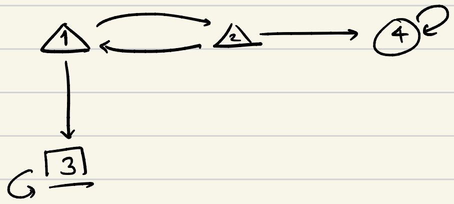
Tutorial on MDPs

1.



Solve for:

$$v_2 = \tfrac{1}{2} v_1 + 1$$
$$v_1 = \tfrac{1}{2} v_2$$

$$v_1 = \tfrac{1}{4} v_1 + \tfrac{1}{2}$$

$$\tfrac{3}{4} v_1 = \tfrac{1}{2} \qquad v_1 = \tfrac{2}{3}$$

$$v_2 = \tfrac{1}{3}$$

2.

Equations for primed variables would be:

$$v'_i = \tfrac{1}{2} v_j + \tfrac{1}{2} v'_k$$

Notice that substituting $v'_i = \bar{\lambda}(i)$ for all i satisfies the equations.

$\therefore$ In G': prob to reach n equals $\bar{\lambda}(i)$ for both i and i'.

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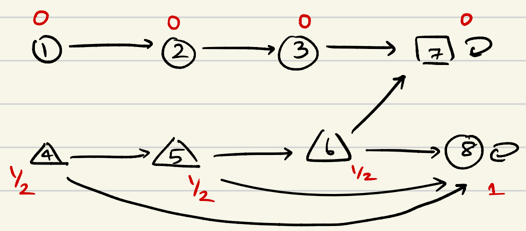
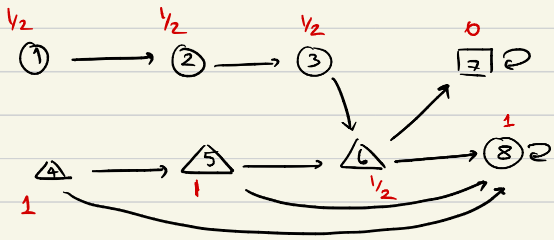
3. Initial Markov chain: G_{10}



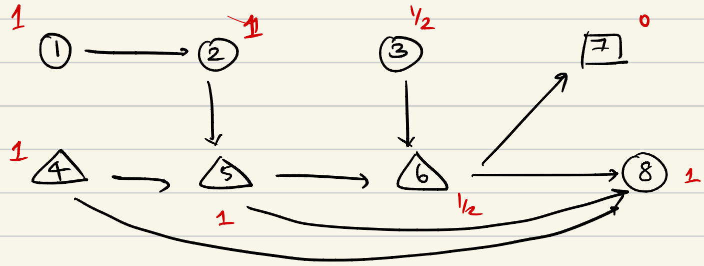
Switch 3 to 6.

G_1:



Switch 2 to 5

G_2:



Final strategy.

4.

$$\begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8 \end{array} \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\1 \end{bmatrix} - \begin{bmatrix} 0\\0\\0\\1/2\\1/2\\1/2\\0\\1 \end{bmatrix} - \begin{bmatrix} 1/2\\1/2\\1/2\\3/4\\3/4\\1/2\\0\\1 \end{bmatrix} \cdots$$

$\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}$

At i^{th} iteration, $\frac{1}{2} + \frac{1}{2^2} + \cdots \frac{1}{2^i}$

Will converge in the limit to 1.

LP: minimize $x_1 + x_2 + \cdots + x_6$

s.t. $x_7 = 0, \quad x_8 = 1$

$$x_6 = \frac{1}{2} x_7 + \frac{1}{2} x_8$$

$$x_5 = \frac{1}{2} x_6 + \frac{1}{2} x_8$$

$$x_4 = \frac{1}{2} x_5 + \frac{1}{2} x_8$$

$$x_3 \geq x_6, x_7$$

$$x_2 \geq x_3, x_5$$

$$x_1 \geq x_2, x_4$$

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5. Similar to the proof the normal "switch-one" algo done in class.