Automata for Real-time Systems

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Lecture 2: Closure properties of timed languages

Timed regular languages



Definition

A timed language is called **timed regular** if it can be **accepted** by a timed automaton



 $A = (Q, \Sigma, X, T, Q_0, F)$ $A' = (Q', \Sigma, X', T', Q'_0, F')$

 $A_{\cup} = (Q \cup Q', \Sigma, X \cup X', T \cup T', Q_0 \cup Q'_0, F \cup F')$ $\mathcal{L}(A) \cup \mathcal{L}(A') = \mathcal{L}(A_{\cup})$

Timed regular languages are closed under union



 $A = (Q, \Sigma, X, T, Q_0, F) \qquad A' = (Q', \Sigma, X', T', Q'_0, F')$ $A_{\cap} = (Q \times Q', \Sigma, X \cup X', T_{\cap}, Q_0 \times Q'_0, F \times F')$ $T_{\cap} : \quad (q_1, q'_1) \xrightarrow[R \cup R']{} (q_2, q'_2) \text{ if }$ $q_1 \xrightarrow[R \cup R']{} q_2 \in T \text{ and } q'_1 \xrightarrow[R']{} q'_2 \in T'$

Timed regular languages are closed under intersection

L: a timed language over Σ

Untime(L) $\equiv \{w \in \Sigma^* \mid \exists \tau. (w, \tau) \in L\}$

Untiming construction

For every timed automaton A there is a finite automaton A_u s.t.

Untime($\mathcal{L}(A)$) = $\mathcal{L}(A_u)$

more about this later . . .

Complementation

 $\Sigma : \{a, b\}$

 $L = \{ (w, \tau) \mid \text{ there is an } a \text{ at some time } t \text{ and} \\ \text{no action occurs at time } t + 1 \}$

$$\overline{L} = \{ (w, \tau) \mid \text{ every } a \text{ has an action at} a \text{ distance 1 from it } \}$$

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$$\overline{L} = \{ (w, \tau) \mid \text{ every } a \text{ has an action at} \\ a \text{ distance 1 from it } \}$$

Claim: No timed automaton can accept \overline{L}

Decision problems for timed automata: A survey

Alur, Madhusudhan. SFM'04: RT

Suppose \overline{L} is timed regular

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Step 2: Let $L' = \{ (a^*b^*, \tau) \mid all a's occur before time 1 and no two a's happen at same time \}$

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Step 4: But, Untime($\overline{L} \cap L'$) = { $a^n b^m | m \ge n$ }, not regular!

Therefore \overline{L} cannot be timed regular \Box



Timed regular languages are not closed under complementation

Timed automata

Runs

 $1 \operatorname{clock} < 2 \operatorname{clocks} < \dots$

Role of max constant

Timed regular lngs.

Closure under \cup , \cap

Non-closure under complement



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ε -transitions



Claim: No timed automaton can accept L_6





 $x = 1, \varepsilon, \{x\}$ $\xrightarrow{q_0}$ $x = 1, a, \{x\}$

ε -transitions

 ε -transitions add expressive power to timed automata.

Characterization of the expressive power of silent transitions in timed automata

Bérard, Diekert, Gastin, Petit. Fundamenta Informaticae'98

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More expressive

 $\xrightarrow{\varepsilon}$ without reset \equiv TA