Unit-3: Linear-time properties

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Chennai Mathematical Institute

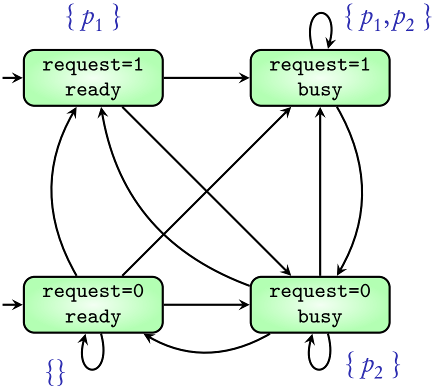
NPTEL-course

July - November 2015

Module 3:
Invariants

Atomic propositions AP $= \{ p_1, p_2 \}$

p_1: `request=1` $\quad$ p_2: `status=busy`



Atomic propositions $\mathrm{AP} = \{ p_1, p_2, p_3, p_4 \}$

p_1: `pr1.location=crit` p_2: `pr1.location=wait`

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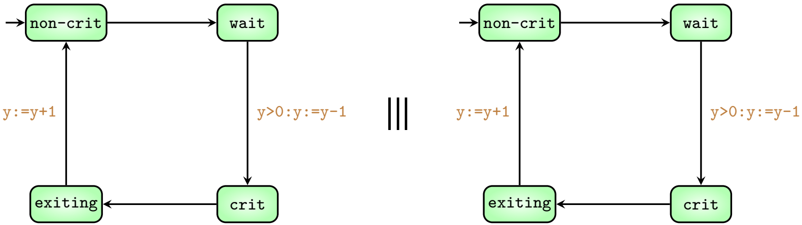

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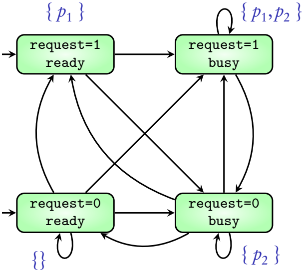

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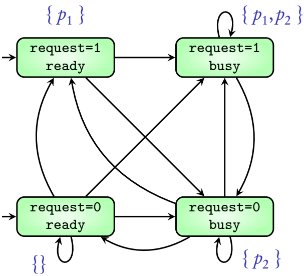

AP-INF = set of **infinite words** over *PowerSet*(**AP**)

Property 1: p_1 is always true

$\{ A_0 A_1 A_2 \cdots \in$ AP-INF $\mid$ **each** A_i contains $p_1 \}$

$\{p_1\}\{p_1\}\{p_1\}\{p_1\}\{p_1\}\{p_1\}\{p_1\}\ldots$

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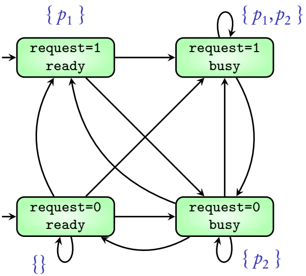

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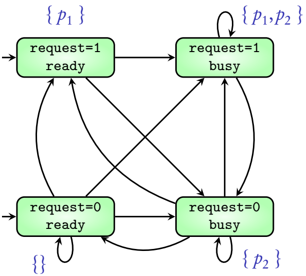

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Above TS **does not satisfy** G p_1

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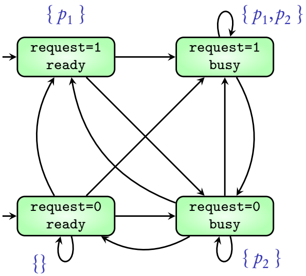

AP-INF = set of **infinite words** over *PowerSet*(**AP**)

Property 2: $p_1 \wedge \neg p_2$ is always true

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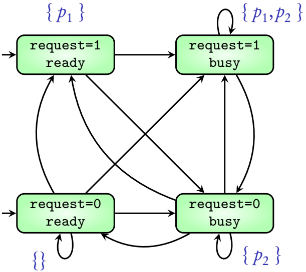

