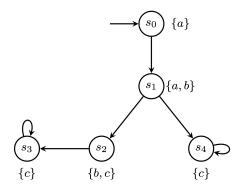
- 1. Let the set of atomic propositions be  $\{a, b, c\}$ .
  - (a) Rewrite the CTL formula A [a U (AF c)] in existential normal form (that is, using only EX, EU and EG).
  - (b) Which states of the transition system below satisfy the formula EFAG c?



2. (a) Let TS be a transition system, and let TS' be a transition system obtained by removing some state of TS and its associated transitions. Assume that TS' has at least one state, and there are no terminal states.

Show that if TS satisfies an LTL property  $\phi$ , then TS' satisfies  $\phi$ .

- (b) Use the above observation to show that there is no equivalent LTL formula for the CTL property EFAGp.
- 3. The F operator in LTL is used to say that a property is true sometime in the *future*. Let us now introduce the O operator (short form for Once) to say that a property was true sometime in the past.

The formal semantics of  $\boldsymbol{O}$  can be defined as follows. For an  $\omega$ -word  $\alpha$ , let  $\alpha^i$  denote the suffix of  $\alpha$  starting from the  $i^{th}$  position. Then:

$$\alpha^i \models \mathbf{O}\phi$$
 if  $\exists j \leq i \text{ s.t. } \alpha^j \models \phi$  and  $\alpha \models \mathbf{O}\phi$  if  $\alpha^0 \models \mathbf{O}\phi$ 

Let  $p_1$  and  $p_2$  be atomic propositions. Take the alphabet  $\mathbb{B}^2 = \{\binom{0}{0}, \binom{0}{1}, \binom{1}{0}, \binom{1}{0}, \binom{1}{1}\}$  where the top element indicates the value for  $p_1$  and the bottom one indicates the value of  $p_2$ .

Let 
$$\Psi := \boldsymbol{G} (p_1 \to \boldsymbol{O} p_2).$$

- i) Give two examples of  $\omega$ -words over  $\mathbb{B}^2$ : one which satisfies  $\Psi$  and one which does not satisfy  $\Psi$ .
- ii) Show that  $\Psi$  can be rewritten into an equivalent LTL formula which uses only the standard Until operator U and the boolean connectives  $(\neg, \land, \lor, \rightarrow)$ .
- 4. Draw the ROBDD for the following boolean functions, with the specified order for variables:

(a) 
$$x.\overline{y} + \overline{x}.y$$
 with order  $[x, y]$  (b)  $(x + y).\overline{z}$  with order  $[x, y, z]$ 

5. Represent the following transition system as an ROBDD.

