Unit-5: ω -regular properties

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Module 2: ω-regular expressions

Languages over finite words Σ : finite alphabet $\Sigma^* = \text{ set of all words over } \Sigma$ Language: A set of finite words $\{ab, abab, ababab, \ldots\}$ finite words starting with an a finite words starting with a b $\{\epsilon, b, bb, bbb, \ldots\}$ $\{\epsilon, ab, abab, ababab, \ldots\}$ $\{\epsilon, bbb, bbbbbb, (bbb)^3, \ldots\}$ words starting and ending with an a $\{\epsilon, ab, aabb, aaabbb, a^4b^4 \dots\}$

$$\label{eq:setofallwords} \begin{split} \Sigma : \text{finite alphabet} \quad \Sigma^* \ = \ \text{set of all words over } \Sigma \\ & \text{Language: A set of finite words} \end{split}$$

- $ab(ab)^* \{ab, abab, ababab, \ldots\}$
- $a\Sigma^*$ finite words starting with an *a*
- $b\Sigma^*$ finite words starting with a b

 $b^* \quad \{\epsilon, b, bb, bbb, \ldots\}$

- $(ab)^* \{\epsilon, ab, abab, ababab, \ldots\}$
- $(bbb)^* \{\epsilon, bbb, bbbbbb, (bbb)^3, \ldots\}$
- $a\Sigma^*a$ words starting and ending with an a

 $\{\epsilon, ab, aabb, aaabbb, a^4b^4 \dots\}$

Alphabet $\Sigma = \{a, b\}$

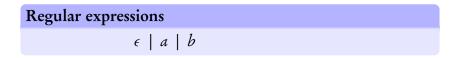
$$\Sigma \cdot \Sigma = \{a, b\} \cdot \{a, b\}$$
$$= \{aa, ab, ba, bb\}$$

 $\Sigma^0 = \{ \epsilon \}$ (empty word, with length 0) Σ^1 = words of length 1 Σ^2 = words of length 2 Σ^3 = words of length 3 : $\Sigma^k =$ words of length k: $\Sigma^* = \bigcup_{i>0} \Sigma^i$ = set of all finite length words

$$aba \cdot \epsilon = aba$$

 $\epsilon \cdot bbb = bbb$
 $w \cdot \epsilon = w$
 $\epsilon \cdot w = w$

 ϵ



$$\epsilon \mid a \mid b \mid r_1 r_2$$

$$\epsilon \mid a \mid b \mid r_1 r_2 \mid r_1 + r_2$$

$$\epsilon \mid a \mid b \mid r_1 r_2 \mid r_1 + r_2 \mid r^*$$

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where r_1, r_2, r are regular expressions themselves

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$$a^* + b^*$$

 $ab + bb + baa$
 $(a + b)^*ab(ba + bb)$
 $(ab + bb)^*$

:

Theorem

- 1. Every regular expression can be converted to an NFA accepting the language of the expression
- 2. Every NFA can be converted to a regular expression describing the language of the NFA

Coming next: Languages over infinite words

$$\Sigma = \{a, b\}$$

$$\Sigma = \{ a, b \}$$

{ *aaaaaaaaaaaaaaaa* ... }

$$\Sigma = \{a, b\}$$

{ aaaaaaaaaaaaaa . . . }

Example 2: Infinite words containing only *a* or only *b*

$$\Sigma = \{a, b\}$$

{ aaaaaaaaaaaaaa . . . }

Example 3: a word in $aa\Sigma^*aa$ followed by only *b*-s { $aaaabbbbbbbb..., aababaabbbbbbb..., aabbbbaabbbbbbbb..., ... }$

$$\Sigma = \{a, b\}$$

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$$\Sigma = \{a, b\}$$

Example 5: Infinite words where *b* occurs infinitely often { *abababababab...*, *bbbabbbabbbabbbabbab...*, *...* }

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Example 3: a word in $aa\Sigma^*aa$ followed by only *b*-s $aa\Sigma^*aa \cdot b^{\omega}$ { $aaaabbbbbbbb \dots$, $aababaabbbbbbb \dots$, $aabbbbaabbbbbbb \dots$, \dots }

Example 5: Infinite words where *b* occurs infinitely often
{ abababababab..., bbbabbbabbbabbbabbba..., ... }

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ω -regular expressions

$$G = E_1 \cdot F_1^{\omega} + E_2 \cdot F_2^{\omega} + \cdots + E_n \cdot F_n^{\omega}$$

$$E_1, \ldots, E_n, F_1, \ldots, F_n$$
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$$L(F^{\omega}) = \{ w_1 w_2 w_3 \dots \mid \text{each } w_i \in L(F) \}$$

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- $((a+b)^*c)^{\omega}$ words where *c* occurs infinitely often

$$\mathbf{AP} = \{ p_1, p_2, \dots, p_k \}$$

$$\Sigma = PowerSet(\mathbf{AP}) = \{ \{ \}, \{ p_1 \}, \dots, \{ p_k \}, \{ p_1, p_2 \}, \{ p_1, p_3 \}, \dots, \{ p_{k-1}, p_k \}, \dots, \{ p_{k-1}, p_k \}, \dots, \{ p_1, p_2, \dots, p_k \} \}$$

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The property is ω -regular if it can be described by an ω -regular expression

$$\begin{split} \mathbf{AP} &= \{ \text{ wait, crit } \} \\ \Sigma &= \textit{PowerSet}(\mathbf{AP}) = \{ \{ \}, \{ \text{wait} \}, \{ \text{crit} \}, \{ \text{wait, crit} \} \} \end{split}$$

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Property: Process enters critical section infinitely often

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Property: Process enters critical section infinitely often $((\{\}+\{wait\})^*(\{crit\}+\{wait, crit\}))^{\omega}$ ω -regular properties

 ω -regular expressions

Next goal: Find algorithms to model-check ω -regular properties