

DUAL OF AN LP - CONTINUED

$$\begin{array}{ll} \max & c^T x \\ \text{Subj. to} & Ax \leq b \\ & x \geq 0 \end{array}$$

Primal

$$\begin{array}{ll} \min & b^T y \\ \text{Subj. to} & A^T y \geq c \\ & y \geq 0 \end{array}$$

Dual

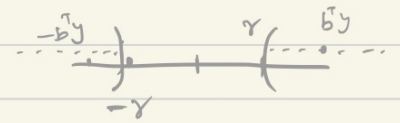
Part 1: Dual of a dual

Part 2: Writing the dual for LPs in different forms

REFERENCE: Section 6.2 of text:

Understanding and Using Linear Programming
- Matoušek & Gärtner

Part 1: dual of the dual



$\begin{aligned} \min \quad & b^T y \\ \text{subj. to} \quad & A^T y \geq c \\ & y \geq 0 \end{aligned}$	$\begin{aligned} \min \quad & b^T y \\ & (-A^T) y \leq -c \\ & y \geq 0 \end{aligned}$	$\begin{aligned} \max \quad & (-b^T) y \\ & (-A^T) y \leq -c \\ & y \geq 0 \end{aligned}$
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Feasible reg: R R R
 optimum: unbounded \longleftrightarrow unbounded \longleftrightarrow unbounded
 γ γ $-\gamma$

$\begin{aligned} \max \quad & c^T x \\ A x \leq b \\ x \geq 0 \end{aligned}$	$\begin{aligned} \min \quad & (-c^T) x \\ A x \leq b \\ x \geq 0 \end{aligned}$	$\begin{aligned} \min \quad & (-c^T) x \\ (-A) x \geq -b \\ x \geq 0 \end{aligned}$
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R' R' R'
 unbounded \longleftrightarrow unbounded \longleftrightarrow unbounded
 γ $-\gamma$ $-\gamma$

$$\text{minimize } 12y_1 + 3y_2 + 4y_3$$

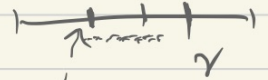
$$\text{subject to } x_1 \times 4y_1 + 2y_2 + 3y_3 \geq 2$$

$$x_2 \times 8y_1 + y_2 + 2y_3 \geq 3$$

$$y_1, y_2, y_3 \geq 0$$

$$12y_1 + 3y_2 + 4y_3 \geq$$

$$(4x_1 + 8x_2)y_1 + (2x_1 + x_2)y_2 + (3x_1 + 2x_2)y_3 \geq 2x_1 + 3x_2$$



$$\begin{aligned} 4x_1 + 8x_2 &\leq 12 \\ 2x_1 + x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

$$\text{maximize } 2x_1 + 3x_2$$

$$\max \quad c^T x$$

$$\text{Subj. to } \begin{aligned} Ax &\leq b \\ x &\geq 0 \end{aligned}$$

$$\min \quad b^T y$$

$$\text{sub. to } \begin{aligned} A^T y &\geq c \\ y &\geq 0 \end{aligned}$$

PRIMAL - DUAL Pairs

Part 2: Writing the dual for different LP forms

$$\begin{array}{ll} \text{maximize} & 2x_1 + 3x_2 \\ \text{subj. to} & y_1 \times 4x_1 + 8x_2 \leq 12 \\ & y_2 \times 2x_1 + x_2 \leq 3 \\ & y_3 \times 3x_1 + 2x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{array}$$

$$\text{minimize } 12y_1 + 3y_2 + 4y_3$$

$$4y_1 + 2y_2 + 3y_3 \geq 2$$

$$8y_1 + y_2 + 2y_3 \geq 3$$

$$\underline{y_1}, y_2, y_3 \geq 0$$

$$\text{maximize } 2x_1 + 3x_2$$

$$\begin{array}{ll} \text{Subj. to} & 4x_1 + 8x_2 \leq 12 \\ & 2x_1 + x_2 \leq 3 \\ & 3x_1 + 2x_2 \leq 4 \\ & x_1 \text{ unconstrained} \\ & x_2 \geq 0 \end{array}$$

$$x_1 = x_1^+ - x_1^-$$

$$x_1^+, x_1^- \geq 0$$

↓

$$\text{maximize } 2x_1^+ - 2x_1^- + 3x_2$$

$$\text{minimize } 12y_1 + 3y_2 + 4y_3$$

$$\begin{array}{l} \text{Subj. to:} \\ 4x_1^+ - 4x_1^- + 8x_2 \leq 12 \\ 2x_1^+ - 2x_1^- + x_2 \leq 3 \\ 3x_1^+ - 3x_1^- + 2x_2 \leq 4 \end{array}$$

$$4y_1 + 2y_2 + 3y_3 = 2$$

$$8y_1 + y_2 + 2y_3 \geq 3$$

$$x_1^+, x_1^- \geq 0$$

$$x_2 \geq 0$$

$$y_1, y_2, y_3 \geq 0$$

$$\begin{aligned} \max \quad & c^T x \\ \text{Subj. to} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

$$A_i x \geq b_i$$

$$y_i \leq 0$$

$$A_i x = b_i$$

y_i is unconstrained

$$x_j \leq 0$$

$$A_j^T y \leq c_j$$

x_j unconstr.

$$A_j^T y = c_j$$

$$\begin{aligned} \min \quad & b^T y \\ \text{Subj. to} \quad & A^T y \geq c \\ & y \geq 0 \end{aligned}$$

$$\max \quad 8x_1 + 3x_2 - 2x_3$$

$$\begin{aligned} \text{Subj. to:} \quad & x_1 - 6x_2 + x_3 \geq 2 \\ & 5x_1 + 7x_2 - 2x_3 = -4 \end{aligned}$$

$$x_1 \leq 0, x_2 \geq 0, x_3 \text{ unrestricted}$$

$$\text{minimize} \quad 2y_1 - 4y_2$$

$$\begin{aligned} & y_1 + 5y_2 \leq 8 \\ & -6y_1 + 7y_2 \geq 3 \\ & y_3 - 2y_2 = -2 \end{aligned}$$

$$y_1 \leq 0$$

y_2 unconstrained