1. Solve the following LPs using the simplex method

   (a)

$$\begin{array}{llcccccl} \text{Maximize} & x_1 & - & 2x_2 & + & x_3 & & \\ \text{Subject to} & x_1 & + & 2x_2 & + & x_3 & \leq 12 \\ & 2x_1 & + & x_2 & - & x_3 & \leq 6 \\ & -x_1 & + & 3x_2 & & & \leq 9 \\ & x_1, & & x_2, & & x_3, & \geq 0 \end{array}$$

   (b)

$$\begin{array}{llcccl} \text{Maximize} & 3x_1 & + & 5x_2 & \\ \text{Subject to} & x_1 & - & 2x_2 & \leq 6 \\ & x_1 & & & \leq 10 \\ & & & x_2 & \geq 1 \\ & x_1, & & x_2, & \geq 0 \end{array}$$

2. Provide an algorithm based on the simplex method to check if a given system of inequalities is feasible.

3. Can a variable which just left the basis in a simplex tableau reenter in the very next pivot? Explain your answer.

4. Give an example of A, c, b such that the following two LPs are unbounded:

   - maximize $c^T x$ subject to $Ax = b$, $x \geq 0$
   - maximize $-c^T x$ subject to $Ax = b$, $x \geq 0$

   The only difference in the two LPs is in the objective function.

5. Write an LP for the following problem:

   minimize $x_1 + 2|x_2| + 3|x_3 - 10|$ subject to $|x_1| + |x_2 + x_3| \leq 10$

   Justify your answer: show why the optimum value of the LP you write is equal to the optimum value of the above problem.