DUAL OF AN LP - CONTINUED

Primal	Dual
$\max \; c^T x$	$\min \; b^T y$
Subj. to $Ax \leq b$	Subj. to $A^T y \geq c$
$x \geq 0$	$y \geq 0$

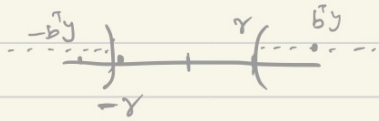
Part 1: Dual of a dual

Part 2: Writing the dual for LPs in different forms

REFERENCE: Section 6.2 of text:

Understanding and Using Linear Programming
– Matoušek & Gärtner

Part 1: dual of the dual



$\min \; b^Ty$	$\min \; b^Ty$	$\max \; (-b^T)y$
subj. to $A^Ty \geq c$, $y \geq 0$	$(-A^T)y \leq -c$, $y \geq 0$	$(-A^T)y \leq -c$, $y \geq 0$

Feasible reg: R $\longleftrightarrow$ R $\longleftrightarrow$ R

Optimum: unbounded $\longleftrightarrow$ unbounded $\longleftrightarrow$ unbounded

r $\longleftrightarrow$ r $\longleftrightarrow$ $-r$

$\max \; c^Tx$	$\min \; (-c^T)x$	$\min \; (-c^T)x$
$Ax \leq b$, $x \geq 0$	$Ax \leq b$, $x \geq 0$	$(-A)x \geq -b$, $x \geq 0$

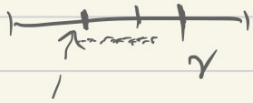
R' $\longleftrightarrow$ R' $\longleftrightarrow$ R'

unbounded $\longleftrightarrow$ unbounded $\longleftrightarrow$ unbounded

r $\longleftrightarrow$ $-r$ $\longleftrightarrow$ $-r$

$$\text{minimize} \quad 12y_1 + 3y_2 + 4y_3$$

$$\text{subject to} \quad \begin{aligned} x_1 \times \quad & 4y_1 + 2y_2 + 3y_3 \geq 2 \\ x_2 \times \quad & 8y_1 + y_2 + 2y_3 \geq 3 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$



$$12y_1 + 3y_2 + 4y_3 \geqslant (4x_1 + 8x_2)y_1 + (2x_1 + x_2)y_2 + (3x_1 + 2x_2)y_3 \geqslant 2x_1 + 3x_2$$

$$\begin{aligned} 4x_1 + 8x_2 &\leq 12 \\ 2x_1 + x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

$$\text{maximize} \quad 2x_1 + 3x_2$$

$\max \quad c^T x$	$\min \quad b^T y$
Subj. to $\quad Ax \leq b$	subj. to $\quad A^T y \geq c$
$x \geq 0$	$y \geq 0$

PRIMAL - DUAL Pairs

Part 2: Writing the dual for different LP forms

maximize	$2x_1 + 3x_2$	minimize $12y_1 + 3y_2 + 4y_3$
subj. to	$y_1 \times \quad 4x_1 + 8x_2 \leq 12$	$4y_1 + 2y_2 + 3y_3 \geq 2$
	$y_2 \times \quad 2x_1 + x_2 \leq 3$	$8y_1 + y_2 + 2y_3 \geq 3$
	$y_3 \times \quad 3x_1 + 2x_2 \leq 4$	
	$x_1, x_2 \geq 0$	$y_1, y_2, y_3 \geq 0$

