1. Prove or disprove the following statement: if the primal has a unique optimum, then the dual has a unique optimum (an LP is said to have a unique optimum if there is exactly one feasible solution giving the optimum value).

2. Consider the following problem.

$$\begin{array}{lrcrcrcrcrll}
\text{Maximize} & 10x_1 & + & 24x_2 & + & 20x_3 & + & 20x_4 & + & 25x_5 & & \\
\text{Subject to} & x_1 & + & x_2 & + & 2x_3 & + & 3x_4 & + & 5x_5 & \leq 19 & (C1) \\
 & 2x_1 & + & 4x_2 & + & 3x_3 & + & 2x_4 & + & x_5 & \leq 57 & (C2) \\
 & x_1, & & x_2, & & x_3, & & x_4, & & x_5 & \geq 0 &
\end{array}$$

   (a) Write its dual with two variables w_1, w_2 (corresponding to the constraints (C1) and (C2)) and verify that $(w_1, w_2) = (4, 5)$ is a feasible solution.

   (b) Use complementary slackness to show that $(w_1, w_2) = (4, 5)$ gives the optimal solution to the dual.

3. Consider the following problem:

$$\begin{array}{lrcrcr}
\text{Maximize} & 9x_1 & + & 8x_2 & & \\
\text{subject to} & x_1 & - & 2x_2 & \leq & -1 \\
 & 4x_1 & + & 3x_2 & \leq & 4 \\
 & -x_1 & + & 2x_2 & \leq & 3 \\
 & 2x_1 & - & x_2 & \leq & -4
\end{array}$$

   Verify, using complementary slackness, whether $x_1 = -3, x_2 = -1$ is optimal. Verify using complementary slackness whether $x_1 = -\frac{5}{3}, x_2 = \frac{2}{3}$ is optimal.