TIMED AUTOMATA

LECTURE 16

Tutorial 5 - Solutions:

1.

a) Λ = power set of $\{q_0, q_1, q_2\} \times \{r_0, r_{01}, r_1, r_{12}, r_2, r_{200}\}$

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b) $W = \{(q_1, r_0)\} \{(q_2, r_{01})\}$

$C_1 : \{(q_1, 0), (q_2, 0.5)\}$

$C_2 : \{(q_1, 0), (q_2, 0.6)\}$

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c) $C_1 \xrightarrow{1.3} \{(q_1, 1.3), (q_2, 1.8)\} \xrightarrow{a} \{(q_1, 1.3), (q_2, 0), (q_2, 1.8)\}$

$C_2 \xrightarrow{1.3} \{(q_1, 1.3), (q_2, 1.9)\} \xrightarrow{a} \{(q_1, 1.3), (q_2, 0), (q_2, 1.9)\}$

$1 < \delta_2 < 1.4$

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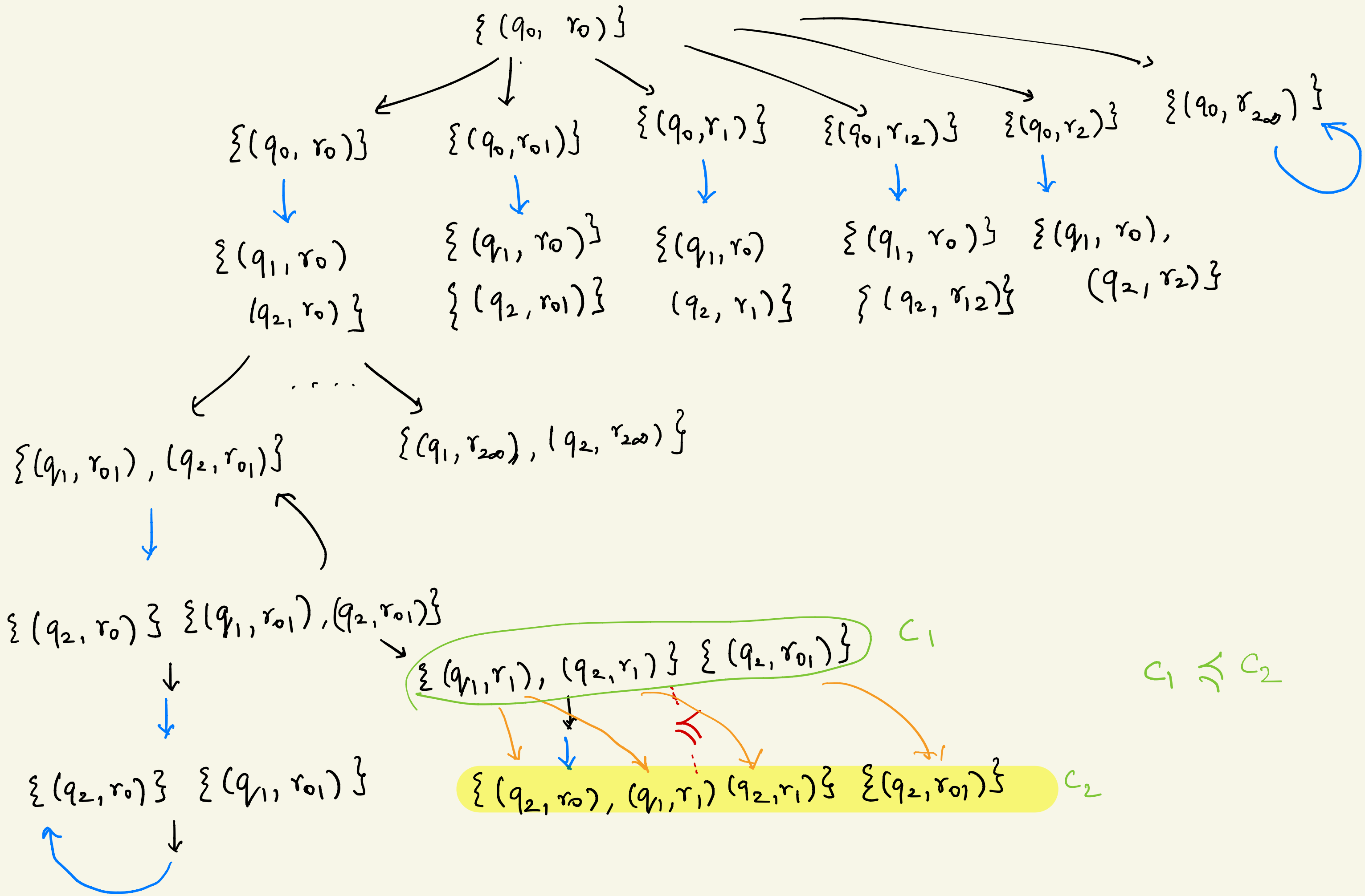
1. (d)


