1. Let ϕ, ψ and χ be LTL formulas. We say two formulas are *equivalent*, written as $\phi \equiv \psi$ if they define the same language. For each of the following, prove or disprove the equivalences:

   (a) $\boldsymbol{G}(\phi \wedge \psi) \equiv (\boldsymbol{G}\phi) \wedge (\boldsymbol{G}\psi)$

   (b) $\boldsymbol{GFG}\phi \equiv \boldsymbol{FGF}\phi$

   (c) $\boldsymbol{X}(\phi\, \boldsymbol{U} \psi) \equiv (\boldsymbol{X}\phi)\, \boldsymbol{U} (\boldsymbol{X}\psi)$

   (d) $(\phi\, \boldsymbol{U} \psi)\, \boldsymbol{U} \chi \equiv \phi\, \boldsymbol{U}(\psi\, \boldsymbol{U} \chi)$

2. Let ϕ and ψ be LTL formulas recognizing ω-languages over an alphabet Σ. The LTL formula $\neg(\phi\, \boldsymbol{U} \psi)$ is language equivalent to one of the following formulas. Which one is it?

   For the formulas which it is not equivalent to, exhibit a word which is in the language of one but not in the other.

   i) $(\neg\phi)\, \boldsymbol{U}(\neg\psi)$

   ii) $(\neg\psi)\, \boldsymbol{U}(\neg\phi)$

   iii) $((\neg\psi)\, \boldsymbol{U}(\neg\phi \wedge \neg\psi)) \vee G(\neg\psi)$

   iv) $((\neg\psi)\, \boldsymbol{U}(\neg\phi \wedge \neg\psi)) \vee G(\neg\phi)$

3. Let $\Sigma = \{0, 1\}$ and let $L = \{\alpha \in \Sigma^\omega \mid \alpha \text{ does not contain } 01\}$.

   (a) Give an ω-regular expression for L.

   (b) Give an NBA for L.

4. Construct an NBA for the LTL formula $\neg(a\, \boldsymbol{U}\, (\boldsymbol{X} b))$.

5. Let $\mathcal{A} = (Q, \{q_0\}, \Sigma, \delta, F)$ be an NFA for a language U. We now give a method for constructing an NBA $\mathcal{B}$ for the language U^ω:

   - $\mathcal{B}$ has the same set of states Q; and the initial state is q_0;
   - All transitions in $\mathcal{A}$ are present in $\mathcal{B}$;
   - In addition, for each transition $q \xrightarrow{a} q_F$ with $q_F \in F$, add a transition $q \xrightarrow{a} q_0$ in $\mathcal{B}$
   - Make q_0 as the only final state in $\mathcal{B}$.

   Will the above procedure result in an NBA for U^ω? If yes, give a proof. If not, give a counterexample.

6. A property (language) $L \subseteq \Sigma^\omega$ is said to be a *safety property* if there exists a set $P \subseteq \Sigma^*$ such that $L = \overline{P.\Sigma^\omega}$ (i.e. L equals complement of $P.\Sigma^\omega$).

   For each of the following properties (which are over a suitable Σ), determine whether it is a safety property or not. Justify your answer. i) $\boldsymbol{G}\,(p_1 \wedge p_2)$ ii) $\boldsymbol{GF}\,(p_1)$ iii) $(00)^*1^\omega$ iv) $p_1\, \boldsymbol{U} p_2$

7. The Release operator $\boldsymbol{R}$ in LTL has the following semantics. A path π satisfies $\phi \boldsymbol{R} \psi$ if:

   - either there is some $i \geq 0$ such that π^i satisfies ϕ and for all $j = 0, \ldots, i$, we have π^j satisfies ψ,
   - or for all $k \geq 1$, we have π^k satisfies ψ.

   where π^i denotes the suffix of π starting from position i.

   (a) Write $\boldsymbol{G}\,\phi$ and $\boldsymbol{F}\,\phi$ using only $\boldsymbol{R}$ and standard boolean operators $(\neg, \vee, \wedge, \bot, \top)$.

   (b) Write $\boldsymbol{R}$ in terms of $\boldsymbol{U}$ and standard boolean operators.