# Automata for Real-time Systems

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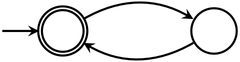
# Overview

Automata (*Finite State Machines*) are **good abstractions** of many real systems

hardware circuits, communication protocols, biological processes, . . .

# Automata can model many **properties** of systems




every request is followed by a response

System ↓ Automaton $\mathcal{A}$

Property ↓ Automaton $\mathcal{B}$

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Property → Automaton $\mathcal{B}$

Does system **satisfy** property?

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$$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})?$$

Does system **satisfy** property?

# Model-checking

System $\downarrow$ Automaton $\mathcal{A}$

Property $\downarrow$ Automaton $\mathcal{B}$

$$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})?$$

Does system **satisfy** property?

# In practice...

Huge system

Property

# In practice...

Huge system
↓
Higher-level description

Property
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Higher-level description

# In practice...

Huge system → Higher-level description → (translation) → Automaton $\mathcal{A}$

Property → Higher-level description → (translation) → Automaton $\mathcal{B}$

**Model-Checker**

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$$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})?$$

Some model-checkers: SMV, NuSMV, SPIN, ...

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Huge system → Higher-level description → (translation) → Automaton $\mathcal{A}$

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**Model-Checker**

$$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})?$$

Some model-checkers: SMV, NuSMV, SPIN, . . .

**Turing Awards:** Clarke, Emerson, Sifakis and Pnueli

Automata are **good abstractions** of many real systems

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Our course: Automata for **real-time** systems


*Picture credits: F. Herbreteau*

pacemaker, vehicle control systems, air traffic controllers, . . .

# Timed Automata

R. Alur and D. Dill in early 90s

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Some model-checkers: UPPAAL, KRONOS, RED, . . .

# Goals of our course

Study **language theoretic** and **algorithmic** properties of timed automata

## Lecture 1:

# Timed languages and timed automata

$\Sigma$ : alphabet $\{a, b\}$

$\Sigma^*$ : words $\{\varepsilon, a, b, aa, ab, ba, bb, aab, \dots\}$

$L \subseteq \Sigma^*$ : language $\longrightarrow$ *property over words*

$L_1 :=$ {set of words starting with an " $a$ "}

$\{a, aa, ab, aaa, aab, \dots\}$

$L_2 :=$ {set of words with a non-zero even length }

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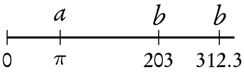
**Finite automata, pushdown automata, Turing machines, ...**

$\Sigma$ : alphabet $\{a, b\}$

$T\Sigma^*$ : timed words


$(aa;\ 0.8, 2.5)$




$(abb;\ \pi, 203, 312.3)$

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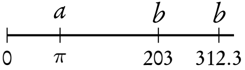



$(abb;\ \pi, 203, 312.3)$

$(w, \tau)$

Word

Time sequence

$w = a_1 \dots a_n$

$a_i \in \Sigma$

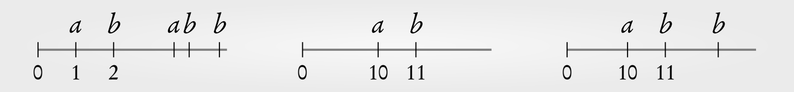
$\tau = \tau_1 \dots \tau_n$

$\tau_i \in \mathbb{R}_{\geq 0}$

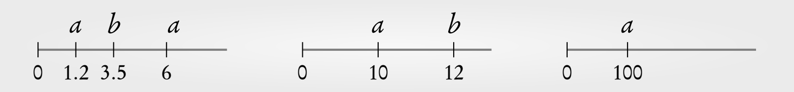
$\tau_1 \leq \cdots \leq \tau_n$

$L \subseteq T\Sigma^*$ : Timed language $\longrightarrow$ *property over timed words*

$$L_1 := \{(\, ab(a+b)^*, \tau\,) \mid \tau_2 - \tau_1 = 1\}$$




$$L_2 := \{\, (w, \tau) \mid \tau_{i+1} - \tau_i \geq 2 \text{ for all } i < |w|\}$$