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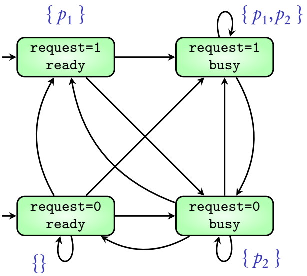

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Above TS **does not satisfy** $\mathrm{G}\ p_1 \wedge \neg p_2$

Invariants

AP-INF = set of **infinite words** over *PowerSet*(**AP**)

Property: ϕ is **always true**

(where ϕ is a boolean expression over AP)

$\{ A_0 A_1 A_2 \cdots \in$ AP-INF | **each** A_i satisfies ϕ $\}$

Invariants

AP-INF $=$ set of **infinite words** over *PowerSet*(**AP**)

Property: ϕ is **always true**
(where ϕ is a boolean expression over AP)

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A property of the above form is called **invariant** property

It is written as G ϕ

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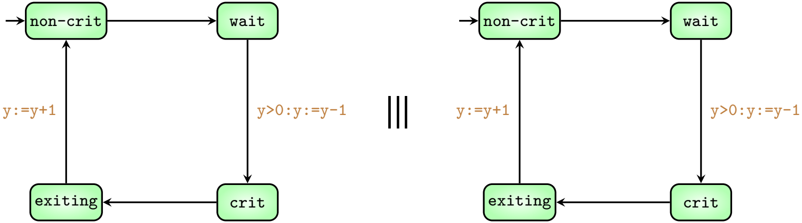


Above TS satisfies invariant property $\mathbf{G}\ \neg (p_1 \wedge p_3)$

Algorithm

Input: A TS and property $\mathrm{G}\ \phi$

Output: Does TS satisfy invariant $\mathrm{G}\ \phi$?

Algorithm

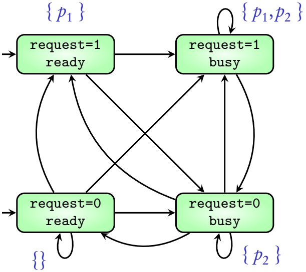
Input: A TS and property G ϕ

Output: Does TS satisfy invariant G ϕ?

A TS satisfies an invariant ϕ

if and only if

every **reachable state** of the TS satisfies ϕ



Property to check: $\mathbf{G}\, p_1$

set R, **stack** U, **bool** b
for all initial states s
  if $s \notin R$ **then**
    visit(s)
  endif
return b

procedure visit (states s)
  $push(s, U)$; $R := R \cup \{ s \}$
  while (U is not empty)
    $s' := top(U)$
    if $Post(s') \subseteq R$ **then**
      $pop(U)$
      $b := b \wedge (s' \models \phi)$
    else
      let $s'' \in Post(s') \setminus R$
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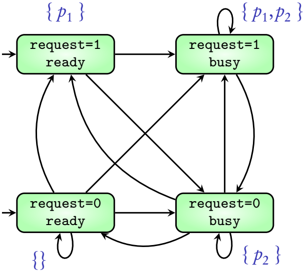
  endwhile

1

R U b



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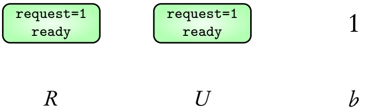

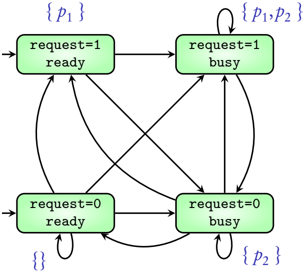

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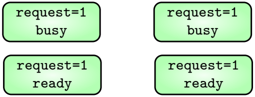

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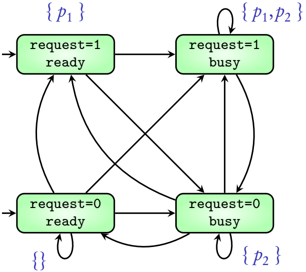

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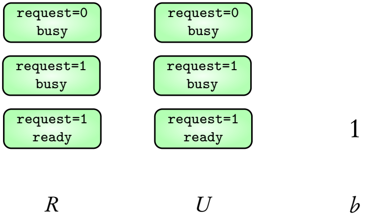