maximize $2x_1 + 3x_2$

Subj. to
$$4x_1 + 8x_2 \leq 12$$
$$2x_1 + x_2 \leq 3$$
$$3x_1 + 2x_2 \leq 4$$

x_1 unconstrained

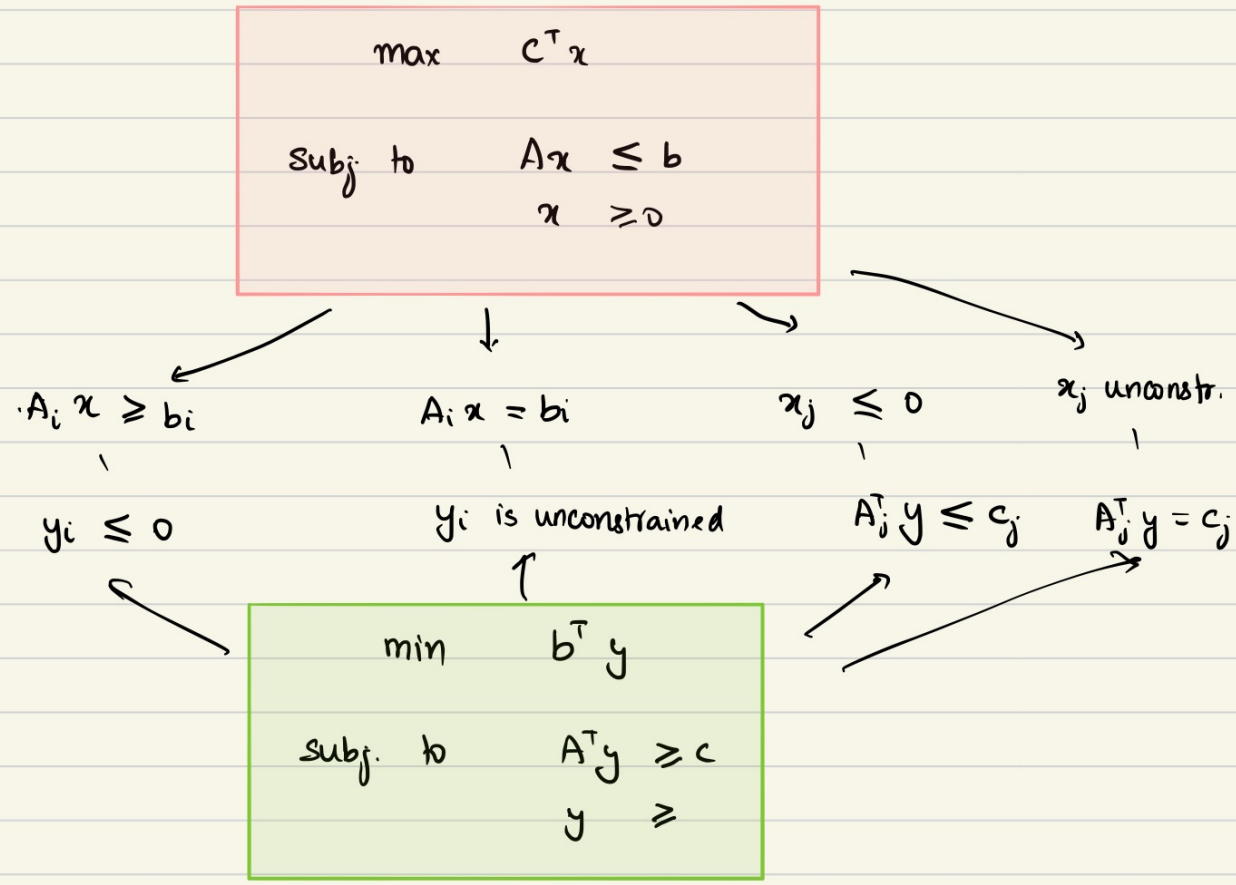
$x_2 \geq 0$

$x_1 = x_1^+ - x_1^-$

$x_1^+, x_1^- \geq 0$

↓

maximize	$2x_1^+ - 2x_1^- + 3x_2$	minimize $12y_1 + 3y_2 + 4y_3$
Subj. to:	$4x_1^+ - 4x_1^- + 8x_2 \leq 12$	$4y_1 + 2y_2 + 3y_3 = 2$
	$2x_1^+ - 2x_1^- + x_2 \leq 3$	$8y_1 + y_2 + 2y_3 \geq 3$
	$3x_1^+ - 3x_1^- + 2x_2 \leq 4$	
	$x_1^+, x_1^- \geq 0$	$y_1, y_2, y_3 \geq 0$
	$x_2 \geq 0$	



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max $8x_1 + 3x_2 - 2x_3$

subj. to:
$x_1 - 6x_2 + x_3 \geq 2$
$5x_1 + 7x_2 - 2x_3 = -4$

$x_1 \leq 0$, $x_2 \geq 0$, x_3 unrestricted

minimize $2y_1 - 4y_2$

$y_1 + 5y_2 \leq 8$
$-6y_1 + 7y_2 \geq 3$
$y_3 - 2y_2 = -2$

$y_1 \leq 0$
y_2 unconstrained