1. Write the basic feasible solutions of the following LP:

$$\begin{array}{cccccccccc} x_1 & + & x_2 & + & x_3 & & & = & 6 \\ & & x_2 & & & + & x_4 & = & 3 \\ x_1, & & x_2, & & x_3, & & x_4 & \geq & 0 \end{array}$$

   For the following questions, assume we are given an LP in equational form: maximize $c^T x$ subject to $Ax = b$ and $x \geq 0$. Assume that the LP is feasible and A is an $m \times n$ matrix with m linearly independent rows.

2. Suppose x and y are feasible solutions. Let $w = x - y$ and $K = \{i \mid w(i) \neq 0\}$.

   Show that the set of columns of A indexed by K are linearly dependent.

3. Prove the following statement.

   Let x be a basic feasible solution. Let y be a feasible solution satisfying: $y(i) = 0$ iff $x(i) = 0$. Then $x = y$.

4. Show that the statement in the previous question does hold when x is not a basic feasible solution.

5. Prove that if an LP is unbounded, there exists a basic feasible point x and a vector w such that $x + tw$ is feasible for all $t \geq 0$ and cost increases as t increases.