1. Consider a parity game in which every vertex is controlled by Player 1. Does he win from every vertex?

2. A (positional) strategy graph for Player 1 is a game graph in which every Player 1 vertex has exactly one outgoing edge. Design a polynomial time algorithm to decide if Player 1 wins the parity game starting from a given vertex v in this strategy graph.

3. Consider a parity game G whose arena is finite. Pick a vertex v_0 in G (need not necessarily be Player 1 vertex). We will now define a new game G' starting from v_0 where all plays are finite. The game stops as soon as a vertex is visited twice. A play is thus a finite path $v_0, \ldots, v_n$ such that $v_0, \ldots, v_{n-1}$ are pairwise distinct and $v_n = v_j$ for some $j < n$. Player P_1 wins if the maximum colour in the loop: $\max\{\chi(v_j), \chi(v_{j+1}), \ldots, \chi(v_n)\}$ is odd.

   Show that the parity game G starting in v_0 and the game G' are equivalent (that is, P_1 wins in G from v_0 iff she wins G'):

   i) Show that if P_1 has a winning strategy in G starting from v_0, then she has a winning strategy in G'.

   ii) Show that if P_1 has a winning strategy in G', then she has a winning strategy in G starting from v_0.