1. Write the duals for the following LPs:

   - $$\begin{array}{lrcrcrl} \text{Maximize} & 2x_1 & - & 12x_2 & + & 20x_3 & \\ \text{Subject to} & 6x_1 & + & 9x_2 & + & 25x_3 & \leq 25 \\ & 2x_1 & - & 6x_2 & + & 3x_3 & = 15 \\ & 4x_1 & + & 7x_2 & - & 20x_3 & \geq 4 \\ & & & & & x_1 & \geq 0 \\ & & & & & x_2 & \leq 0 \\ & & & & & x_3 & \text{unrestricted} \end{array}$$

   - $$\begin{array}{lrcrcrl} \text{Maximize} & 8x_1 & + & 3x_2 & - & 2x_3 & \\ \text{Subject to} & x_1 & - & 6x_2 & + & x_3 & \geq 2 \\ & 5x_1 & + & 7x_2 & - & 2x_3 & = -4 \\ & & & & & x_1 & \leq 0 \\ & & & & & x_2 & \geq 0 \end{array}$$

   - $$\begin{array}{lrcrcrl} \text{Minimize} & -2x_1 & + & 3x_2 & + & 5x_3 & \\ \text{Subject to} & -2x_1 & + & x_2 & + & 3x_3 & \geq 5 \\ & 2x_1 & & & + & x_3 & \leq 4 \\ & & & 2x_2 & + & x_3 & = 4 \\ & & & & & x_1 & \leq 0 \\ & & & & & x_2 & \geq 0 \\ & & & & & x_3 & \text{unrestricted} \end{array}$$

2. Give an example of a primal-dual pair such that both are infeasible.

3. Take primal to be maximize $c^T x$ subject to $Ax = b$. The dual is then to minimize $b^T y$ subject to $A^T y = c$. Show that for every feasible solution $\overline{x}$ of primal and every feasible solution $\overline{y}$ of dual, we have $c^T \overline{x} = b^T \overline{y}$.

4. Show by duality that if the problem - minimize $c^T x$ subject to $Ax = b, x \geq 0$ - has a finite optimal solution, then the new problem - minimize $c^T x$ subject to $Ax = b', x \geq 0$ - cannot be unbounded, no matter what value the vector b' might take.

5. Consider the LP: minimize $c^T x$ subject to $A_i x = b_i$, $i = 1, 2, \ldots, m$, $x \geq 0$. Here we assume that x and c are $n \times 1$ matrices and A_i is $1 \times n$ for every i.

   Suppose x^* is an optimum for the above LP. Let y^* be an optimum for the dual.

   Show that x^* is also an optimum to the LP: minimize $(c^T - y_k^* A_k)x$ s.t. $A_i x = b_i$, $i = 1, \ldots, m$, $i \neq k$, $x \geq 0$ where y_k^* is the k^{th} component of y^*.

6. For the following game, write the LPs for finding max-min and min-max over mixed strategies:

2	-1	4	10
13	7	-5	2
-3	15	9	0

7. Consider a zero-sum game as discussed in class. Suppose x' is the optimum of the maxmin LP and y' is the optimum of the minmax LP. Let maxmin and minmax have value K.

   Prove that Payoff$(x', y') = K$. Use this to show that y' is a best response to x', and x' is a best response to y'.