1. Show that for any function $f : \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}$:

$$\sup_{x \in \mathbb{R}} \inf_{y \in \mathbb{R}} f(x, y) \leq \inf_{y \in \mathbb{R}} \sup_{x \in \mathbb{R}} f(x, y)$$

2. A game is said to be determined if:

$$\sup_{\chi} \inf_{\mu} \pi(\text{Outcome}(v, \mu, \chi)) = \inf_{\mu} \sup_{\chi} \pi(\text{Outcome}(v, \mu, \chi)) \qquad \text{for all vertices } v \tag{1}$$

   where μ, χ range over strategies of Minimizer and Maximizer respectively, $\text{Outcome}(v, \mu, \chi)$ is the play starting at v induced by μ and χ, and π is a payoff function. A game is said to be *positionally determined* if the sup on the left hand side and the inf on the right hand side range over positional strategies of Maximizer and Minimizer respectively. Show that if a game is positionally determined, it is determined.

3. Exercises 3.1, 3.2 and 3.3 from the text *"Lectures in Game theory for Computer Scientists"* by Apt and Grädel.

4. Show that (1) holds for reachability games.