

Notations: Fix a game graph $G = (V, E)$. Let n_q be the number of vertices in V with priority q . Let M_V be the set of tuples of the form $(x_{d-1}, x_{d-3}, \dots, x_1)$ with $0 \leq x_q \leq n_q$. The algorithm *progress-measure-lifting*(G) associates a value from $M_V \cup \{\top\}$ to each vertex. Let $N_b \subseteq M_V$ be the set of tuples that such $\sum_{\text{odd } q} x_q \leq b$. The algorithm *progress-measure-lifting*[†](G, b) is the progress measure lifting algorithm restricted to $N_b \cup \{\top\}$.

1. What can you say about the 0-dominions obtained using *progress-measure-lifting*[†](G, b) when $b = 1$?
2. Let \mathcal{T}_n be the (directed) binary tree of depth n . Assume that the leaves of \mathcal{T}_n have self loops. All non-leaf nodes have priority 1. Among the leaf nodes, some (arbitrary number) of them have priority 1 and the others have priority 2.
 - i) Assuming that all nodes are Player 0's nodes, what is the progress measure computed by *progress-measure-lifting*(\mathcal{T}_n).
 - ii) Assuming that all nodes are Player 1's nodes, what is the progress measure computed by *progress-measure-lifting*(\mathcal{T}_n).

How does your answer depend on the arrangement of priorities 1 and 2 among leaf nodes?

3. For a graph G with $2n$ vertices, let D_0 be the 0-dominion obtained by running *progress-measure-lifting*[†](G, n). Give an example of a G for which D_0 and $\text{reach}_0(D_0)$ are different sets.