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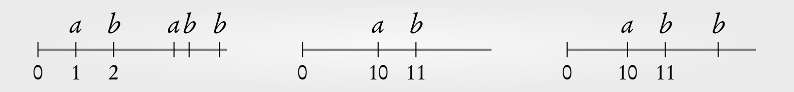


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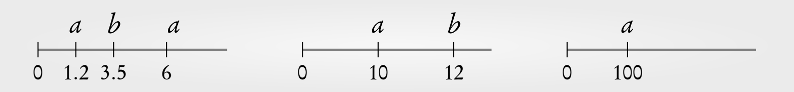
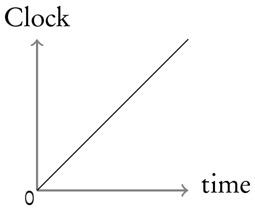



**Timed automata**

Timed automaton: Finite automaton + Finite no. of *Clocks*



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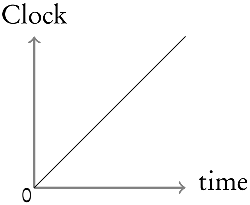
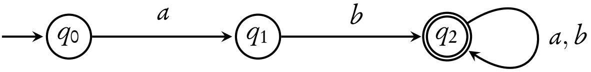



$$\{(\, ab(a+b)^*, \tau) \mid \tau_2 \leq 2\}$$



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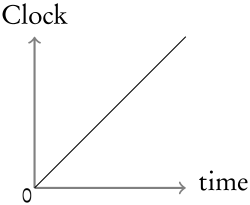


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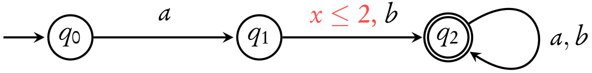

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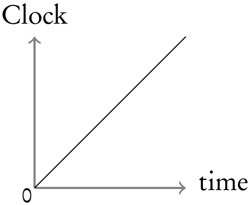


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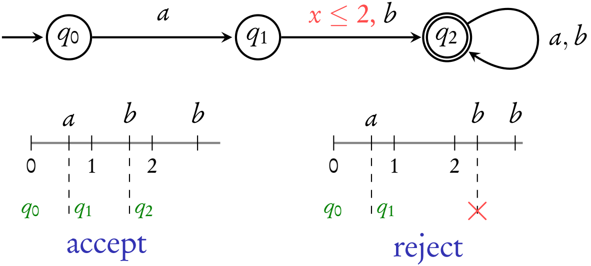

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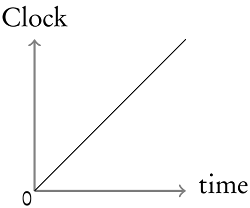


**Guards**

$$\phi := x \leq c \mid x \geq c \mid \neg\phi \mid \phi \wedge \phi$$

$$x \in \mathit{Clocks}\ ,\ \ c \in \mathbb{Q}_{\geq 0}$$

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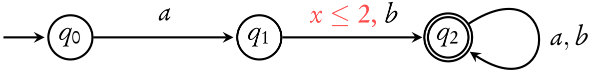



accept

reject

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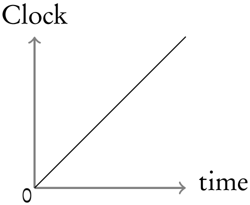


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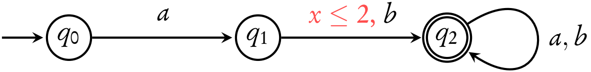

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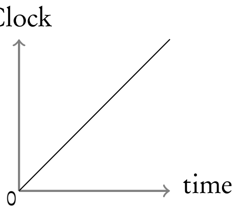


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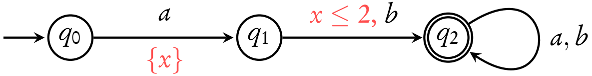

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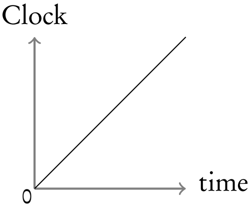


