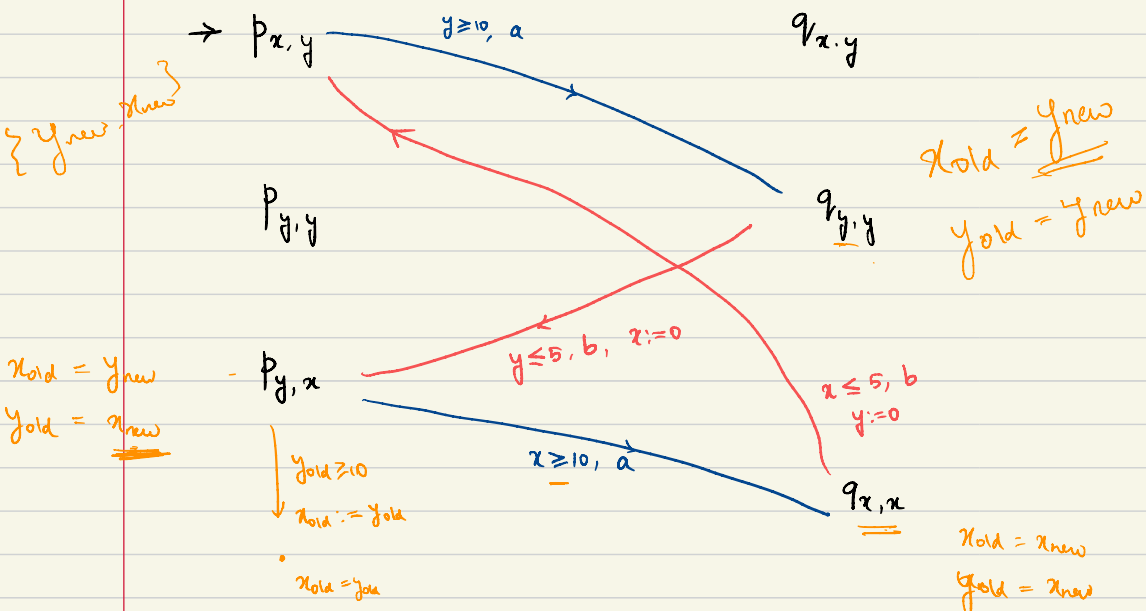
Question 3: Convert the following UTA into a TA:



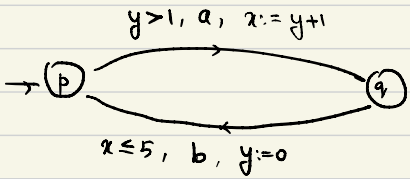


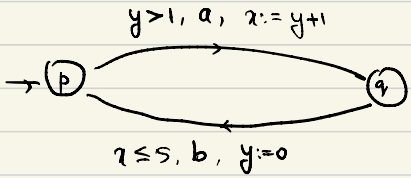
Idea: i) Maintain the last clock to which each clock was reset to.

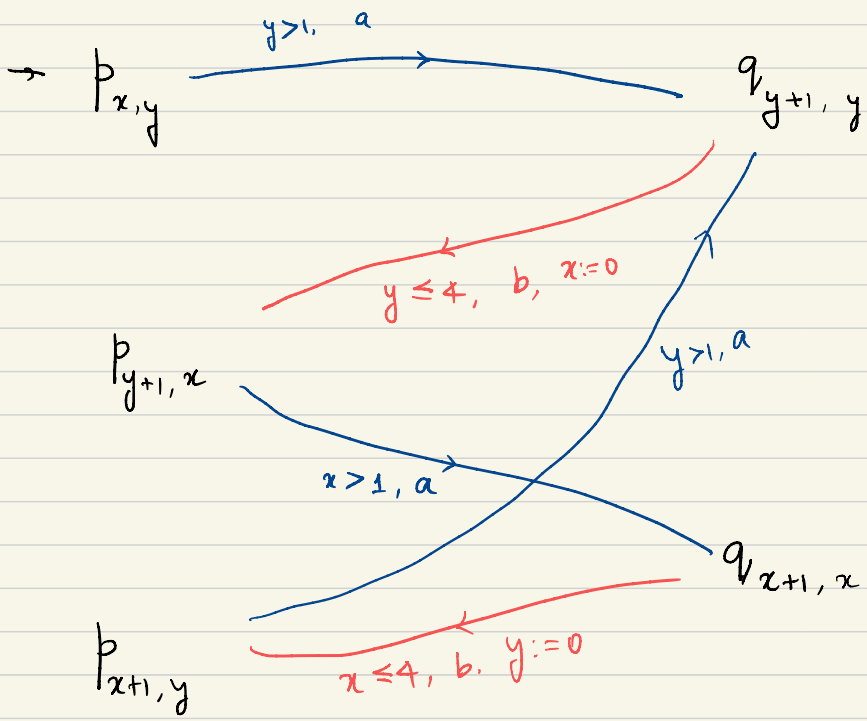
ii) If value of some clock is stored in some other clock, then that clock should not be updated to a constant.



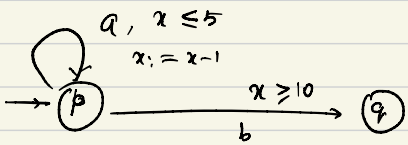
Question 4: Convert the following UTA into a TA:

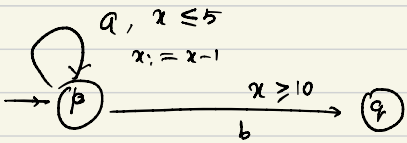



y>1, a, x:= y+1
p
q
x≤5, b, y:=0

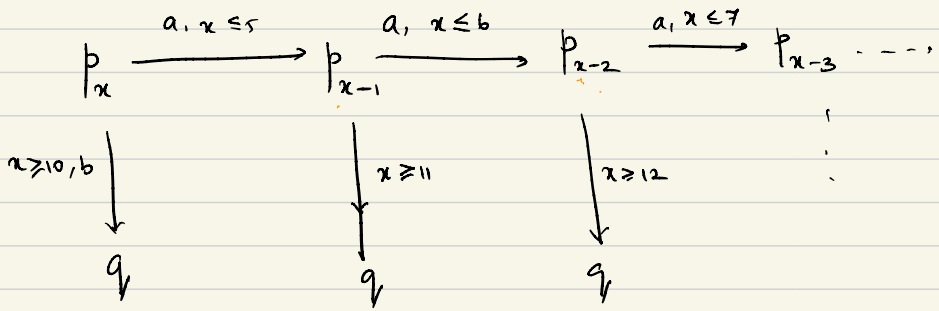

y>1, a
$p_{x,y}$
$q_{y+1,y}$
y≤4, b, x:=0
$p_{y+1,x}$
y>1, a
x>1, a
$q_{x+1,x}$
$p_{x+1,y}$
x≤4, b, y:=0

Question 5: Apply the construction of previous question to the following UTA:





$x_{old} := x_{old} - 1$
$= x_{new} - 1$



Remark 1: This construction does not work always.

The paper on Updatable Timed Automata by Bouyer et al. gives a termination criterion: it generates a system of inequalities; if the system has a solution, then the above construction gives finitely many states.

Remark 2: In our first lecture on UTA we have given an example of a timed language that is accepted by a UTA, but not by TA.

Summary:

	Diagonal-free	Diagonals
$x := c$, $x := y$	TA	TA
$x := y + c$	TA	More expressive
$x := x - 1$	More expressive	More expressive

$x := x+1$

$y - x$

$y - (x+1)$

$y - x - 1$