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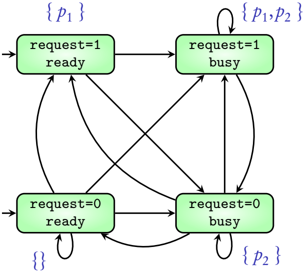

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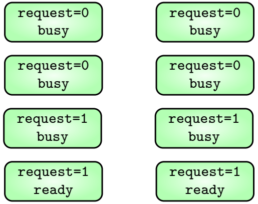

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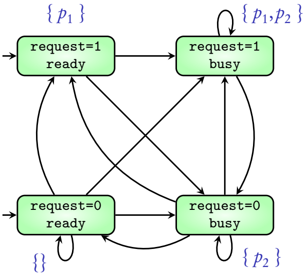

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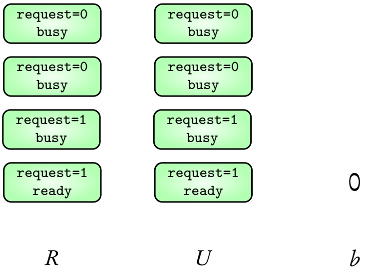

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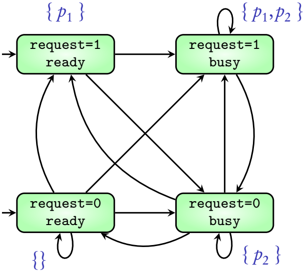

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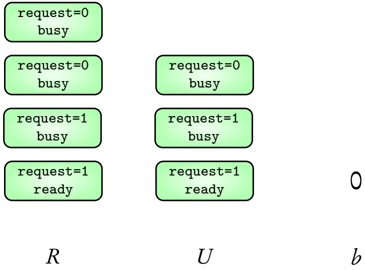


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                let s″ ∈ Post(s′) \ R
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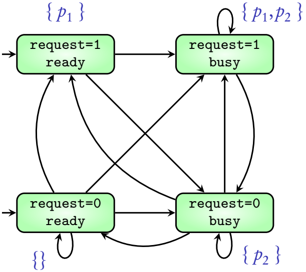

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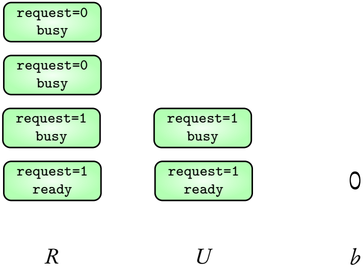

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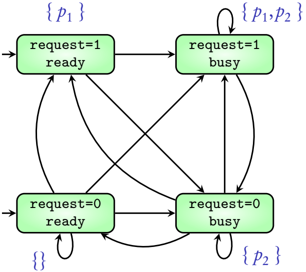

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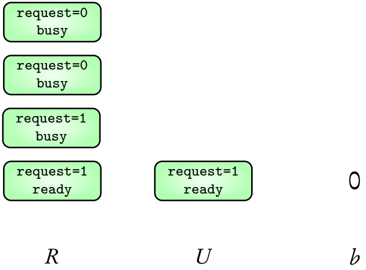

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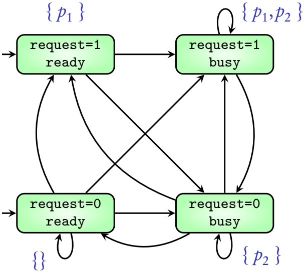

Property to check: G p_1

request=0 busy

request=0 busy

request=1 busy

request=1 ready

0

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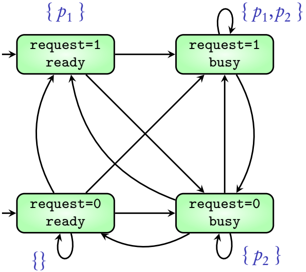

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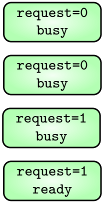
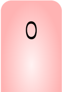


Property **not satisfied**



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Invariants

G ϕ

Algorithm to check invariants