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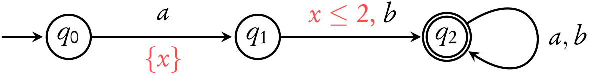
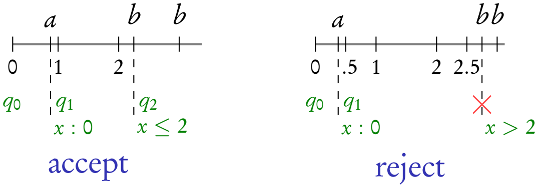
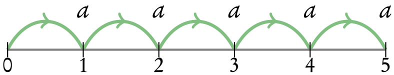


$$L_3 := \{\ (\ a^k, \tau\ ) \mid k > 0,\ \tau_i = i \ \text{ for all } i \leq k\}$$

An "*a*" occurs in every integer from $1, \ldots, k$



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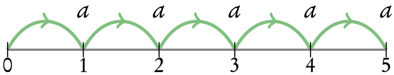
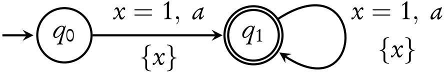
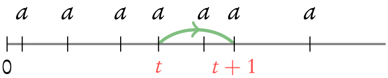
An “$a$” occurs in every integer from $1, \ldots, k$



$$L_4 := \{ \ ( \ a^k, \tau \ ) \mid \text{exist } i, j \text{ s.t. } \tau_j - \tau_i = 1 \}$$

There are 2 "$a$"s which are at distance 1 apart



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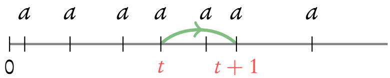
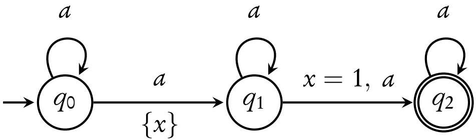
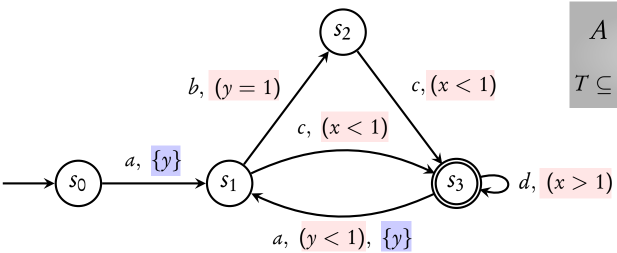


Three **mechanisms** to exploit:

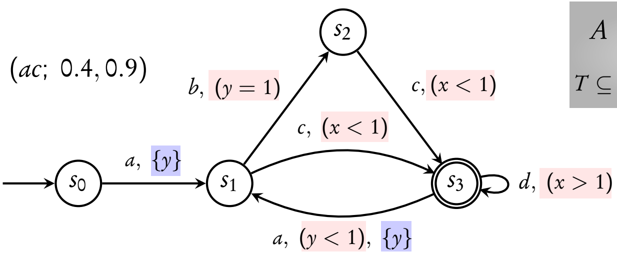
- Reset: to **start** measuring time
- Guard: to **impose** time constraint on action
- Non-determinism: for **existential** time constraints




$$A = (Q, \Sigma, X, T, Q_0, F)$$

$$T \subseteq Q \times \Sigma \times \text{guard} \times \text{reset} \times Q$$

$(ac;\ 0.4, 0.9)$

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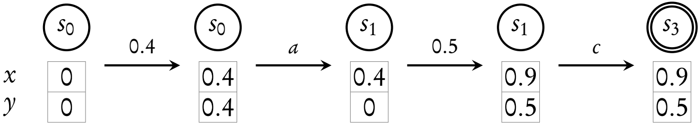

