## Unit-10: Algorithms for CTL

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## Module 2: <br> EX, EU and EG

## CTL model-checking problem

Given transition system $M$ and a CTL formula $\phi$, find all states of $M$ that satisfy $\phi$

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In this unit: Special case when $\phi$ is either E X, E U or E G

## Part 1:

## Algorithm for E X

$\operatorname{EX}\left(p_{1} \wedge p_{2}\right)$

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$\operatorname{EX}\left(p_{1} \wedge p_{2}\right)$


EX $\left(p_{1} \wedge \neg p_{2}\right)$


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$\mathbf{E X}\left(p_{1} \wedge \neg p_{2}\right)$

## EX $\left(p_{1} \wedge \neg p_{2}\right)$



## Algorithm for EX $\phi$



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Suppose states satisfying $\phi$ have been labelled


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State $s$ is labelled with $\mathbf{E X} \phi$ if there exists a successor which is labelled $\phi$

## Part 2:

## Algorithm for E U

## $\mathrm{E}\left(p_{1} \mathrm{U} p_{2}\right)$



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$$
\mathbf{E}\left(\neg p_{1} \mathbf{U} \neg p_{2}\right)
$$



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$\mathrm{E}\left(\neg p_{1} \mathrm{U} \neg p_{2}\right)$


## Algorithm for $\mathrm{E}\left(\phi_{1} \mathrm{U} \phi_{2}\right)$



- If any state is labelled with $\phi_{2}$, label it with $\mathrm{E}\left(\phi_{1} \mathrm{U} \phi_{2}\right)$
- Repeat:

Label any state with $\mathrm{E}\left(\phi_{1} \mathrm{U} \phi_{2}\right)$ if it is labelled with $\phi_{1}$ and at least one successor is labelled with $\mathrm{E}\left(\phi_{1} \mathrm{U} \phi_{2}\right)$ until no change

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## Part 3:

## Algorithm for E G

## E G $p_{1}$



## E G $p_{1}$



## E G $p_{1}$



## E G $p_{1}$



## E G $p_{1}$



## E G $p_{1}$

E G $p_{1}$


## E G $p_{1}$



## E G $p_{1}$



No state of the above transition system satisfies E G $p_{1}$

E G $p_{1}$


## E G $p_{1}$



## E G $p_{1}$



## E G $p_{1}$



E G $p_{1}$

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## Summary

## Algorithms

EX, EU, EG