$\{(q_0, r_0)\}$
$\{(q_0, r_0)\}$
$\{(q_0, r_{01})\}$
$\{(q_0, r_1)\}$
$\{(q_0, r_{12})\}$
$\{(q_0, r_2)\}$
$\{(q_0, r_{200})\}$
$\{(q_1, r_0)\ (q_2, r_0)\}$
$\{(q_1, r_0)\}$ $\{(q_2, r_{01})\}$
$\{(q_1, r_0)\ (q_2, r_1)\}$
$\{(q_1, r_0)\}$ $\{(q_2, r_{12})\}$
$\{(q_1, r_0), (q_2, r_2)\}$
$\{(q_1, r_{01}), (q_2, r_{01})\}$
$\{(q_1, r_{200}), (q_2, r_{200})\}$
$\{(q_2, r_0)\}$ $\{(q_1, r_{01}), (q_2, r_{01})\}$
$\{(q_1, r_1), (q_2, r_1)\}$ $\{(q_2, r_{01})\}$
C_1
$C_1 \preccurlyeq C_2$
$\{(q_2, r_0)\}$ $\{(q_1, r_{01})\}$
$\{(q_2, r_0), (q_1, r_1)\ (q_2, r_1)\}$ $\{(q_2, r_{01})\}$
C_2

2. To find: does there exist a word $w: (\delta_1, a_1)(\delta_2, a_2) \dots (\delta_n, a_n)$

s.t: 1) $\mathcal{B}$ has an accepting run on w

and

2) every run of A on w is non-accepting

We know how to check (2) since A is a one-clock T.A.

Idea is to augment the configurations used in the universality algorithm with a state of automaton B.

Configuration: $(q^B, y_1^B, y_2^B, \dots y_n^B) \cup \{(q_1^A, x_1^A), (q_2^A, x_2^A) \dots (q_n^A, x_n^A)\}$

a state of B union a set of states of A.

Idea:

Time! $\{(q_B, 1.7, 2.2)\ (q_1^A, 0.7)\ (q_2^A, 2.5)\} \xrightarrow{0.4} \{(q_B, 2.1, 2.6)\ (q_1^A, 1.1), (q_2^A, 2.9)\}$

Successors: one successor for B and all successors of A

Discrete

Encoding:

Think of $(q^B, y_1^B, y_2^B, \dots, y_n^B)$

as $\{(q^B, y_1^B), (q^B, y_2^B), \dots (q^B, y_n^B)\}$

$y_1^B : r_0$

and interleave these states into the configuration of A.

- Encode as usual.

Bad node: Configuration where q^B is accepting, but every state from A is non-accepting.

$$\{(q_B, \overset{y_1}{1.7}, \overset{y_2}{2.2})\ (q_1^A, 0.7)\ (q_2^A, 2.5)\}$$

$$(q_B, y_1, r_{12}, 0.7)\ (q_B, y_2, r_{23}, 0.2)\ (q_1^A, r_{12}, 0.7)\ (q_2^A, r_{23}, 0.5)$$

$$\{(q_B, y_2, r_{23})\}\ \{(q_2^A, r_{23})\}\ \{(q_B, y_1, r_{12}), (q_1^A, r_{12})\}.$$

3. $\mathcal{A}$: an NTA where all guards are of the form $x=0$, $x>0$.

Claim: There is an equivalent DTA for $\mathcal{A}$.

$\mathcal{A}$: Multiple clocks.

$$\xrightarrow{x=0 \wedge y=0 \wedge z>0}$$

For every clock, we need to track whether it is 0 or >0.

$\rightarrow$ One bit of information.

States: $(q, \underline{01101})$ bit vector of length $|X|$ (no. of clocks).

$$(q, 011) \xrightarrow{0,\, a} (q', 010)$$

$$(q, 011) \xrightarrow{1,\, a} (q', 110)$$

$$q \xrightarrow[\{z\}]{a,\, y>0} q'$$

From every state there are two kinds of transitions: $\xrightarrow{0,\, a}$ $\xrightarrow{1,\, a}$

For every transition $q \xrightarrow[R]{a,\, g} q_1$ in $\mathcal{A}$

there is a transition: $(q, \bar{b}) \xrightarrow{0,\, a} (q_1, \bar{b}_1)$

if $\bar{b}$ satisfies g. & $\bar{b}_1(z) = 0 \quad z \in R$

$\bar{b}_1(z) = \bar{b}(z) \quad z \notin R$

$$(q, \bar{b}) \xrightarrow{1, a} (q_1, \bar{b}_1)$$

if $\boxed{111111\ldots1}$ satisfies g and

$$\bar{b}_1(z) = 0 \quad \text{if } z \in R$$
$$= 1 \quad \text{otherwise}$$

The above construction gives a one-clock timed automaton.

Assume that there is a clock z.

- Reset z in every transition.
- $(q, \bar{b}) \xrightarrow{0, a} (q_1, \bar{b}_1)$: $(q, \bar{b}) \xrightarrow[\{z\}]{z=0,\ a} (q_1, \bar{b}_1)$

  $(q, \bar{b}) \xrightarrow{1, a} (q_1, \bar{b}_1)$: $(q, \bar{b}) \xrightarrow[\{z\}]{z>0,\ a} (q_1, \bar{b}_1)$

- This already shows that universality is decidable for such automata.

Further: one can show that a subset construction works for the above one-clock T.A. where z is reset everywhere.