1. An oil refinery can buy two types of oil: light crude oil and heavy crude oil. The cost per barrel of these types of oil is respectively 11 and 9 dollars. The following quantities of gasoline, kerosene and jet fuel are produced per barrel of each type of oil.

	Gasoline	Kerosene	Jet fuel
Light crude oil	0.4	0.2	0.35
Heavy crude oil	0.32	0.4	0.2

   From the above values, note that 5 percent of light crude oil and 8 percent of heavy crude oil are lost in the refining process. The refinery has contracted to deliver 1,000,000 barrels of gasoline, 400,000 barrels of kerosene and 250,000 barrels of jet fuel.

   Formulate a linear program for finding the number of barrels of each crude oil that satisfies the demand and minimizes the total cost (assuming fractional barrels are allowed).

2. Convert the following LP to equational form:

$$\begin{array}{lrcrcrl} \text{Maximize} & 8x_1 & + & 3x_2 & - & 2x_3 & \\ & & & & & & \\ \text{Subject to} & x_1 & - & 6x_2 & + & x_3 & \geq 2 \\ & 5x_1 & + & 7x_2 & - & 2x_3 & = -4 \\ & & & & & x_1 & \leq 0 \\ & & & & & x_2 & \geq 0 \end{array}$$

3. Write the basic feasible solutions of the following LP:

$$\begin{array}{ccccccccc} x_1 & + & x_2 & + & x_3 & & & = & 6 \\ & & x_2 & & & + & x_4 & = & 3 \\ x_1, & & x_2, & & x_3, & & x_4 & \geq & 0 \end{array}$$

For the following questions, assume we are given an LP in equational form: maximize $c^T x$ subject to $Ax = b$ and $x \geq 0$. Assume that the LP is feasible and A is an $m \times n$ matrix with m linearly independent rows.

4. Suppose x and y are feasible solutions. Let $w = x - y$ and $K = \{i \mid w(i) \neq 0\}$.

   Show that the set of columns of A indexed by K are linearly dependent.

5. Prove the following statement.

   Let x be a basic feasible solution. Let y be a feasible solution satisfying: $y(i) = 0$ iff $x(i) = 0$. Then $x = y$.

6. Show that the statement in the previous question does not hold when x is not a basic feasible solution.