Problem Set 1
Linear Optimization 2020

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Problem 1. If the feasible set is unbounded, is the solution necessarily unbounded?

Problem 2. If the feasible set is bounded, is the solution bounded?

Problem 3. Show that the set of solutions to $Ax = b$ is an affine subspace.

Problem 4. You have boolean variables $x_1, x_2 \dots x_n$. A literal is x_i or $\bar{x_i}$. A clause is a disjunction (OR) of literals. A CNF formula is a conjuction (AND) of clauses. Given such a formula, the boolean satisfiability problem asks if there is an assignment to the boolean variables which makes the formula True. Example:

$$\Phi = (x_1 \vee x_3 \vee \bar{x_4}) \wedge (x_2 \vee x_1)$$

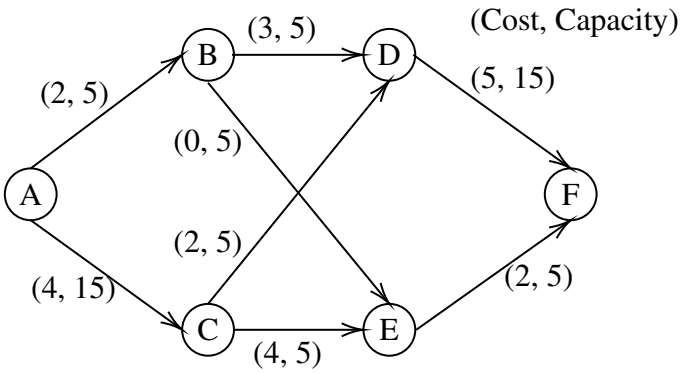
3-SAT is a specific case of the boolean satisfiability problem where each clause has exactly 3 literals. Write the ILP formulation of the 3-SAT problem.

Problem 5. The Moscow Industrial Ghetto (MIG) manufactures large and small iron dragons. The large dragons are used for garden decorations and the small dragons are used for automobile ornaments. Each week upto 1000pounds (lb) of iron can be purchased. MIG employs five workers, each of whom works 30 hours (hr) per week. Their salary is fixed cost that does not depend on productivity. The machinery used is available for a maximum of 300 hours per week. The following table gives data about dragons

	Small dragon	Large Dragon
Iron per dragon (lb)	3	125
Labour per dragon(hr)	2	12
Machine time per dragon (hr)	1.5	9
Selling price per dragon ($)	35	300

Set up the LP problem to determine how many dragons of each size MIG should make to maximize total selling price.

Problem 6. Following is a network, with source A and sink F. Each edge has a cost(first co-ordinate) and a capacity(second co-ordinate) associated to it. If an edge e has the cost co-efficient $cost_e$, and the flow through the edge is $flow_e$, then the total cost of maintaining the flow is $cost_e \times flow_e$.



Write the LP to find a flow of size 11 from A to F, with the smallest cost.