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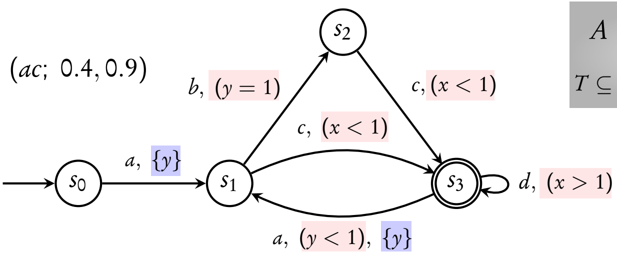


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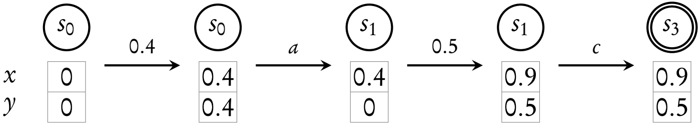


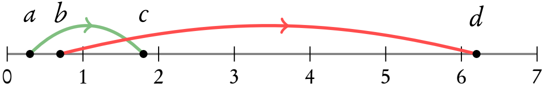

**Run** of $A$ over $(a_1 a_2 \ldots a_k;\ \tau_1 \tau_2 \ldots \tau_k)$ $\qquad \delta_i := \tau_i - \tau_{i-1};\ \tau_0 := 0$

$$(q_0, v_0) \xrightarrow{\delta_1} (q_0, v_0 + \delta_1) \xrightarrow{a_1} (q_1, v_1) \xrightarrow{\delta_2} (q_1, v_1 + \delta_2) \cdots \xrightarrow{a_k} (q_k, v_k)$$

$(w, \tau) \in \mathcal{L}(A)$ if $A$ has an **accepting** run over $(w, \tau)$

$$L_5 := \{ \, ( \, abcd.\Sigma^*, \tau \, ) \mid \tau_3 - \tau_1 \leq 2 \text{ and } \tau_4 - \tau_2 \geq 5 \}$$

Interleaving distances



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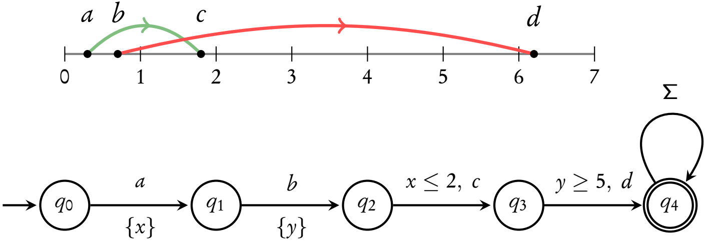


$n$ interleavings $\Rightarrow$ need $n$ clocks

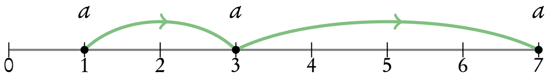
$n + 1$ clocks more expressive than $n$ clocks

# Timed automata

Runs

1 clock $<$ 2 clocks $< \ldots$

$$L_6 := \{\ (\ a^k, \tau\ )\ |\ \tau_i \text{ is some integer for each } i\}$$



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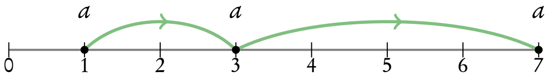


Claim: **No timed automaton** can accept $L_6$

## Step 1: *Suppose* $L_6 = \mathcal{L}(A)$

Let $c_{max}$ be the maximum constant appearing in a guard of $A$

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$$x = \lceil c_{max} \rceil + 1 \text{ and } x = \lceil c_{max} \rceil + 1.1$$

satisfy the same guards

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Step 3: $(a; \lceil c_{max} \rceil + 1) \in L_6$ and so $A$ has an accepting run

$$(q_0, v_0) \xrightarrow{\delta = \lceil c_{max} \rceil + 1} (q_0, v_0 + \delta) \xrightarrow{a} (q_F, v_F)$$

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Step 4: By Step 2, the following is an accepting run

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Hence $(a;\ \lceil c_{max} \rceil + 1.1) \in \mathcal{L}(A) \neq L_6$

Therefore **no timed automaton** can accept $L_6$ □

# Timed automata

Runs

1 clock $<$ 2 clocks $< \dots$

Role of max constant