

Note. Each question carries 3 marks.

1. Write the duals for the following LPs:

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$$\begin{array}{llllll}
 \text{Maximize} & 2x_1 & - & 12x_2 & + & 20x_3 \\
 \\
 \text{Subject to} & 6x_1 & + & 9x_2 & + & 25x_3 & \leq 25 \\
 & 2x_1 & - & 6x_2 & + & 3x_3 & = 15 \\
 & 4x_1 & + & 7x_2 & - & 20x_3 & \geq 4 \\
 & & & & & x_1 & \geq 0 \\
 & & & & & x_2 & \leq 0 \\
 & & & & & x_3 & \text{unrestricted}
 \end{array}$$

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$$\begin{array}{llllll}
 \text{Maximize} & 8x_1 & + & 3x_2 & - & 2x_3 \\
 \\
 \text{Subject to} & x_1 & - & 6x_2 & + & x_3 & \geq 2 \\
 & 5x_1 & + & 7x_2 & - & 2x_3 & = -4 \\
 & & & & & x_1 & \leq 0 \\
 & & & & & x_2 & \geq 0
 \end{array}$$

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$$\begin{array}{llllll}
 \text{Minimize} & -2x_1 & + & 3x_2 & + & 5x_3 \\
 \\
 \text{Subject to} & -2x_1 & + & x_2 & + & 3x_3 & \geq 5 \\
 & 2x_1 & & & + & x_3 & \leq 4 \\
 & & & 2x_2 & + & x_3 & = 4 \\
 & & & & & x_1 & \leq 0 \\
 & & & & & x_2 & \geq 0 \\
 & & & & & x_3 & \text{unrestricted}
 \end{array}$$

2. Give an example of a primal-dual pair such that both are infeasible.

3. Take primal to be maximize $c^T x$ subject to $Ax = b$. The dual is then to minimize $b^T y$ subject to $A^T y = c$. Show that for every feasible solution \bar{x} of primal and every feasible solution \bar{y} of dual, we have $c^T \bar{x} = b^T \bar{y}$.